Integrated Mathematics 1

by

The Mathematics Vision Project:

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Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

Prepared for the Students of

PUSD

Porterville Unified School District

Creating Opportunites: Changing Lives

Modules:

1. Getting Ready
2. Systems of Equations and Inequalities
3. Arithmetic and Geometric Sequences
4. Linear and Exponential Functions
5. Features of Functions
6. Congruence, Constructions and Proof
7. Connecting Algebra and Geometry
8. Modeling Data

Secondary One Mathematics: An Integrated Approach
Module 1
Getting Ready

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In partnership with the Utah State Office of Education

Module 1 – Getting Ready

Classroom Task: Checkerboard Borders - A Develop Understanding Task
Defining quantities and interpreting expressions (**N.Q.2, A.SSE.1**)
Ready, Set, Go Homework: Getting Ready 1.1

Classroom Task: Building More Checkerboard Borders – A Develop Understanding Task
Defining quantities and interpreting expressions (**N.Q.2, A.SSE.1**)
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Solving linear inequalities and representing the solution (**A.REI.1, A.REI.3**)
Ready, Set, Go Homework: Getting Ready 1.10

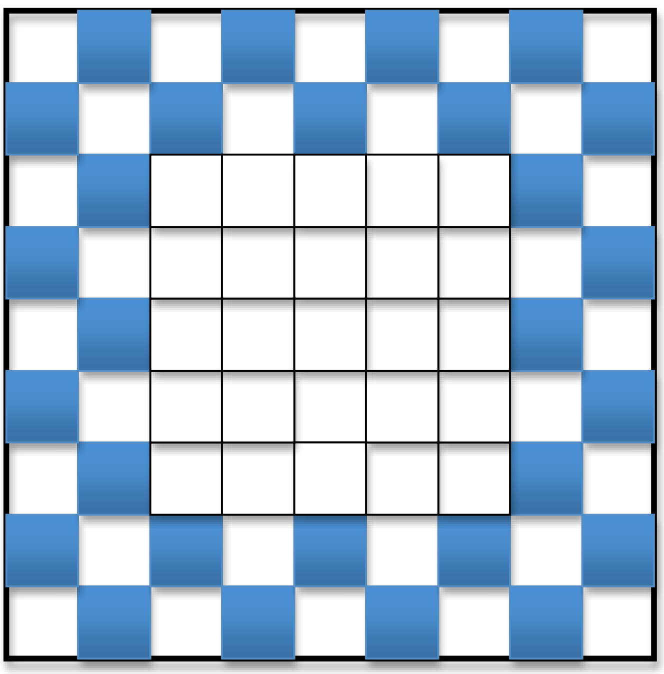
1.1 Checkerboard Borders

A Develop Understanding Task

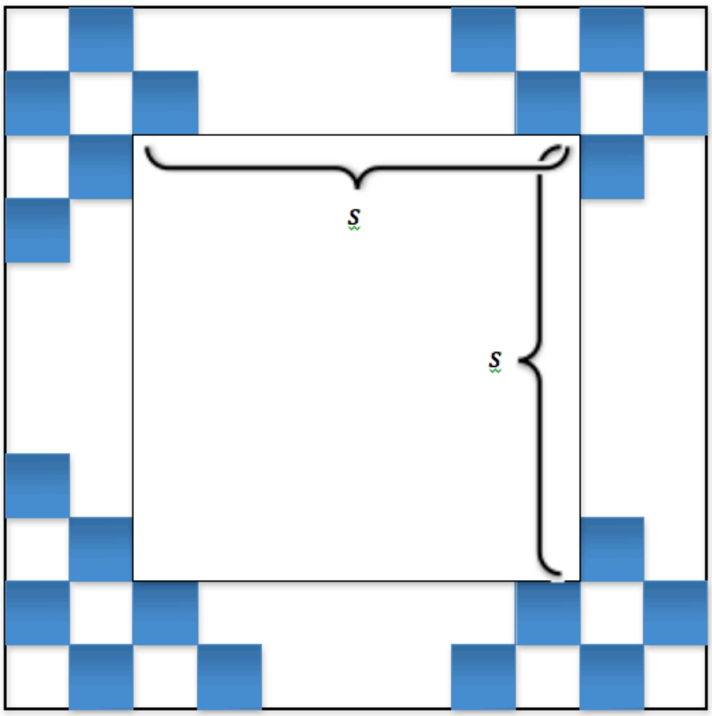
In preparation for back to school, the school administration has planned to replace the tile in the cafeteria. They would like to have a checkerboard pattern of tiles two rows wide as a surround for the tables and serving carts.

Below is an example of the boarder that the administration is thinking of using to surround a square 5 x 5 set of tiles.

A. Find the number of colored tiles in the checkerboard border. Track your thinking and find a way of calculating the number of colored tiles in the border that is quick and efficient. Be prepared to share your strategy and justify your work.



B. The contractor that was hired to lay the tile in the cafeteria is trying to generalize a way to calculate the number of colored tiles needed for a checkerboard border surrounding a square of tiles with dimensions *s* x *s*. Find an expression for the number of colored border tiles needed for any *s* x *s* square center.



Name: Getting Ready | 1.1

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Ready, Set, Go!

Ready

Topic: Solve one variable equations

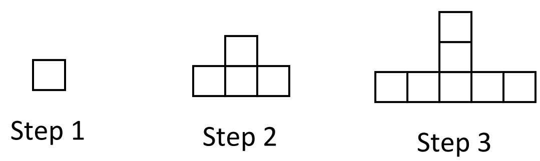
Find the value of *x* that makes each equation true.

1. $6x = 18$
2. $3x - 10 = 2$
3. $8x - 10 = x + 11$
4. $5x - 7 = 7x - 17$
5. $3x + 9 = 44 - 2x$
6. $3x + 6 = x + 2$

Set

Topic: Create and solve equations in one variable.

Use the pictures below to answer questions 7 – 12.



7. Each square represents one tile, how many total tiles are in Step 5? Step 6?

8. What might you do to determine the number of tiles in Step 25?

9. Write a rule to predict the total number of tiles for any step. Show how your rule relates to the pattern.

10. Try to think of a different rule that you can use to predict the total number of tiles for any step. Show how your rule relates to the pattern.

11. Andrew also solved this problem and came up with following equation: s = 1 + 3(n-1). How does each piece of his expression show up in the pattern?

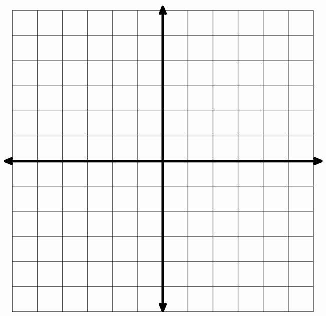
12. Tami came up with the equation s = 3n – 2. How does each piece of her expression show up in the pattern?

Go

Topic: Graph linear equations

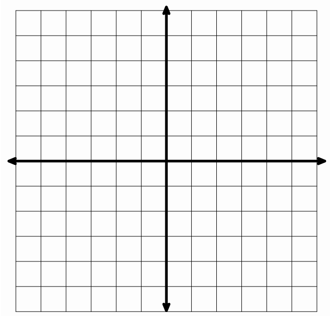
For the following problems two points and a slope are given. Use the graph to plot these points, draw the line, and *clearly* label the slope on the graph.

13. (2 , -1) and (4 , 2)



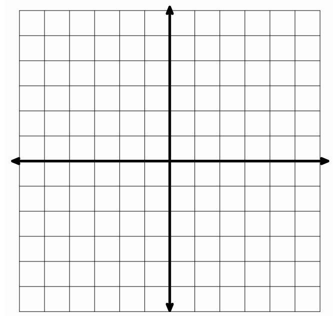
Slope: m = $\frac{3}{2}$

14. (-2 , 1) and (2 , 5)



Slope: m = 1

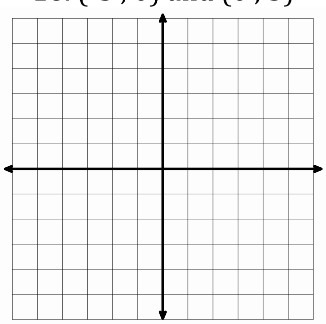
15. (0 , 0) and (3 , 6)



Slope: m = 2

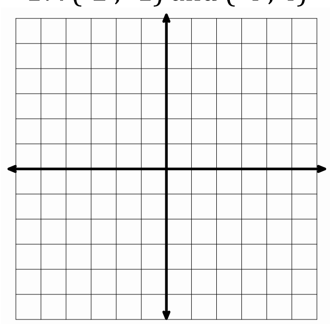
For the following problems, two points are given. Use the graph to plot these points *and* find the slope.

16. (-3 , 0) and (0 , 5)



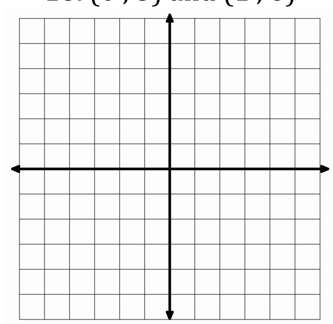
Slope: m =

17. (-2 , -1) and (-4 , 4)



Slope: m =

18. (0 , 3) and (1 , 6)



Slope: m =

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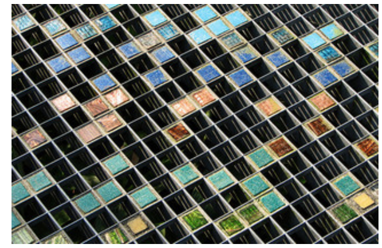
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-a-line-in-slope-intercept-form

http://www.youtube.com/watch?v=WXzpisUh0AU

1.2 Building More Checkerboard Borders

A Develop Understanding Task



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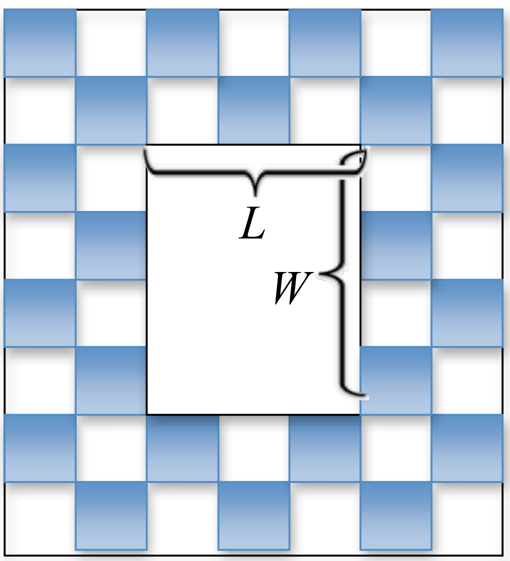
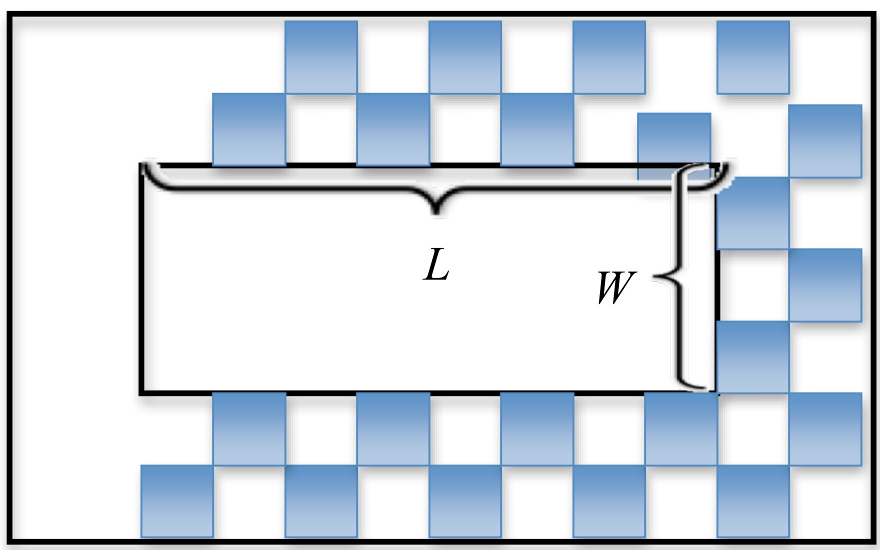
As the tile workers started to look more deeply into their work they found it necessary to develop a way to quickly calculate the number of colored border tiles for not just square arrangements but also for checkerboard borders to surround any $L \times W$ rectangular tile center.

Find an expression to calculate the number of colored tiles in the two row checkerboard border for any rectangle. Be prepared to share your strategy and justify your work. Create models to assist you in your work.





Name: Getting Ready | 1.2

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Ready, Set, Go!

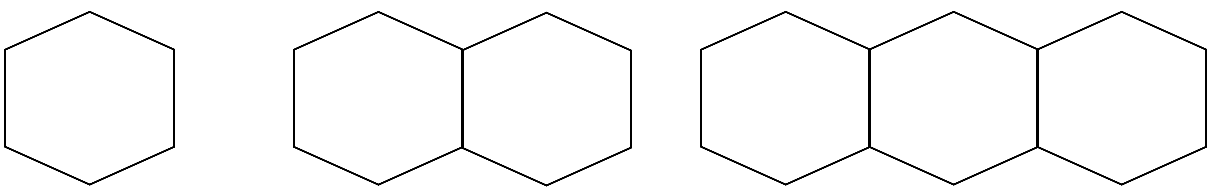
Ready

Solve the following equations for the unknown variable.

1. $4(x+3)=1$
2. $q-13=-13$
3. $21s=3$
4. $\frac{7f}{11}=\frac{7}{11}$
5. $5q-7=\frac{2}{3}$
6. $5x-(3x+2)=1$

Set

For the growing pattern below, each line segment is one unit in length.



Step 1 Step 2 Step 3

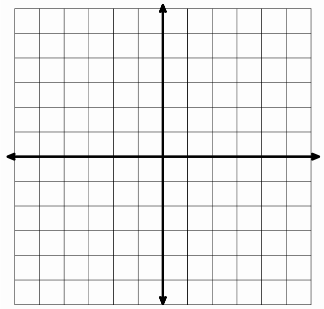
7. How much total **perimeter** in Step 5? Step 6? (Remember to focus on the perimeter.)
8. How can you determine the amount of perimeter in Step 25?
9. Write a rule to predict the total amount of perimeter for any step. Show how your rule relates to the pattern.
10. Marsha also solved this problem and came up with following expression: 1 + 5n -(n-1). How does each piece of her expression show up in the pattern?
11. Tyler came up with the expression 6n – 2(n-1). How does each piece of his expression show up in the pattern?

Name: Getting Ready | 1.2

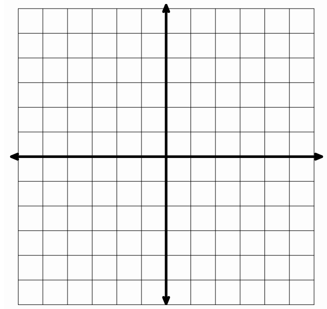
Go

For problems 12 and 13, the y-intercept and the slope of a line are given. Graph the line on the coordinate axes, clearly labeling the slope and y-intercept.

12. (0, 2); $m = \frac{3}{4}$



13. (0, -3); m = 4



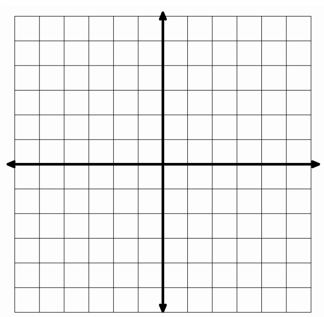
The equations below are represented in the above graphs. Explain how the slope and y-intercept show up in both the graph and algebraic representations.

$y = \frac{3}{4}x + 2$

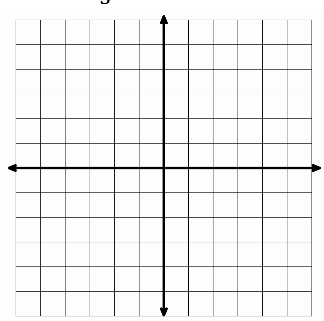
$y = 4x - 3$

For problems 14-16, graph the following equations on the provided coordinate axes.

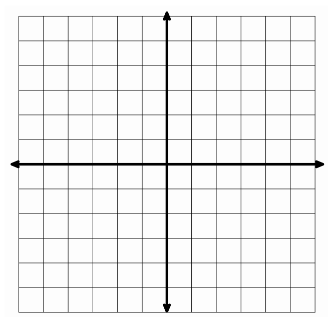
14. y = 2x-1



15. $y = \frac{1}{3}x + 2$



16. y = -3x + 5



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http://www.youtube.com/watch?v=WXzpisUh0AU

1.3 Serving Up Symbols
A Develop Understanding Task


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As you look around your school cafeteria, you may see many things that could be counted or measured. To increase the efficiency of the cafeteria, the cafeteria manager, Elvira, decided to take a close look at the management of the cafeteria and think about all the components that affect the way the cafeteria runs. To make it easy, she assigned symbols for each count or measurement that she wanted to consider, and made the following table:

Symbol	**Meaning** (description of what the symbol means in context)	**Units** (what is counted or measured)
S	Number of students that buy lunch in the cafeteria each day	*students* or $students/day$
S_M	Number of students who have passed through a line in M minutes	
C	Number of classes per lunch period	
P	Number of lunch periods per day	
B	Number of boys that buy lunch each day	*boys* or *students* or $boys/day$
G	Number of girls that buy lunch each day	
F	Number of food servers in the cafeteria	
T	Total number of food items in one lunch (Each entrée, side dish, or beverage counts as 1 item.)	
M	Number of minutes passed since the beginning of the lunch period	
N_e	Number of entrees in each lunch	
N_s	Number of side dishes in each lunch	
N_b	Number of beverages in each lunch	
C_e	Cost of each entrée	
C_s	Cost of each side dish	
C_b	Cost of each beverage	
L	Number of lines in the cafeteria	
W	The number of food servers per line	
i	Average number of food items that a server can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)	
H	Number of hours each food server works each day	
P_L	Price per lunch	

Using the given symbols, it is possible to write many different algebraic expressions.

1. Using these symbols, what would the expression $\frac{G+B}{C \times P}$ mean?

2. Using these symbols, what would the expression $S + F + L$ mean?

Elvira hopes to use the symbols in the chart to come up with some meaningful expressions that will allow her to analyze her cafeteria. Your job is to help her by writing as many expressions as you can and describe what they mean. Put each of your expressions in the following chart, adding lines if you need to:

Expression	Description

Write an expression for the average number of lunches served in a line each day.

Write an expression for the total price of the items served in a line.

Name: Getting Ready 1.3

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Ready, Set, Go!

Ready

Topic: Evaluating Expressions

Evaluate the following expressions for $a = -3, b = 2, c = 5, and\ d = -4$.

1. $2a + 3b$

2. $4c + d$

3. $5ac - 2b$

4. $\frac{2a}{c-d}$

5. $\frac{3b}{d}$

6. $\frac{a-4b}{3c+2d}$

The equation $c = 2000 + 3r$ represents the cost in dollars (c) of producing remote controls (r).

7. What is the cost of producing 1000 remote controls?

8. What is the cost of producing 2000 remote controls?

9. What is the cost of producing 2500 remote controls?

Name: Getting Ready | 1.3

Set

Solve each equation, justifying each step you use.

10.

$3x = 15$	Justification

11.

$x - 10 = 2$	Justification

12.

$-16 = x + 11$	Justification

13.

$6 - x = 10$	Justification

14.

$6x + 3 = 15$	Justification

15.

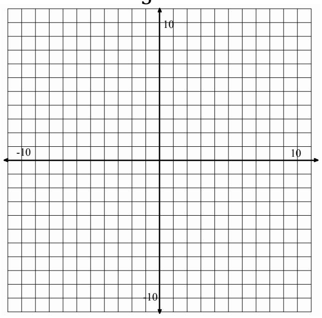
$3x - 10 = 2$	Justification

Name: Getting Ready | **1.3**

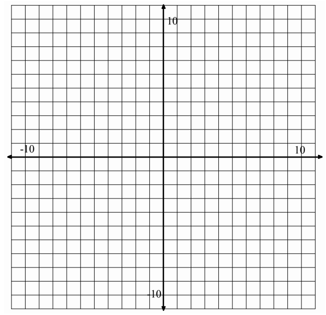
Go

Graph the following equations on the provided coordinate grids.

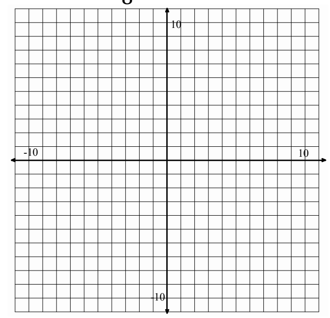
16. $y = -\frac{3}{5}x + 7$



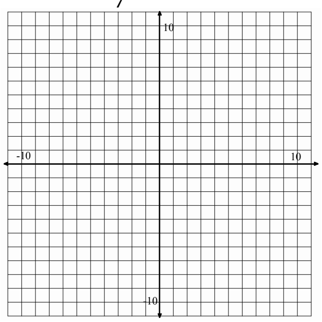
17. $y = -2x + 1$



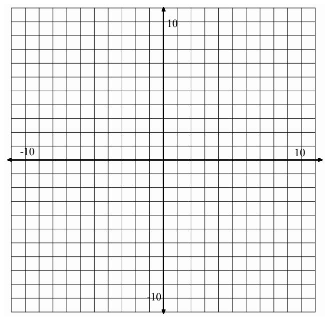
18. $y = \frac{5}{8}x + 1$



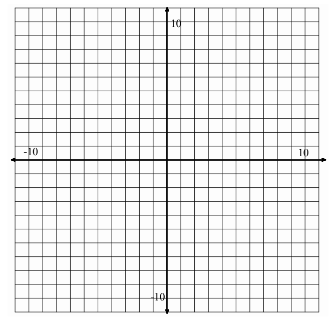
19. $y = \frac{6}{7}x$



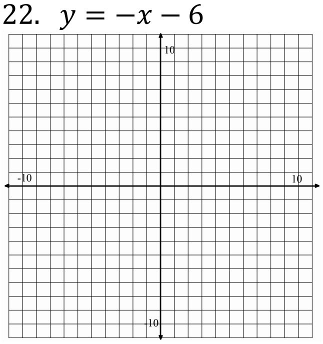
20. $y = x - 3$



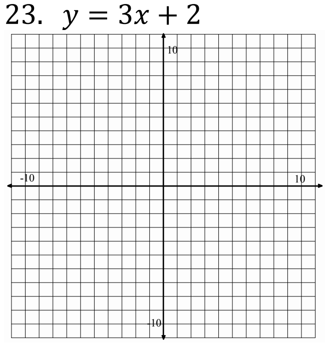
21. $y = 4x$



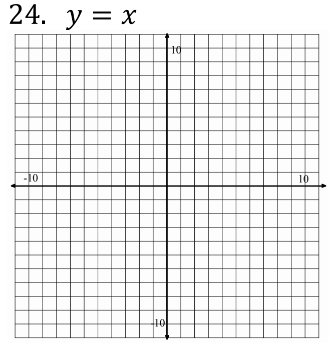
22. $y = -x - 6$



23. $y = 3x + 2$



24. $y = x$



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http://www.youtube.com/watch?v=WXzpisUh0AU

1.4 Examining Units
A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

Units in Addition and Subtraction

1. Why can you add $N_e + N_s + N_b$ and you can add $B + G$, but you can't add $M + W$?

2. We measure real-world quantities in units like feet, gallons, students and miles/hour (miles per hour).

   a. What units might you use to measure N_e, N_s and N_b?
   What about the sum $N_e + N_s + N_b$?

   b. What units might you use to measure B? G?
   What about the sum $B + G$?

   c. What units might you use to measure M? W?
   What about the sum $M + W$?

3. State a rule about how you might use units to help you think about what types of quantities can be added. How would you use or modify your rule to fit subtraction?

Units in Multiplication, scenario 1

1. Why can you multiply $N_e \times C_e$ and you can multiply $L \times W$, but you can't multiply $G \times C$?

2. Units in multiplication often involve rates like miles/gallon (miles per gallon), feet/second (feet per second), or students/table (students per table).

   a. What units might you use to measure N_e? C_e?
   What about the product $N_e \times C_e$?

   b. What units might you use to measure L? W?
   What about the product $L \times W$?

   c. What units might you use to measure G? C?
   What about the product $G \times C$?

3. State a rule about how you might use units to help you think about what types of quantities can be multiplied.

Units in Multiplication, scenario 2

1. Let ℓ represent the length of the cafeteria in feet and w represent its width in feet. What does $\ell + w + \ell + w$ represent? What about $\ell \times w$?

2. Why can we add $\ell + w$ and multiply $\ell \times w$? What is it about these variables that allow them to be added or multiplied?

3. How might you modify your rule for using units to guide your thinking when multiplying?

Units in Division, scenario 1

1. What are the units for the **dividend** (what you are dividing up), the **divisor** (what you are dividing by) and the **quotient** (the result of the division) in the following expressions:

   a. $\dfrac{S}{P}$

   b. $\dfrac{F}{L}$

   c. $\dfrac{S}{F}$

   d. $\dfrac{S_M}{M}$

2. State a rule about the units in division problems like those represented above.

Units in Division, scenario 2

1. What are the units for the dividend (what you are dividing up), the divisor (what you are dividing by) and the quotient (the result of the division) in the following expressions:

   a. $\dfrac{F}{W}$

   b. $\dfrac{P_L}{T}$

2. State a rule about the units in division problems like those represented above.

Name:

Ready, Set, Go!

Ready

Topic: Solve and justify one variable equations

Solve each equation, justifying each step you use.

1.

$8x - 10 = x + 11$	Justification

2.

$5p - 2 = 32$	Justification

3.

$10(y + 5) = 10$	Justification

4.

$3x + 9 = 44 - 2x$	Justification

Set

Topic: Understanding variables

Use the task Serving Up Symbols to complete the table below.

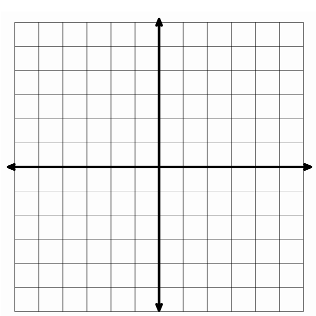
Expression	Description
$\frac{C}{L}$	
$\frac{C}{W}$	
$\frac{F}{L}$	
	Total beverages served in the cafeteria per day
	Average number of food items per week
	The average number of food items served per minute

Go

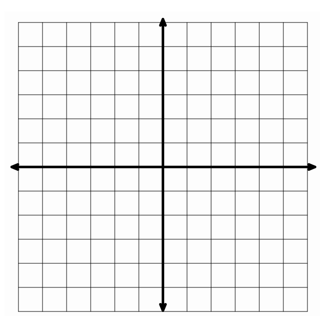
Topic: Graph linear equations

Graph each equation.

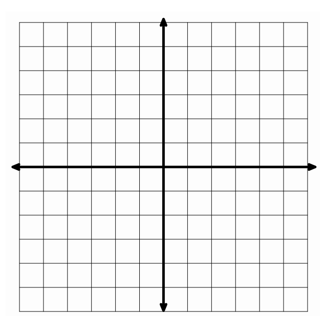
6. $y = 3x + 1$



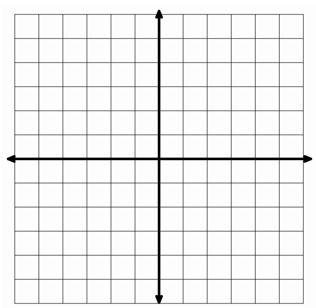
7. $y = -2x + 3$



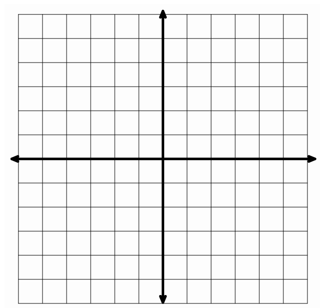
8. $y = \frac{1}{2}x - 5$



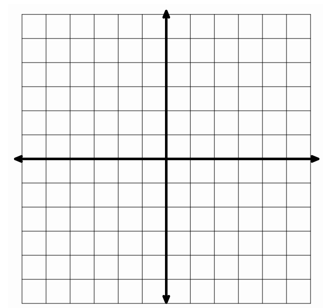
9. $y = -\frac{2}{3}x + 2$



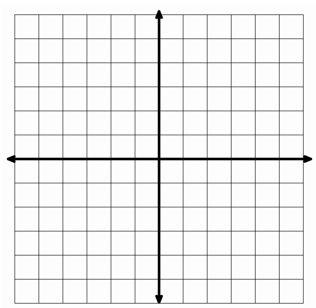
10. $y = 2x - 1$



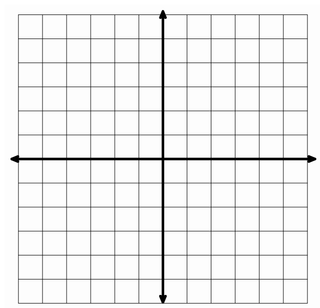
11. $y = -\frac{1}{2} + 4$



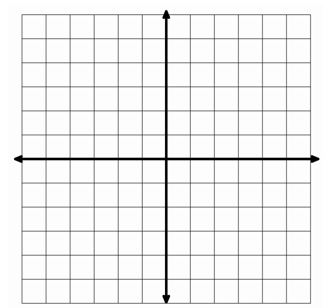
12. $y = 4x + 2$



13. $y = 2x$



14. $y = -3x + 5$



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http://www.youtube.com/watch?v=WXzpisUh0AU

1.5 Cafeteria Actions and Reactions

A Develop Understanding Task

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Elvira, the cafeteria manager, has just received a shipment of new trays with the school logo prominently displayed in the middle of the tray. After unloading 4 cartons of trays in the pizza line, she realizes that students are arriving for lunch and she will have to wait until lunch is over before unloading the remaining cartons. The new trays are very popular and in just a couple of minutes 24 students have passed through the pizza line and are showing off the school logo on the trays. At this time, Elvira decides to divide the remaining trays in the pizza line into 3 equal groups so she can also place some in the salad line and the sandwich line, hoping to attract students to the other lines. After doing so, she realizes that each of the three serving lines has only 12 of the new trays.

"That's not many trays for each line. I wonder how many trays there were in each of the cartons I unloaded?"

1. Can you help the cafeteria manager answer her question using the data in the story about each of the actions she took? Explain how you arrive at your solution.

Elvira is interested in collecting data about how many students use each of the tables during each lunch period. She has recorded some data on Post-It Notes to analyze later. Here are the notes she has recorded:

- Some students are sitting at the front table. (I got distracted by an incident in the back of the lunchroom, and forgot to record how many students.)
- Each of the students at the front table has been joined by a friend, doubling the number of students at the table.
- Four more students have just taken seats with the students at the front table.
- The students at the front table separated into three equal-sized groups and then two groups left, leaving only one-third of the students at the table.
- As the lunch period ends, there are still 12 students seated at the front table.

Elvira is wondering how many students were sitting at the front table when she wrote her first note. Unfortunately, she is not sure what order the middle three Post-It Notes were recorded in since they got stuck together in random order. She is wondering if it matters.

2. Does it matter which order the notes were recorded in? Determine how many students were originally sitting at the front table based on the sequence of notes that appears above. Then rearrange the middle three notes in a different order and determine what the new order implies about the number of students seated at the front table at the beginning.

3. Here are three different equations that could be written based on a particular sequence of notes. Examine each equation, and then list the order of the five notes that is represented by each equation. Find the solution for each equation.

- $\dfrac{2(x+4)}{3}=12$

- $2\left(\dfrac{x}{3}+4\right)=12$

- $\dfrac{2x+4}{3}=12$

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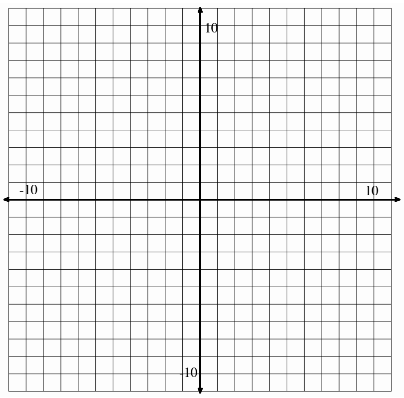
Ready, Set, Go!

Ready

Topic: Solutions to an equation

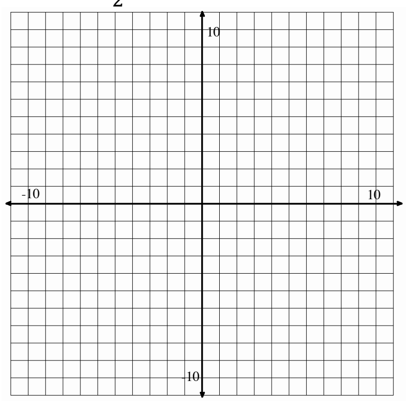
Graph the following equations using the coordinate graph, and then say if the given point is a solution to the equation.

1. $y = 5x - 2$



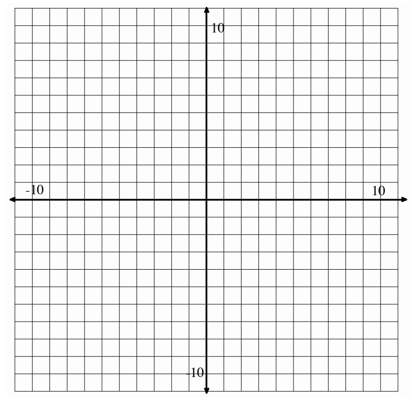
Point: (1 , 3) Yes / No

2. $y = -\frac{1}{2}x + 8$



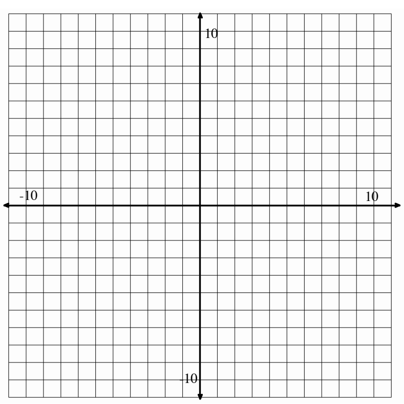
Point: (0 , 4) Yes / No

3. $y = x + 4$



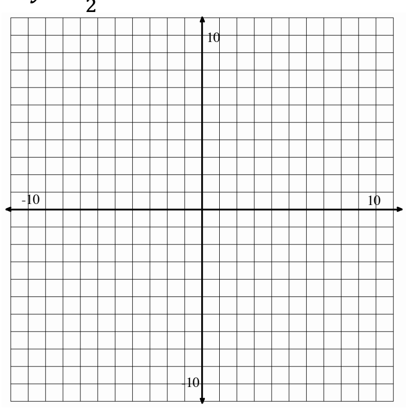
Point: (-2 , 2) Yes / No

4. $y = x + 2$



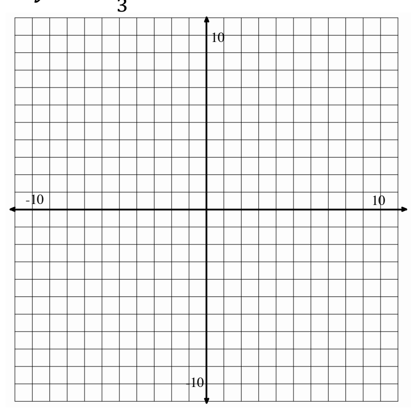
Point: (1 , 3) Yes / No

5. $y = \frac{5}{2}x - 7$



Point: (2 , -2) Yes / No

6. $y = -\frac{4}{3}x$



Point: (4 , -6) Yes / No

Set

7. The solution to an equation is n = -5. The equation has parentheses on at least one side of the equation and has variables on both sides of the equation. What could the equation be?

8. Create a two-step equation that is true by expanding the given solution using properties of equality. Draw a model to represent your expanded equation.

   a. $x = 3$ $\quad$ $m = -2$ $\quad$ $a = 0$

9. Without solving, determine if the two expressions are equivalent. Explain your reasoning.

   a. $14 - (3a + 2) = 14 - 3a - 2$

   b. $4a - 10 = 2(2a - 5)$

10. Without solving, determine if these two equations have the same solution. $3(x - 5) = 35$ *and* $3x - 5 = 35$. Why or why not?

11. Which of the following expressions are equivalent?

    $\frac{4t-10}{2}$ $\quad$ $\frac{4t}{2} - 10$ $\quad$ $2t - 10$ $\quad$ $4t - 5$

Go

Check whether the given number is a solution to the corresponding equation.

12. $a = -3;\ \ 4a + 3 = -9$

13. $x = \frac{4}{3};\ \ \frac{3}{4}x + \frac{1}{2} = \frac{3}{2}$

14. $y = 2;\ \ 2.5y - 10.0 = -0.5$

15. $z = -5;\ \ 2(5 - 2z) = 20 - 2(z - 1)$

Need Help? Check out these related videos:

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http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-a-line-in-slope-intercept-form

http://www.youtube.com/watch?v=WXzpisUh0AU

http://patrickjmt.com/an-intro-to-solving-linear-equations-what-does-it-mean-to-be-a-solution/

1.6 Elvira's Equations

A Solidify Understanding Task

(Note: This task refers to the same set of variables as used in *Serving Up Symbols*)

Elvira, the cafeteria manager, has written the following equation to describe a cafeteria relationship that seems meaningful to her. She has introduced a new variable A to describe this relationship.

$$A = \frac{S}{CP}$$

1. What does A represent in terms of the school and the cafeteria?

2. Using what you know about manipulating equations, solve this equation for S. Your solution will be of the form S = *an expression written in terms of the variables A, C* and *P.*

3. Does your expression for S make sense in terms of the meanings of the other variables? Explain why or why not.

4. Now solve the above equation for C and explain why the solution makes sense in terms of the variables.

Here is another one of Elvira's equations.

$$T_S = \frac{S(N_e + N_s + N_b)}{i}$$

5. What does T_S represent in terms of the school and the cafeteria?

   (Hint: Elvira was really clever here. She recognized that the expression $N_e + N_s + N_b$ counted the number of *items/lunch*. She also noticed that since S represented the number of students that eat lunch each day, S also counted the number of *lunches* served. Using these new units, what would the product $S(N_e + N_s + N_b)$ mean? What would the quotient named T_s mean?)

6. Using what you know about manipulating equations, solve this equation for *S.*

7. Does your expression for S make sense in terms of the meanings of the other variables? Explain why or why not.

8. Now solve the above equation for N_e and explain why the solution makes sense in terms of the variables.

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Ready, Set, Go!

Ready

Topic: Solve literal equations

Solve each equation for *x*. Justify each step.

1. $ax = d$
2. $b + cx = d$
3. $ab + cx = d$

Set

Topic: Solve literal equations or writing linear equations in slope-intercept form

Rearrange the following equations to solve for y (slope-intercept form).

4. $4x + y = 3$
5. $2y = 6x + 9$
6. $5x - 2y = 10$
7. $3x + 6y = 25$
8. $x - 8y = 12$
9. $3x - 7y = 20$

Go

Topic: Create and solve equations for real world problems

Create an equation that describes each of the situations below, then solve.

10. The cost of a birthday party at Classic Boon is $200 plus $4 per person. The cost for Fletcher's party came to $324. How many people came to his party?

11. A cell phone company charges $55 per month for unlimited minutes plus $0.25 per text sent. If the charges to Dayne's cell phone for last month came to $100, how many texts did Aly send?

12. Aly has baked an apple pie and wants to sell it in her bakery. She is going to cut it into 12 slices and sell them individually. She wants to sell it for three times the cost of making it. The ingredients cost $8.50, and she allowed $1.25 to cover the cost of electricity to bake it. Find the values for each of the following questions:

a) What is the amount Aly will charge for each slice of pie?

b) What is the total amount she will gross if she sells the entire pie?

c) What will be the profit if she sells the entire pie?

Need Help? Check out these related videos:

Solve and justify two step equations using properties of equality http://www.youtube.com/watch?v=WXzpisUh0AU

Rearrange equations to highlight a variable: solve literal equations
http://www.khanacademy.org/math/algebra/solving-linear-equations/v/example-of-solving-for-a-variable

1.7 Solving Equations, Literally

A Practice Understanding Task

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Solve each of the following equations for *x*:

1. $\frac{3x+2}{5} = 7$

2. $\frac{3x+2y}{5} = 7$

3. $\frac{4x}{3} - 5 = 11$

4. $\frac{4x}{3} - 5y = 11$

5. $\frac{2}{5}(x+3) = 6$

6. $\frac{2}{5}(x+y) = 6$

7. $2(3x+4) = 4x+12$

8. $2(3x+4y) = 4x+12y$

Write a verbal description for each step of the equation solving process used to solve the following equations for *x*. Your description should include statements about how you know what to do next. For example, you might write, "*First I ________________, because ____________________*. . ."

9. $\frac{ax+b}{c} - d = e$

10. $r \cdot \sqrt{\frac{mx}{n}} + s = t$

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Ready, Set, Go!

Ready

Topic: Inequalities

Use the inequality 4 < 6 to complete each row in the table.

Apply each operation to the original inequality 4 < 6	Result	Is the inequality true or false?
1. Add 4 to both sides		
2. Add -4 to both sides		
3. Subtract 10 from both sides		
4. Multiply both sides by 4		
5. Divide both sides by 2		
6. Multiply both sides by -3		
7. Divide both sides by -2		

In general, what operations, when performed on an inequality, *reverse* the inequality?

Set

Topic: Solve literal equations

Solve for the indicated variable.

8. Solve the following equation to isolate F: $C = \frac{5}{9}(F - 32)$

9. For $V = \frac{1}{3}\pi r^2 h$, rewrite the formula to isolate the variable h.

10. The area formula of a regular polygon is $A = \frac{1}{2}Pa$. The variable a represents the apothem and P represents the perimeter of the polygon. Rewrite the equation to highlight the value of the perimeter, P.

11. The equation $y = mx + b$ is the equation of a line. Isolate the variable m.

12. The equation $y = mx + b$ is the equation of a line. Isolate the variable x.

13. $Ax + By = C$ is the standard form for a line. Isolate the equation for x.

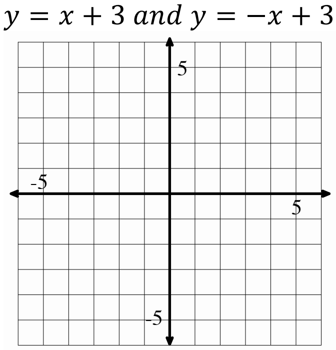
14. $Ax + By = C$ is the standard form for a line. Isolate the equation for y.

Go

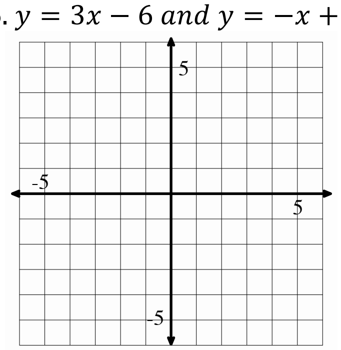
Topic: Solve systems of linear equations

Solve linear equations and pairs of simultaneous linear equations (simple, with a graph only) by graphing both lines and finding where they intersect. Justify the solution numerically.

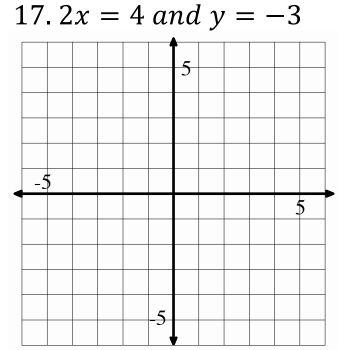
15. $y = x + 3 \text{ and } y = -x + 3$



16. $y = 3x - 6 \text{ and } y = -x + 6$



17. $2x = 4 \text{ and } y = -3$



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http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-for-a-variable

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-linear-systems-by-graphing

1.8 Cafeteria Conundrums

A Solidify Understanding Task

Between serving and preparing delicious school lunches, our cafeteria manager, Elvira, is busy analyzing the business of running the cafeteria. We previously saw the symbols for some of the things that she measured. Now she plans to use those symbols. Help Elvira to consider the pressing questions of the lunch room.

Symbol	Meaning
S	Number of students that buy lunch in the cafeteria each day
S_m	Number of students who have passed through a line in m minutes
C	Number of classes per lunch period
P	Number of lunch periods per day
B	Number of boys that buy lunch each day
G	Number of girls that buy lunch each day
F	Number of food servers in the cafeteria
T	Total number of food items in one lunch (Each entrée, side dish, or beverage counts as 1 item.)
M	Number of minutes passed since the beginning of the lunch period
N_e	Number of entrees in each lunch
N_s	Number of side dishes in each lunch
N_b	Number of beverages in each lunch
C_e	Cost of each entrée
C_s	Cost of each side dish
C_b	Cost of each beverage
L	Number of lines in the cafeteria
W	The number of food workers (servers) per line
i	Average number of food items that a worker can serve each minute (Each entrée, side dish, or beverage counts as 1 item.)
H	Number of hours each food worker works each day
P_L	Price per lunch

Write equations or inequalities to express some of the conditions that Elvira sees in the cafeteria.

1. Each lunch can have no more than 4 side dishes.

2. More boys eat school lunch than girls.

3. There can be no more than 7 food items in each lunch.

4. In each lunch, there are 3 more side dishes than entrees and twice as many beverages as entrees. Write an inequality in **one variable** that shows that the total number of food items in a lunch cannot be more than 7.

5. The cost of food in the lunch is the total of the cost of the entrée, the side dishes, and the beverages. Write an inequality that shows that the cost of the food in the lunch must be less than $1.50.

6. To meet district guidelines, the total price of a lunch must be more than $2.25, but less than $3.50.

7. Elvira knows that the number of lines that she can open in the cafeteria depends on how many food servers she has in the cafeteria each day and how many workers are needed in each line. Write an inequality that shows this relationship.

8. Food workers are paid $11.50 per hour. Elvira can't spend more than $500 per day on employees. Write an inequality that relates the number of food workers to the amount spent each day on employees.

9. Elvira knows that the money she gets from selling lunches has to be greater than her costs.
   a. Write an expression for the cost of employees each day
   b. Write an expression for the cost of food each day
   c. Write an expression that shows that the total cost of food and employees each day must be less than the amount she brings in from selling lunches.

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Ready, Set, Go!

Ready

Topic: Solving equations

Jesse was asked to solve an algebra problem. She submitted the following solution

$4(x+3)=1$

$4x+3=1$

$4x=-2$

$x=-2$

1. Is Jesse's solution correct?
2. If it is correct, justify each step of her solution.
3. If it is incorrect, correct her solution, and explain to Jesse what she did wrong.

Set

Topic: Creating and solving real world problems

4. Jade is stranded downtown with only $10 to get home. Taxis cost $0.75 per mile, but there is an additional $2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money.

5. Jasmin's Dad is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs $150 for the afternoon, and the food will cost $3 per person. Andrew, Jasmin's Dad, has a budget of $300. Write an equation and use it to determine the maximum number of guests he can invite.

6. Jane is baking cookies for a large party. She has a recipe that will make one batch of two dozen cookies, and she decides to make five batches. To make five batches, she finds that she will need 12.5 cups of flour and 15 eggs. Write an equation to describe each of the following situations. Then solve the problem.

   a. How many cookies will she make in all?

   b. How many cups of flour go into one batch?

   c. How many eggs go into one batch?

   d. If Jane only has a dozen eggs on hand, how many more does she need to make five batches?

   e. If she doesn't go out to get more eggs, how many batches can she make? How many cookies will that be?

Go

Topic: Solve systems of equations

Solve the following systems of equations by graphing. You may use a graphing calculator.

7. Mary's car has broken down and it will cost her $1200 to get it fixed—or, for $4500, she can buy a new, more efficient car instead. Her present car uses about $2000 worth of gas per year, while gas for the new car would cost about $1500 per year. After how many years would the total cost of fixing the car equal the total cost of replacing it?

8. Juan is considering two cell phone plans. The first company charges $120 for the phone and $30 per month for the calling plan that Juan wants. The second company charges $40 for the same phone but charges $45 per month for the calling plan that Juan wants. After how many months would the total cost of the two plans be the same?

9. A tortoise and hare decide to race 30 feet. The hare, being much faster, decides to give the tortoise a 20 foot head start. The tortoise runs at 0.5 feet/sec and the hare runs at 5.5 feet per second. How long until the hare catches the tortoise?

Need Help? Check out these related videos:

http://www.youtube.com/watch?v=EWcllbr8Hqs&feature=related

http://www.youtube.com/watch?v=ldYGiXSHa_Q

1.9 Greater Than?
A Solidify Understanding Task

For each situation you are given a mathematical statement and two expressions beneath it.

1. Decide which of the two expressions is greater, if the expressions are equal, or if the relationship cannot be determined from the statement.
2. Write an equation or inequality that shows your answer.
3. Explain why your answer is correct.

Watch out—this gets tricky!

Example:
Statement: $x = 8$
Which is greater? $x + 5$ or $3x + 2$
Answer: $3x + 2 > x + 5$ because if $x = 8$, $3x + 2 = 26$, $x + 5 = 13$ and $26 > 13$.

Try it yourself:

1. Statement: $y < x$
   Which is greater? $x - y$ or $y - x$

2. Statement: $2x - 3 > 7$
   Which is greater? 5 or x

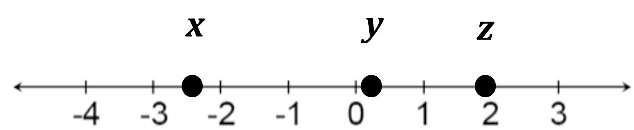
3. Statement: $10 - 2x < 6$
   Which is greater? x or 2

4. Statement: $4x = 0$
   Which is greater? 1 or x

5. Statement: $a > 0$, $b < 0$
   Which is greater? ab or $\frac{a}{b}$

6. Statement: n is an integer
   Which is greater? n or $-n$

7. Statement:
   Which is greater? *1* or yz



8. Statement: Use the number line in #7 and $x < w < y$
   Which is greater? w or $-y$

9. Statement: $0 < x < 10$ and $0 < y < 12$
   Which is greater? x or y

10. Statement: $3^{n+2} = 27$
    Which is greater? n or 3

11. Statement: $5 > 4$
    Which is greater? $5x$ or $4x$

12. Statement $x > y$
    Which is greater? $x + a$ or $y + a$

13. Statement: $5 > 4$
    Which is greater? $\frac{5}{x}$ or $\frac{4}{x}$

14. Statement: $x > y$ and $a > b$
    Which is greater? $x - a$ or $y - b$

15. Statement: $x > 0, y > 0, \frac{x}{y} > 2$
    Which is greater? $2y$ or x

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Ready, Set, Go!

Ready

Topic: Solve inequalities, create and solve equations.

Solve the following inequalities for x.

1. $2x - 9 < 3$
2. $4x - 3 < 13$
3. $6x - 4 < 26$
4. $3x - 5 \geq 10$

Create and solve the equations for the following problems.

5. Virginia's Painting Service charges $10 per job and $0.20 per square foot. If Virginia earns $50 for painting one job, how many square feet did she paint at the job?

6. Renting the ice-skating rink for Dayne's birthday party costs $200 plus $4 per person. If the cost was $324, how many people were at Dayne's birthday party?

Set

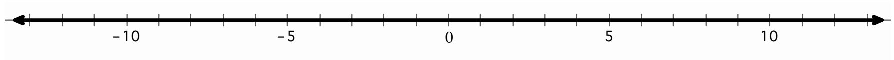
Topic: Solve inequalities

Solve each inequality. Write the solution as an inequality.

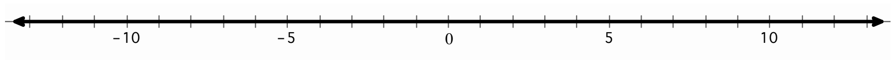
7. $x + 15 < 12$
8. $x - 4 \geq 13$
9. $9x > -\frac{3}{4}$
10. $3x - 7 \geq 3(x - 7)$
11. $x - 12 \geq 80$

Solve each inequality and graph the solution on the number line.

12. $x - 2 \leq 1$

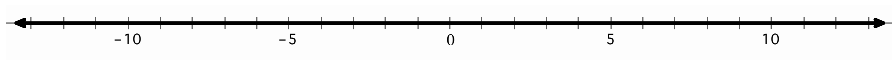


13. $x - 8 > -20$

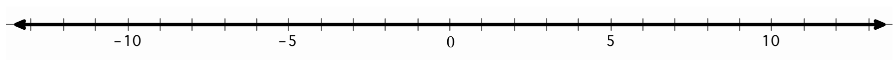


Solve each inequality. Write the solution as an inequality and graph it.

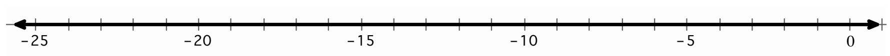
14. $3x \leq 6$



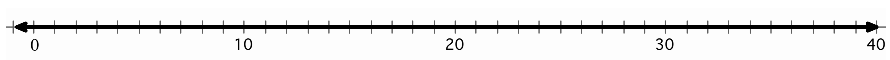
15. $\frac{x}{5} > -\frac{3}{10}$



16. $-10x > 150$



17. $\frac{x}{7} \geq -5$



Solve each multi-step inequality.

18. $x - 5 > 2x + 3$

19. $\frac{3(x-4)}{12} \leq \frac{2x}{3}$

20. $2(x - 3) \leq 3x - 2$

Go

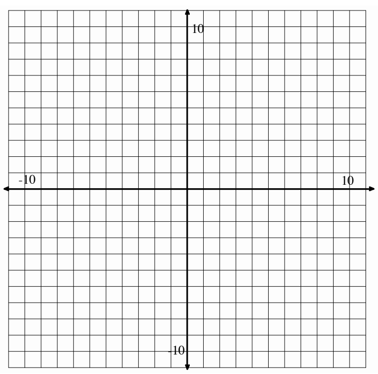
Topic: Solve systems of linear equations

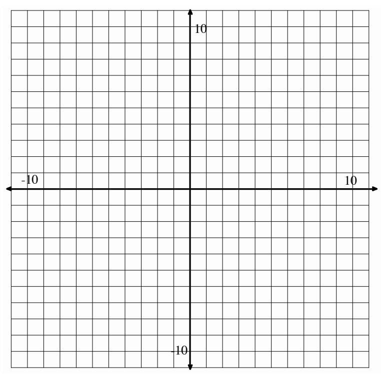
Solve linear equations and pairs of simultaneous linear equations (simple, with a graph only). Justify the solution numerically.

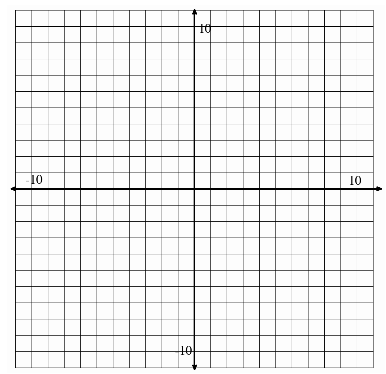
21. $\begin{cases} y = -x + 5 \\ -x + y = 1 \end{cases}$

22. $\begin{cases} x + 2y = 8 \\ 5x + 2y = 0 \end{cases}$

23. $\begin{cases} 3x + 2y = 12 \\ 4x - y = 5 \end{cases}$







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1.10 Taking Sides

A Practice Task

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Joaquin and Serena work together productively in their math class. They both contribute their thinking and when they disagree, they both give their reasons and decide together who is right. In their math class right now, they are working on inequalities. Recently they had a discussion that went something like this:

> Joaquin: The problem says that "6 less than a number is greater than 4." I think that we should just follow the words and write $6 - x > 4$.
>
> Serena: I don't think that works because if x is 20 and you do 6 less than that you get $20 - 6 = 14$. I think we should write $x - 6 > 4$.
>
> Joaquin: Oh, you're right. Then it makes sense that the solution will be $x > 10$, which means we can choose any number greater than 10.

The situations below are a few more of the disagreements and questions that Joaquin and Serena have. Your job is to decide how to answer their questions, decide who is right, and give a mathematical explanation of your reasoning.

1. Joaquin and Serena are assigned to graph the inequality $x \geq -7$.
   Joaquin thinks the graph should have an open dot -7.
   Serena thinks the graph should have a closed dot at -7.
   Explain who is correct and why.

2. Joaquin and Serena are looking at the problem $3x + 1 > 0$.
   Serena says that the inequality is always true because multiplying a number by three and then adding one to it makes the number greater than zero.
   Is she right? Explain why or why not.

3. The word problem that Joaquin and Serena are working on says, "4 greater than x".
   Joaquin says that they should write: $4 > x$.
   Serena says they should write: $x + 4$.
   Explain who is correct and why.

4. Joaquin is thinking hard about equations and inequalities and comes up with this idea:
If $45 + 47 = t$, then $t = 45 + 47$.
So, if $45 + 47 < t$, then $t < 45 + 47$.
Is he right? Explain why or why not.

5. Joaquin's question in #4 made Serena think about other similarities and differences in equations and inequalities. Serena wonders about the equation $-\frac{x}{3} = 4$ and the inequality $-\frac{x}{3} > 4$. Explain to Serena ways that solving these two problems are alike and ways that they are different. How are the solutions to the problems alike and different?

6. Joaquin solved $-15q \leq 135$ by adding 15 to each side of the inequality. Serena said that he was wrong. Who do you think is right and why?

Joaquin's solution was $q \leq 150$. He checked his work by substituting 150 for q in the original inequality. Does this prove that Joaquin is right? Explain why or why not.

Joaquin is still skeptical and believes that he is right. Find a number that satisfies his solution but does not satisfy the original inequality.

7. Serena is working is checking her work with Joaquin and finds that they disagree on a problem. Here's what Serena wrote:

$$3x + 3 \leq -2x + 5$$
$$3x \leq -2x + 2$$
$$x \leq 2$$

Is she right? Explain why or why not?

8. Joaquin and Serena are having trouble solving $-4(3m - 1) \geq 2(m + 3)$.
Explain how they should solve the inequality, showing all the necessary steps and identifying the properties you would use.

9. Joaquin and Serena know that some equations are true for any value of the variable and some equations are never true, no matter what value is chosen for the variable. They are wondering about inequalities. What could you tell them about the following inequalities? Do they have solutions? What are they? How would you graph their solutions on a number line?

   a. $4s + 6 \geq 6 + 4s$

   b. $3r + 5 > 3r - 2$

   c. $4(n + 1) < 4n - 3$

10. The partners are given the literal inequality $ax + b > c$ to solve for x. Joaquin says that he will solve it just like an equation. Serena says that he needs to be careful because if a is a negative number, the solution will be different. What do you say? What are the solutions for the inequality?

Name: Getting Ready | 10

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Ready, Set, Go!

Ready

Topic: Solving equations and inequalities

1. The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.

   a. If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?

   b. How many visits can he make if he does not?

   c. If he increases his budget to $160, how many visits can he make as a member?

   d. How many can he make as a non-member?

2. Jae just took a math test with 20 questions, each worth an equal number of points. The test is worth 100 points total.

   a. Write an equation relating the number of questions Jae got right to the total score he will get on the test.

   b. If a score of 70 points earns a grade of *C*-, how many questions would Jae need to get right to get a *C*- on the test?

   c. If a score of 83 points earns a grade of *B*, how many questions would Jae need to get right to get a *B* on the test?

   d. Suppose Jae got a score of 60% and then was allowed to retake the test. On the retake, he got all the questions right that he got right the first time, and also got half the questions right that he got wrong the first time. What percent did Jae get right on the retake?

Set

Topic: Solve and justify one variable inequalities

Solve each inequality, justifying each step you use.

3.

$x - 5 < 35$	Justification

4.

$x + 68 \geq 75$	Justification

5.

$2x - 4 \leq 10$	Justification

6.

$5 - 4x \leq 17$	Justification

7.

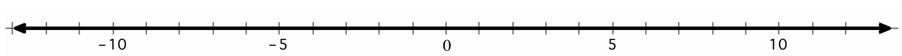
$\frac{x}{-3} > -\frac{10}{9}$	Justification

8.

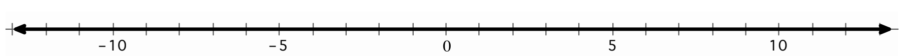
$2(x - 3) \leq 3x - 2$	Justification

Solve each inequality and graph the solution on the number line.

9. $x - 8 > -20$



10. $x + 11 > 13$



Solve each multi-step inequality.

11. $4x + 3 < -1$

12. $4 - 6x \leq 2(2x + 3)$

13. $5(4x + 3) \geq 9(x - 2) - x$

14. $\frac{2}{3}x - \frac{1}{2}(4x - 1) \geq x + 2(x - 3)$

Topic: Solve literal equations

15. Solve the following equation to isolate C: $F = \frac{9}{5}C + 32$

16. For $V = \frac{1}{3}\pi r^2 h$, rewrite the formula to isolate the variable r.

17. The area formula of a regular polygon is $A = \frac{1}{2}Pa$. The variable a represents the apothem and P represents the perimeter of the polygon. Rewrite the equation to highlight the value of the apothem, a.

18. The equation $y = mx + b$ is the equation of a line. Isolate the variable b

19. The equation for the circumference c of a circle with radius r is $c = 2\pi r$. Solve the equation for the radius, r.

20. The equation for the area of a circle A with diameter d is $A = \pi\frac{d^2}{4}$. Solve the equation to isolate the diameter, d.

Go

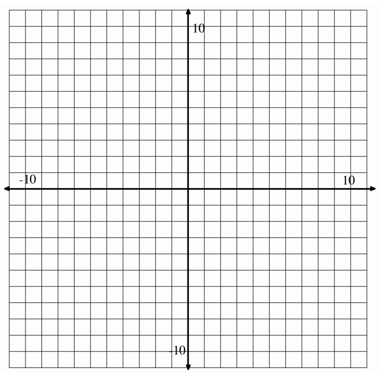
Topic: Solve systems of equations

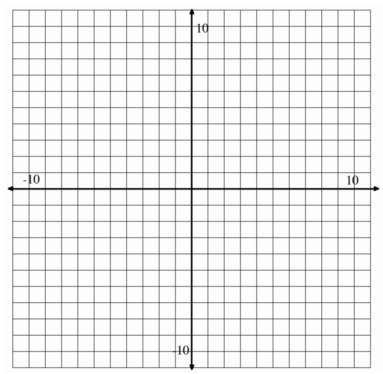
Solve linear equations and pairs of simultaneous linear equations (simple, with a graph only). Justify the solution numerically.

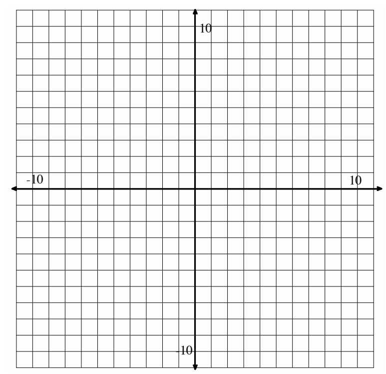
21. $\begin{cases} y = 2x + 5 \\ -x + y = 1 \end{cases}$

22. $\begin{cases} 3x - 2y = 16 \\ x + 2y = 0 \end{cases}$

23. $\begin{cases} 3x + 2y = 11 \\ 4x - y = 10 \end{cases}$







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Secondary One Mathematics: An Integrated Approach
Module 2
Systems

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the
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Module 2 – Systems of Equations and Inequalities

Classroom Task: Pet Sitters- A Develop Understanding Task
An introduction to representing constraints with systems of inequalities (**A.CED.3**)
Ready, Set, Go Homework: Systems 2.1

Classroom Task: Too Big or Not Too Big, That is the Question - A Solidify Understanding Task
Writing and graphing linear inequalities in two variables (**A.CED.2, A.REI.12**)
Ready, Set, Go Homework: Systems 2.2

Classroom Task: Some of One, None of the Other– A Solidify Understanding Task
Writing and solving equations in two variables (**A.CED.2, A.CED.4**)
Ready, Set, Go Homework: Systems 2.3

Classroom Task: Pampering and Feeding Time – A Practice Understanding Task
Writing and graphing inequalities in two variables to represent constraints (**A.CED.2, A.CED.3, A.REI.12**)
Ready, Set, Go Homework: Systems 2.4

Classroom Task: All for One, One for All – A Solidify Understanding Task
Graphing the solution set to a linear system of inequalities (**A.CED.3, A.REI.12**)
Ready, Set, Go Homework: Systems 2.5

Classroom Task: Get to the Point – A Solidify Understanding Task
Solving systems of linear equations in two variables (**A.REI.6**)
Ready, Set, Go Homework: Systems 2.6

Classroom Task: Shopping for Cats and Dogs – A Develop Understanding Task
An introduction to solving systems of linear equations by elimination (**A.REI.5, A.REI.6**)
Ready, Set, Go Homework: Systems 2.7

Classroom Task: Can You Get to the Point, Too? – A Solidify Understanding Task
Solving systems of linear equations by elimination (**A.REI.5, A.REI.6**)
Ready, Set, Go Homework: Systems 2.8

Classroom Task: Food for Fido and Fluffy – A Solidify Understanding Task
Solving systems of linear inequalities representing constraints (**A.CED.3**)
Ready, Set, Go Homework: Systems 2.9

Classroom Task: Taken Out of Context – A Practice Understanding Task
Working with systems of linear equations, including inconsistent and dependent systems (**A.REI.6**)
Ready, Set, Go Homework: Systems 2.10

Classroom Task: More Things Taken Out of Context – A Practice Understanding Task
Working with systems of linear inequalities and their boundaries (**A.REi.12**)
Ready, Set, Go Homework: Systems 2.11

Classroom Task: Pet Sitters Revisited – A Develop Understanding Task
Using systems of linear equations and inequalities in a modeling context (**High School Modeling Standard**)
Ready, Set, Go Homework: Systems 2.12

2.1 Pet Sitters
A Develop Understanding Task

The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore they can board more.

What do you think? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

Ready, Set, Go!

Ready

Topic: Determine if given value is a solution and solve systems of equations

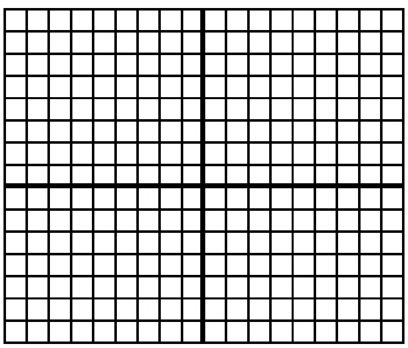
Substitute the given points into the equations to determine which ordered pair satisfies the system of linear equations, then graph both equations and label the point of intersection.

1. $y = 3x - 2$ *and* $y = x$

   a. $(0, -2)$

   b. $(2, 2)$

   c. $(1, 1)$

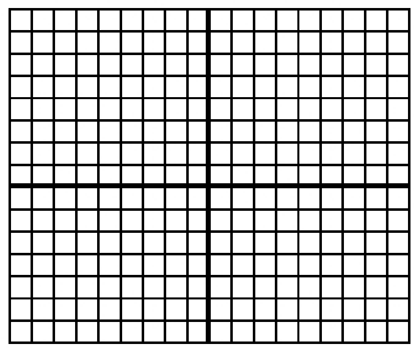


2. $y = 2x + 3$ *and* $y = x + 5$

   a. (2, 7)

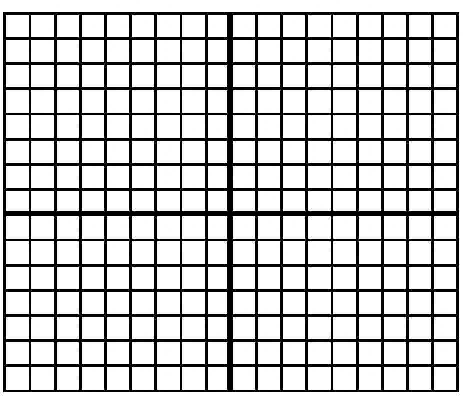
   b. (-7, 11)

   c. (0, 5)

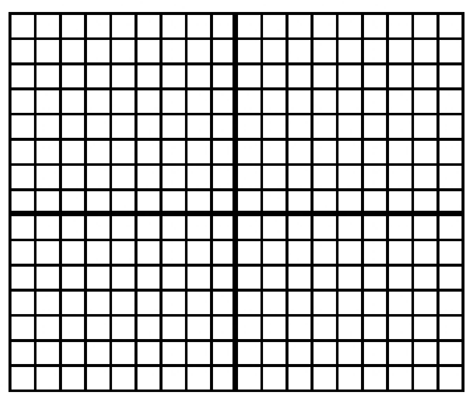


Solve the following systems by graphing. Check the solution by evaluating both equations at the point of intersection.

3. $y = x + 3$ *and* $y = -2x + 3$



4. $y = 3x - 8$ *and* $y = -x$



Name: Systems 2.1

Set Topic: Determining possible solutions

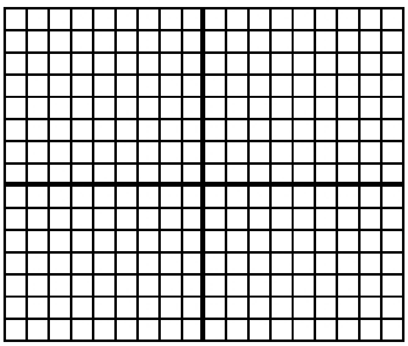
5. A theater wants to take in at least $2000 for a certain matinee. Children's tickets cost $5 each and adult tickets cost $10 each. The theater can seat up to 350 people. Find five combinations of children and adult tickets that will make their goal.

Go

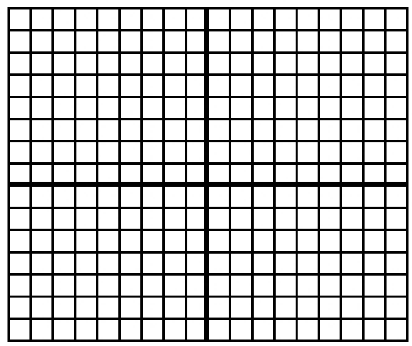
Topic: graphing linear equations and determining if a given value is a solution

Graph each equation below, then determine if the point (3,5) is a solution to the equation. Name two additional points that are solutions to the equation and show these points on the graph.

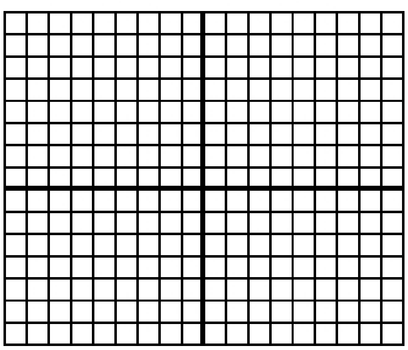
6. $y = 2x - 1$



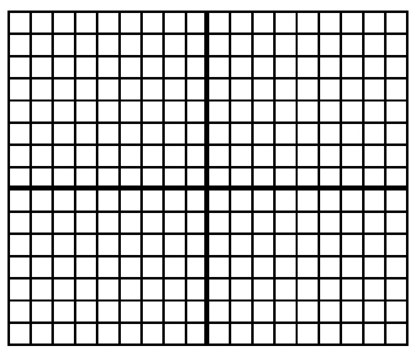
7. $y = \frac{1}{3}x + 2$



8. $y = -3x + 5$



9. $y = \frac{-3}{5}x + 4$



Need help? Check out this related video:

https://www.youtube.com/watch?v=vo-CXaCf1I4

2.2 Too Big or Not Too Big, That is the Question

A Solidify Understanding Task

As Carlos is considering the amount of money available for purchasing cat pens and dog runs (see below) he realizes that his father's suggestion of boarding "the same number of each, perhaps 12 cats and 12 dogs" is too big. Why?

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

1. Find at least 5 more combinations of cats and dogs that would be "too big" based on this *Start-up Cost* **constraint.** Plot each of these combinations as points on a coordinate grid using the same color for each point.

2. Find at least 5 combinations of cats and dogs that would not be "too big" based on this *Start-up Cost* constraint. Plot each of these combinations as points on a coordinate grid using a different color for the points than you used in #1.

3. Find at least 5 combinations of cats and dogs that would be "just right" based on this *Start-up Cost* constraint. That is, find combinations of cat pens and dog runs that would cost exactly $1280. Plot each of these combinations as points on a coordinate grid using a third color.

4. What do you notice about these three different collections of points?

5. Write an equation for the line that passes through the points representing combinations of cat pens and dog runs that cost exactly $1280. What does the slope of this line represent?

Carlos and Clarita don't have to spend all of their money on cat pens and dog runs, unless it will help them maximize their profit.

6. Shade all of the points on your coordinate grid that **satisfy** the *Start-up Costs* constraint.

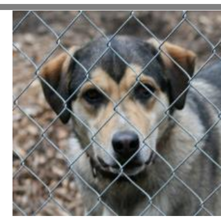
7. Write a mathematical rule to represent the points shaded in #6. That is, write an inequality whose **solution set** is the collection of points that satisfy the *Start-up Costs* constraint.

In addition to *start-up costs,* Carlos needs to consider how much space he has available, base on the following:

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

8. Write an inequality to represent the solution set for the *space* constraint. Shade the solution set for this inequality on a different coordinate grid.

Name: Systems 2.2



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Ready, Set, Go!

Ready

Topic: Determining if given values are solutions to an equation

Identify which of the given points are solutions to the following linear equations.

1. 3x + 2y = 12
   a. (2, 4)
   b. (3, 2)
   c. (4, 0)
   d. (0, 6)

2. 5x – y = 10
   a. (2, 0)
   b. (3, 0)
   c. (0, -10)
   d. (1, 1)

Find the value that will make each ordered pair a solution to the given equations.

3. x + y = 6
   a. (2, __)
   b. (0, __)
   c. (__, 0)

4. 2x + 4y = 8
   a. (2, __)
   b. (0, __)
   c. (__, 0)

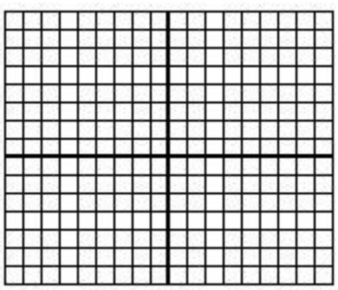
5. 3x – y = 8
   a. (2, __)
   b. (0, __)
   c. (__, 0)

Set

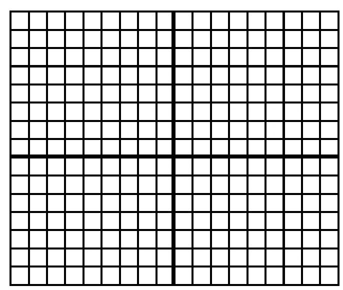
Topic: Graph linear inequalities

Graph the following inequalities on the coordinate plane. Name one point that is a solution to the inequality and one point that is not a solution. Show algebraically and graphically that your points are correct.

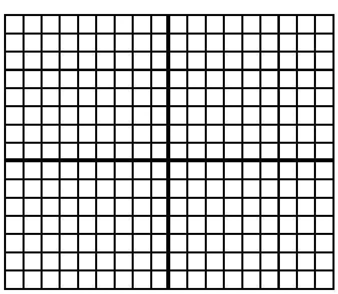
6. $y \le 3x + 4$



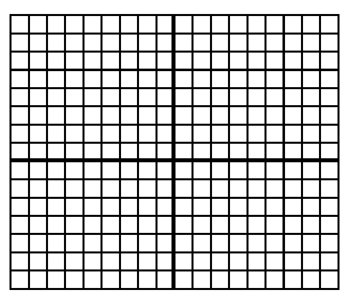
7. $x < 7$



8. $y > \frac{-3}{5}x + 2$



9. $y \ge -6$



Go Topic: Solving inequalities

Follow the directions for each problem below. (Show your work!)

10. $10 - 3x < 28$ a. Solve for x. Then graph the solution on the number line.

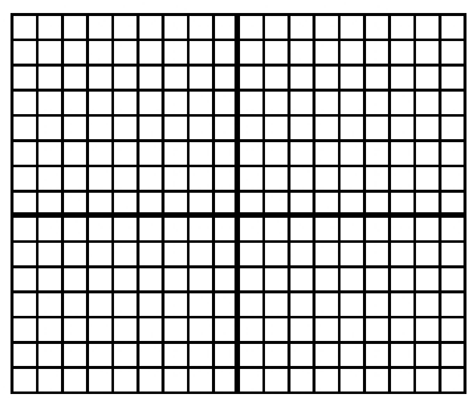
b. Select an x-value from your graph of the solution of the inequality. Replace x in the original inequality $10 - 3x < 28$ with your chosen value. Does the inequality hold true?

c. Select an x-value that is outside of the solution set on your graph. Replace x in the original inequality $10 - 3x < 28$ with your chosen value. Does the inequality still hold true?

11. $4x - 2y \geq 6$

a. Solve for y.

b. Now imagine that your inequality is an equation. In other words, your solution will say y = , instead of y ≥ or y ≤. With the equal sign, it should be the equation of a line. Graph your equation.



c. Find the y – intercept.

d. Find the slope.

e. Select a point that is above the line. (,)
Replace the x and y – values in the inequality $4x - 2y \geq 6$.
Is the inequality still true?

f. Select a point that is below the line. (,)
Replace the x and y – values in the inequality $4x - 2y \geq 6$.
Is the inequality still true?

g. Explain which side of the line should be shaded.

h. Decide whether the line should be solid or dotted. Justify your decision.

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http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-linear-inequalities-in-two-variables-2


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2.3 Some of One, None of the Other

A Solidify Understanding Task

Carlos and Clarita are comparing strategies for writing equations of the boundary lines for the "Pet Sitter" constraints. They are discussing their work on the *space* constraint.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

Carlos' Method: "I made a table. If I don't have any dogs, then I have room for 60 cats. If I use some of the space for 1 dog, then I can have 56 cats. With 2 dogs, I can boards 52 cats. For each additional dog, I can board 4 fewer cats. From my table I know the y-intercept of my line is 60 and the slope is -4, so my equation is $y = -4x + 60$."

Clarita's Method: "I let x represent the number of dogs, and y the number of cats. Since dog runs require 24 ft^2, $24x$ represents the amount of space used by dogs. Since cat pens require 6 ft^2, $6y$ represents the space used by cats. So my equation is
$24x + 6y = 360$."

1. Since both equations represent the same information, they must be equivalent to each other.

    a. Show the steps you could use to turn Clarita's equation into Carlos' equation. Explain why you can do each step.

    b. Show the steps you could use to turn Carlos' equation into Clarita's. Explain why you can do each step.

2. Use both Carlos' and Clarita's methods to write the equation of the boundary line for the *start-up costs* constraint.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

3. Show the steps you could use to turn Clarita's *start-up costs* equation into Carlos' equation. Explain why you can do each step.

4. Show the steps you could use to turn Carlos' *start-up costs* equation into Clarita's. Explain why you can do each step.

In addition to writing an equation of the boundary lines, Carlos and Clarita need to graph their lines on a coordinate grid.

Carlos' equations are written in **slope-intercept form**. Clarita's equations are written in **standard form**. Both forms are ways of writing **linear equations**.

Both Carlos and Clarita know they only need to plot two points in order to graph a line.

Carlos' strategy: How might Carlos use his slope-intercept form, $y = -4x + 60$, to plot two points on his line?

Clarita's strategy: How might Clarita use her standard form, $24x + 6y = 360$, to plot two points on her line? (Clarita is really clever, so she looks for the two easiest points she can find.)

Name: Systems | 2.3

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Ready, Set, Go!

Ready

Topic: Determining points that satisfy equations and solving systems of equations

Three points are given. Each point is a solution to at least one of the equations. Find the point that satisfies both equations. (This is the solution to the system!) Justify that the point is a solution to both equations and that the others are not.

1. $\begin{cases} y = 2x - 3 \\ y = -x + 3 \end{cases}$
   a. (-2 , 5)
   b. (2 , 1)
   c. (4 , 5)

2. $\begin{cases} y = 3x + 3 \\ y = -x + 3 \end{cases}$
   a. (-1 , 0)
   b. (6 , -3)
   c. (0 , 3)

3. $\begin{cases} y = 2 \\ y = -4x - 6 \end{cases}$
   a. (7 , 2)
   b. (2 , -14)
   c. (-2 , 2)

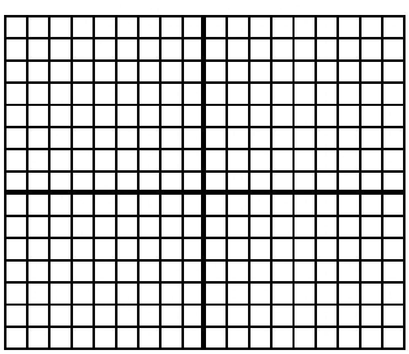
4. $\begin{cases} y = 2x + 4 \\ x + y = -5 \end{cases}$
   a. (1 , 6)
   b. (-3 , -2)
   c. (-3 , 2)

Set

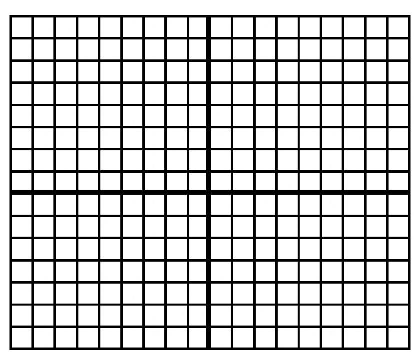
Topic: Graphing linear equations from standard form using intercepts

Graph the following equations by finding the intercepts.

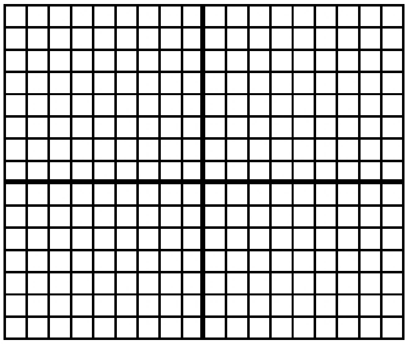
5. $5x - 2y = 10$



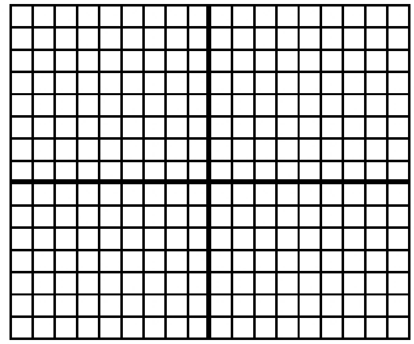
6. $3x - 6y = 24$



7. $6x + 2y = 18$



8. $-2x + 7y = -14$



Go

Topic: Adding and multiplying fractions

Add. Reduce your answers but leave as improper fractions when applicable.

9. $\frac{3}{4} + \frac{1}{8}$

10. $\frac{3}{5} + \frac{7}{10}$

11. $\frac{2}{3} + \frac{1}{4}$

12. $\frac{4}{7} + \frac{8}{21}$

Multiply. Reduce your answers but leave as improper fractions when applicable.

13. $\frac{3}{4} \times \frac{2}{9}$

14. $\frac{4}{7} \times \frac{7}{10}$

15. $\frac{5}{4} \times \frac{2}{9}$

16. $\frac{3}{7} \times \frac{8}{21}$

Need help? Check out these video lessons.

http://www.youtube.com/watch?v=cuNpXve18Pc

http://www.youtube.com/watch?v=6zixwWZ88tk

http://www.youtube.com/watch?v=oHNR0FK_lDE

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2.4 Pampering and Feeding Time
A Practice Understanding Task

Carlos and Clarita have been worried about space and start-up costs for their pet sitters business, but they realize they also have a limit on the amount of time they have for taking care of the animals they board. To keep things fair, they have agreed on the following time constraints.

- *Feeding Time*: Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.

- *Pampering Time*: The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

Write inequalities for each of these additional time constraints. Shade the solution set for each constraint on separate coordinate grids.

Name: Systems 2.4

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Ready, Set, Go!

Ready

Topic: Substitution and Solving Equations

Determine whether $h = 3$ is a solution to each problem.

1. $3(h-4) = -3$

2. $3h = 2(h+2) - 1$

3. $2h - 3 = h + 6$

4. $3h > -3$

5. $\frac{3}{5} = h \times \frac{1}{5}$

Determine the value of x that makes each equation true.

6. $4x - 2 = 8$

7. $3(x+5) = 20$

8. $2x + 3 = 2x - 5$

Set

Topic: Creating equations, solving real world problems, solve systems of equations

A phone company offers a choice of three text-messaging plans. Plan A gives you unlimited text messages for $10 a month; Plan B gives you 60 text messages for $5 a month and then charges you $0.05 for each additional message; and Plan C has no monthly fee but charges you $0.10 per message.

9. Write an equation for the monthly cost of each of the three plans.

10. If you send 30 messages per month, which plan is cheapest?

11. What is the cost of each of the three plans if you send 50 messages per month?

12. Determine the values for which each plan is the cheapest?

Name: Systems | 2.4

Go

Topic: Solve literal equations

Re-write each of the following equations for the indicated variable.

13. $3x + 5y = 30$ for y

14. $24x + 6y = 360$ for x

15. $\frac{1280 - 80d}{32} = c$ for d

16. $C = \frac{5}{9}(F - 32)$ for F

17. $y = mx + b$ for b

18. $Ax + By = C$ for y

Need help? Check out these related videos.

What does it mean to be a solution?
http://patrickjmt.com/an-intro-to-solving-linear-equations-what-does-it-mean-to-be-a-solution/
http://patrickjmt.com/solving-linear-equations/

Solving for a variable.
http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-for-a-variable

2.5 All For One, One For All

A Solidify Understanding Task

Carlos and Clarita have found a way to represent combinations of cats and dogs that satisfy each of their individual "Pet Sitter" constraints, but they realize that they need to find combinations that satisfy all of the constraints simultaneously. Why?

1. Begin by listing the **system of inequalities** you have written to represent the *start-up costs* and *space* "Pet Sitter" constraints.

2. Find at least 5 combinations of cats and dogs that would satisfy both of the constraints represented by this system of inequalities. How do you know these combinations work?

3. Find at least 5 combinations of cats and dogs that would satisfy one of the constraints, but not the other. For each combination, explain how you know it works for one of the inequalities, but not for other?

4. Shade a region on a coordinate grid that would represent the **solution set to the system of inequalities.** Explain how you found the region to shade.

5. Rewrite your systems of inequalities to include the additional constraints for *feeding time* and *pampering time*.

6. Find at least 5 combinations of cats and dogs that would satisfy all of the constraints represented by this new system of inequalities. How do you know these combinations work?

7. Find at least 5 combinations of cats and dogs that would satisfy some of the constraints, but not all of them. For each combination, explain how you know it works for some inequalities, but not for others?

8. Shade a region of a coordinate grid that would represent the solution set to the system of inequalities consisting of all 4 "Pet Sitter" constraints. Explain how you found the region to shade.

9. Shade a region in quadrant 1 of a coordinate grid that would represent all possible combinations of cats and dogs that satisfy the 4 "Pet Sitter" constraints. This set of points is referred to as the **feasible region** since Carlos and Clarita can feasibly board any of the combinations of cats and dogs represented by the points in this region without exceeding any of their constraints on time, money or space.

10. How is the feasible region shaded in #9 different from the solution set to the system of inequalities shaded in #8?

Name: Systems | 2.5

Ready, Set, Go!

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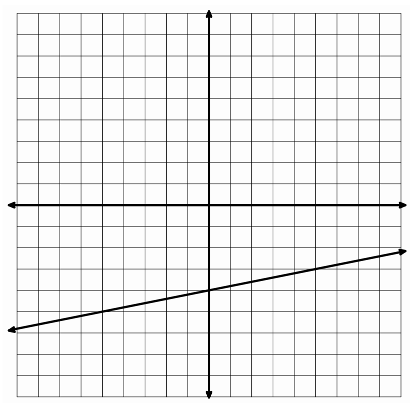
Ready

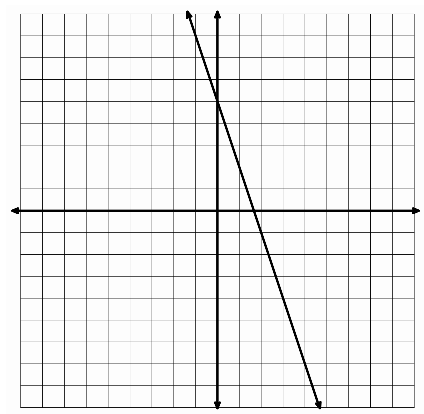
Topic: Graphing two variable inequalities

For each inequality and graph, pick a point and use it to determine which half-plane should be shaded, then shade the correct half-plane.

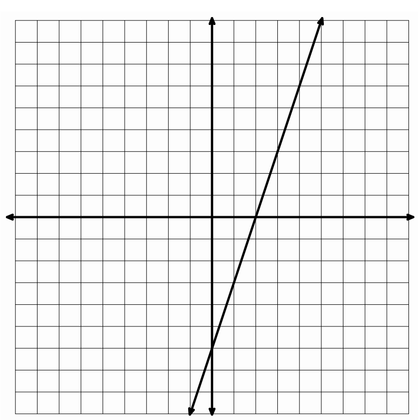
1. $y \leq \frac{1}{5}x - 4$



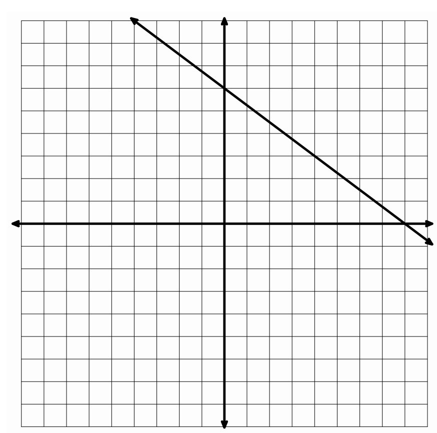
2. $y \geq -3x + 5$



3. $5x - 2y \leq 10$



4. $3x + 4y \geq 24$



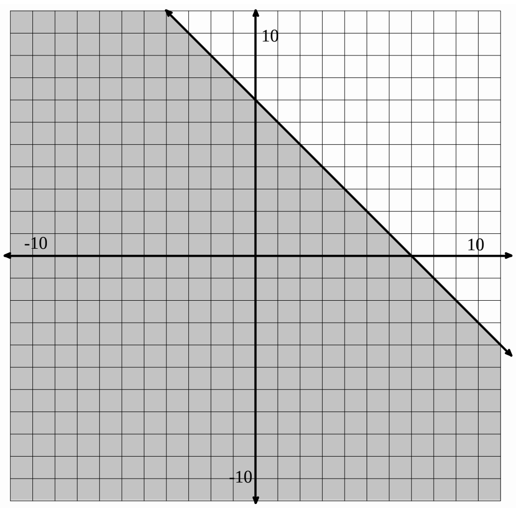
Name:

Set

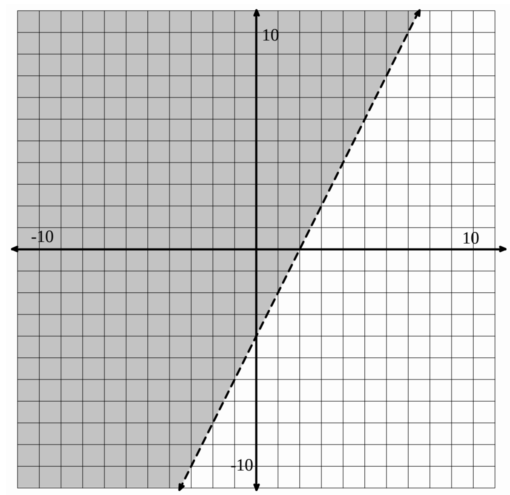
Topic: Writing two variable inequalities

Given the graph with the regions that are shaded write the inequality or system of inequalities.

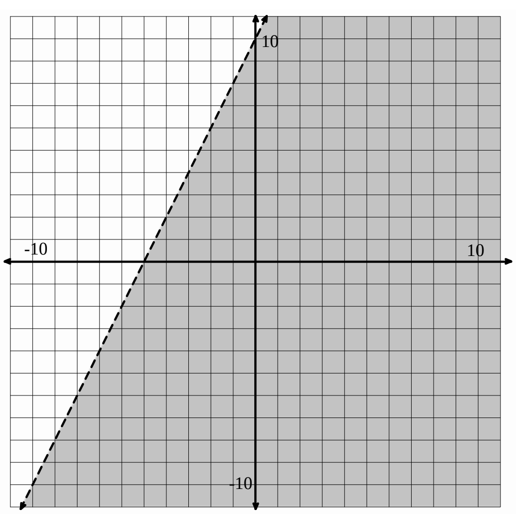
5.



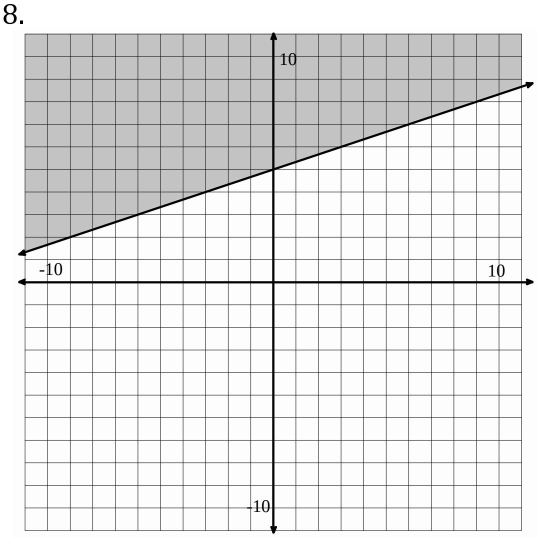
6.



7.



8.



Name: Systems | 2.5

Go

Topic: Proportional relationships

For each proportional relationship below, one representation is provided. Show the remaining representations and explain any connections you notice between representations.

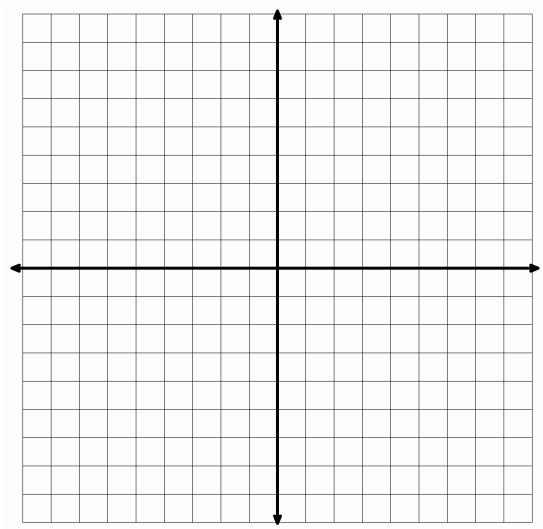
9. **Equation**:

Table

Days	Cost
1	8
2	16
3	24
4	32

Create a context

Graph



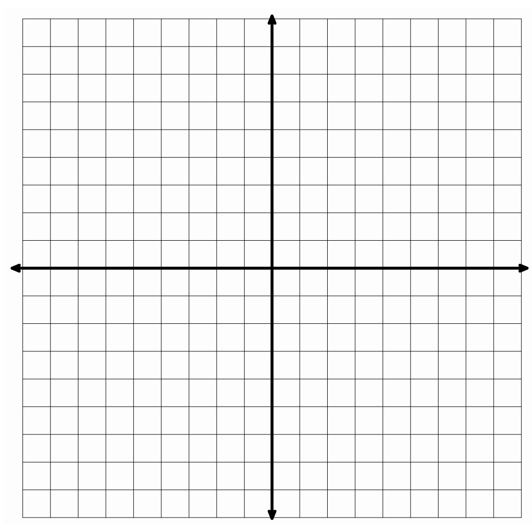
10. **Equation**:

Table

Create a context

Claire earns $9 per week allowance.

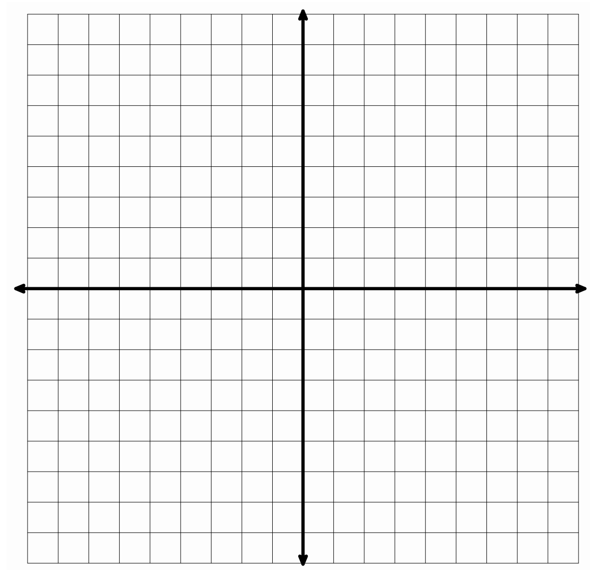
Graph



Name:

11. **Equation**: $y = 3x$

Graph

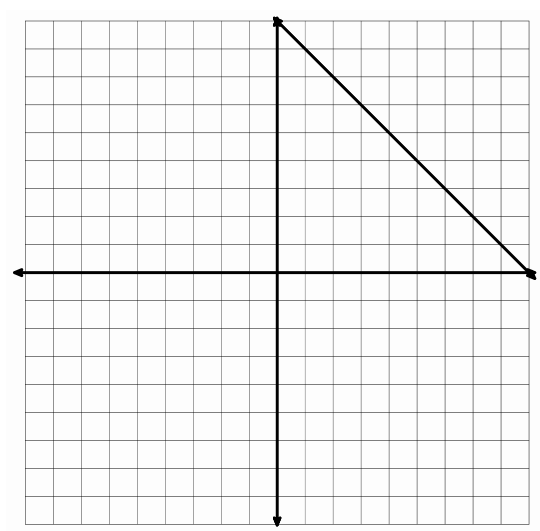


Table

Create a context

12. **Equation**:

Graph



Table

Days	Cost

Create a context

Need Help? Check out these related videos.

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-linear-inequalities-in-two-variables-3

2.6 Get to the Point!
A Solidify Understanding Task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Carlos and Clarita need to clean the storage shed where they plan to board the pets. They have decided to hire a company to clean the windows. After collecting the following information, they have come to you for help deciding which window cleaning company they should hire.

- *Sunshine Express Window Cleaners* charges $50 for each service call, plus $10 per window.
- "*Pane"less Window Cleaners* charges $25 for each service call, plus $15 per window.

1. Which company would you recommend, and why? Prepare an argument to convince Carlos and Clarita that your recommendation is reasonable. (It is always more convincing if you can support your claim in multiple ways. How might you support your recommendation using a table? A graph? Algebra?)

Your presentation to Carlos reminds him of something he has been thinking about—how to find the coordinates of the points where the boundary lines in the "Pet Sitter" constraints intersect. He would like to do this algebraically since he thinks guessing the coordinates from a graph might be less accurate.

2. Write equations for the following two constraints.

    - *Space*
    - *Start-up Costs*

    Find where the two lines intersect algebraically. Record enough steps so that someone else can follow your strategy.

3. Now find the point of intersection for the two time constraints.

    - *Feeding Time*
    - *Pampering Time*

Name: Systems | 2.6

Ready, Set, Go!

Ready

Topic: Determine patterns

Find the next two values in the pattern. Describe how you determined these values.

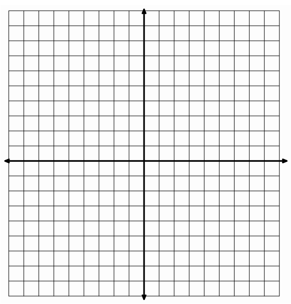
1. 3, 6, 9, 12, ____, ____ Description:
2. 3, 6, 12, 24, ____, ___ Description:
3. 24, 20, 16, 12, ___, ___ Description:
4. 24, 12, 6, 3, ___, ___ Description:

Set

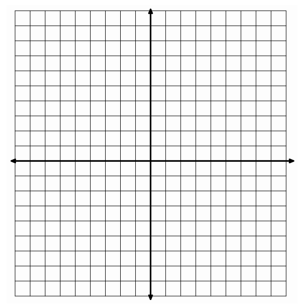
Topic: Solve systems of equations using substitution

For questions 5-8 solve the system of equations using substitution. Check your work by graphing.

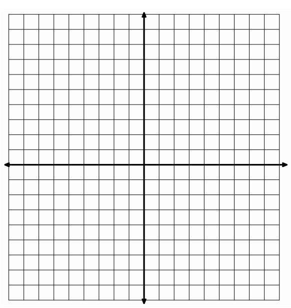
5. $\begin{cases} x + 2y = 9 \\ 3x + 5y = 20 \end{cases}$



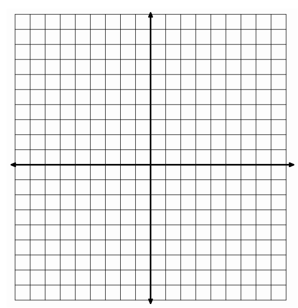
6. $\begin{cases} -4y + 8x = 16 \\ 3y + 21x = 15 \end{cases}$



7. $\begin{cases} x + 2y = -1 \\ 3x + 5y = -1 \end{cases}$



8. $\begin{cases} y = 2x - 3 \\ x + y = -5 \end{cases}$



9. Tickets to a show cost \$10 in advance and \$15 at the door. If 120 tickets are sold for a total of \$1390, how many of the tickets were bought in advance?

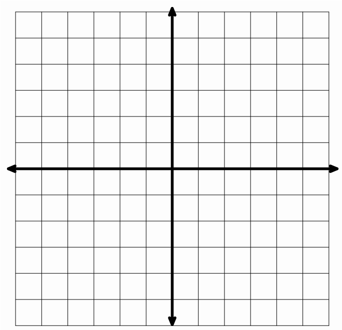
Name:

Go

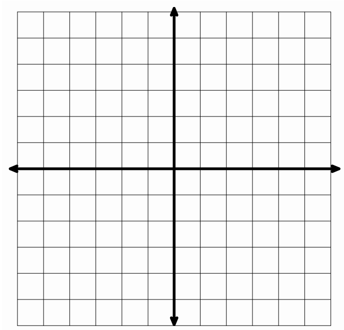
Topic: Graph two variable inequalities

Graph the following inequalities.

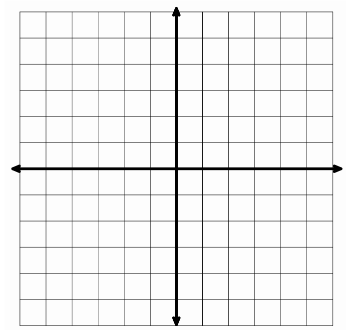
10. $y \leq 3x - 4$



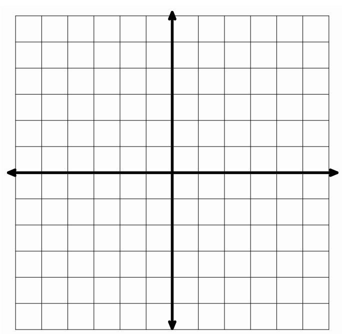
11. $y \leq -2x + 3$



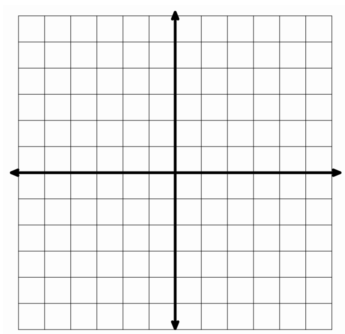
12. $y \geq 4x - 3$



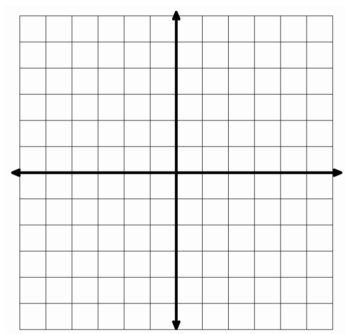
13. $3x + 4y < 12$



14. $6x + 8y \leq 24$



15. $5x + 43 \leq 15$



Need help? Check out these related videos.

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-systems-by-substitution-3

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/solving-and-graphing-linear-inequalities-in-two-variables-1

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-inequalities-2

2.7 Shopping for Cats and Dogs

A Develop Understanding Task

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Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more that one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases. See if you can help him figure out the cost of particular items for each purchase, and be prepared to explain your reasoning to Carlos.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Lucky Licks* for $42.50. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Lucky Licks* for $94.25. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

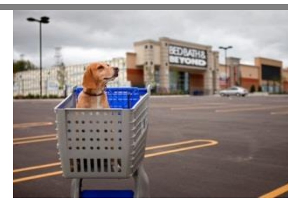
3. Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. Based on this information, figure out the price of each item. Explain your reasoning.

4. One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $18.50. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $1.00 for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Summarize the strategies you have used to reason about the price of individual items in the problems given above. What are some key ideas that seem helpful?

Name: Systems 2.7



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Ready, Set, Go!

Ready

Topic: Exponents

Write the following in exponential notation.

1. $4 \times 4 \times 4 \times 4 \times 4$
2. $3x \cdot 3x \cdot 3x \cdot 3x$

Find each value.

3. 2^3
4. 3^3
5. 2^5
6. $(-2)^3$
7. 4^3

Set

Topic: Solving systems

8. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for \$2.84. Peter also buys three candy bars, but can only afford one additional fruit roll-up. His purchase costs \$1.79. What is the cost of a candy bar and a fruit roll-up individually?

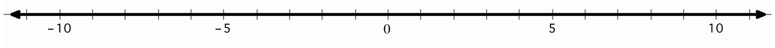
9. A farmer noticed that his chickens were loose and were running around with the cows in the cow pen. He quickly counted 100 heads and 270 legs. How many chickens did he have and how many cows?

Go

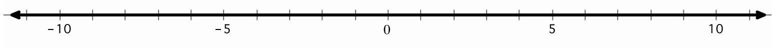
Topic: Solve one variable inequalities.

Solve the following inequalities. Write the solution set in *interval notation* and graph the solution set on a number line.

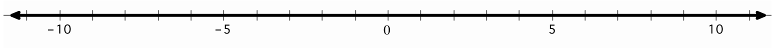
10. $4x + 10 < 2x + 14$



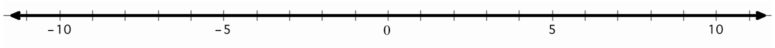
11. $2x + 6 > 55 - 5x$



12. $2(\frac{x}{4} + 3) > 6(x - 1)$



13. $9x + 4 \leq -2(x + \frac{1}{2})$



Solve each inequality. Give the solution in *inequality notation* and *set notation*.

14. $-\frac{x}{3} > -\frac{10}{9}$

15. $5x > 8x + 27$

16. $\frac{x}{4} > \frac{5}{4}$

17. $3x - 7 \geq 3(x - 7)$

18. $2x < 7x - 36$

19. $5 - x < 9 + x$

Need help? Check out these related videos?

Exponential notation:
http://www.khanacademy.org/math/algebra/exponents-radicals/v/understanding-exponents

Solving inequalities:
http://www.khanacademy.org/math/algebra/solving-linear-inequalities/v/solving-inequalities
http://www.khanacademy.org/math/algebra/solving-linear-inequalities/v/multi-step-inequalities-2

Set notation and interval notation:
http://patrickjmt.com/using-interval-notation-to-express-inequalities-ex1/

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2.8 Can You Get to the Point, Too?

A Solidify Understanding Task

Part 1

In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

Part 2

Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how each scenario in "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

Part 3

Carlos also realized that the algebraic strategy he created in part 1 could be used to find the points of intersection for the "Pet Sitters" constraints. Use the **elimination of variables** method developed in part 1 to find the point of intersection for each of the following pairs of "Pet Sitter" constraints.

- *Start-up costs* and *space* constraints
- *Pampering time* and *feeding time* constraints
- Any other pair of "Pet Sitter" constraints of your choice

Name: Systems | 2.8

Ready, Set, Go!

Ready

Topic: Evaluate exponents

Simplify and evaluate the following.

1. 3^{-2}

2. $(0.5)^{-2}$

3. 2^{4}

4. 4^{-2}

Write the following expression three different ways (one way can include the simplified value).

5. $(2^3)(4)$

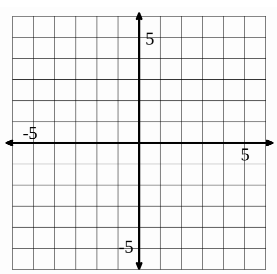
6. $(3^3)(2^3)$

Set

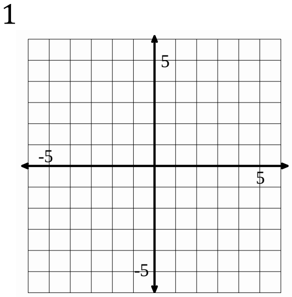
Topic: Solve systems of equations

Solve the following systems of equations using *elimination* of variables, then justify graphically.

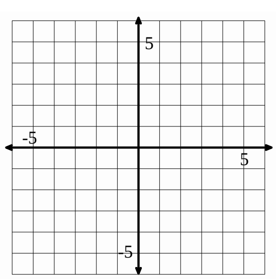
7. $\begin{cases} 2x + 0.5y = 3 \\ x + 2y = 8.5 \end{cases}$



8. $\begin{cases} 3x + 5y = -1 \\ x + 2y = -1 \end{cases}$



9. $\begin{cases} 3x + 5y = -3 \\ x + 2y = -\dfrac{4}{3} \end{cases}$



10. A 150-yard pipe is cut to provide drainage for two fields. If the length of one piece (a) is three yards less than twice the length of the second piece (b), what are the lengths of the two pieces?

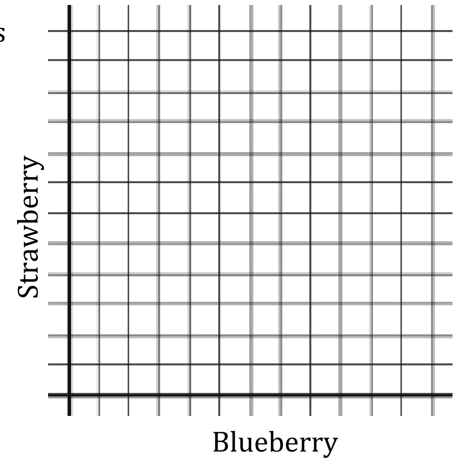
Name: Systems | 2.8

Go

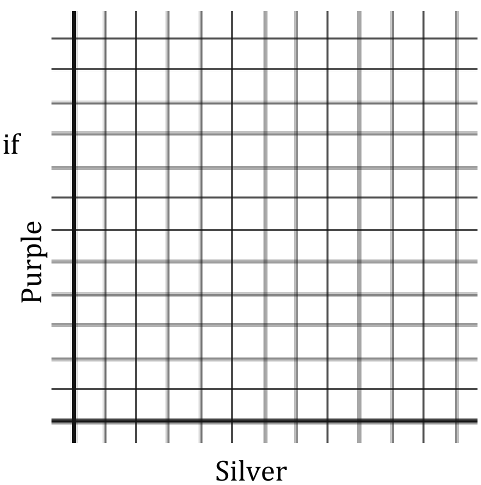
Topic: Graph two variable linear inequalities

Graph the following linear inequalities on the graphs below. Include constraints.

11. Ben has enough money to buy up to eight yogurts. If his favorite flavors are blueberry and strawberry, what are all the possible combinations he can buy? Graph the inequality that shows all possible combinations of his favorite flavors.



12. Peggy is buying a balloon bouquet. Her favorite colors are silver and purple. The silver balloons are $1 and the purple balloons are $0.80. Graph an inequality that shows how many of each color balloon she can put in her bouquet if she doesn't spend more than $20.



Need help? Check out these related videos.

Negative exponents
http://patrickjmt.com/negative-exponents/
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/zero--negative--and-fractional-exponents

Solving systems by elimination
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-systems-by-elimination-2

Solving systems by graphing
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-linear-systems-by-graphing

2.9 Food for Fido and Fluffy

A Solidify Understanding Task

Carlos and Clarita have found two different cat foods that seem to appeal to even the most finicky of cats: *Tabitha Tidbits* and *Figaro Flakes.* Each ounce of *Tabitha Tidbits* contains 2 grams of protein, 4 grams of carbohydrates and 4 grams of fat. Each ounce of *Figaro Flakes* contains 3 grams of protein, 4 grams of carbohydrates and 2 grams of fat. Since *Tabitha Tidbits* is fairly expensive, while *Figaro Flakes* is very cheap, the twins have decided to create a new cat food by mixing the two. After studying some nutritional guidelines for cats, Carlos and Clarita have decided to create a mixture based on the following constraints.

- *Amount of Protein*: Each meal should contain at least 12 grams of protein.
- *Amount of Carbohydrates:* Each meal should contain more than 16 grams of carbohydrates.
- *Amount of Fats:* Each meal should contain no more than 18 grams of fat.
- *Size of a Feeding:* Each meal should consist of less than 10 ounces of food.

For the work that follows, let *T* represent the number of ounces of *Tabitha Tidbits* in a meal and let *F* represent the number of ounces of *Figaro Flakes.*

1. Write an inequality for each of the constraints.

2. On separate coordinate grids, graph the solution set for each of the inequalities you wrote in #1. How do you know on which side of the boundary line you should shade the half-plane that represents the solution set?

3. Decide if the boundary line for each inequality represented in #2 should be a solid line or a dotted line. Which words or phrases in the constraints suggested a solid line? A dotted line?

4. Find at least 5 combinations of *Tabitha Tidbits* and *Figaro Flakes* Carlos and Clarita can mix together to create a nutritious cat meal. Show that these points lie within a feasible region for these constraints.

5. *Brutus Bites* is a brand of dog food that contains 4 grams of protein and 6 grams of fat per ounce. *Lucky Licks* is another brand of dog food that contains 12 grams of protein and 4 grams of fat per ounce. Carlos wants to make a meal for dogs that contains at least 8 grams of protein and no more than 6 grams of fat. Write and solve a system of inequalities that Carlos can use to determine a combination of *Brutus Bites* and *Lucky Licks* that will satisfy these constraints.

Name: Systems | 2.9

Ready, Set, Go!

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Ready

Topic: Solving two variable inequalities

1. A theater wants to take in at least $2000 for a certain matinee. Children's tickets cost $5 each and adult tickets cost $10 each.

a. Write an inequality describing the number of tickets that will allow the theater to meet their goal of $2000.

b. If the theater has a maximum of 350 seats, write an inequality describing the number of both types of tickets the theater can sell.

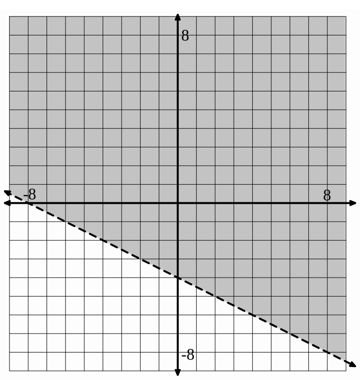
c. Find the number of children and adult tickets that can be sold so that all seats are sold and the $2000 goal is reached.

Set

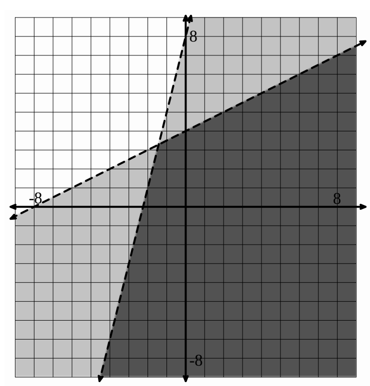
Topic: Writing equations of two variable inequalities

Given the graph with the regions that are shaded write the inequality or system of inequalities.

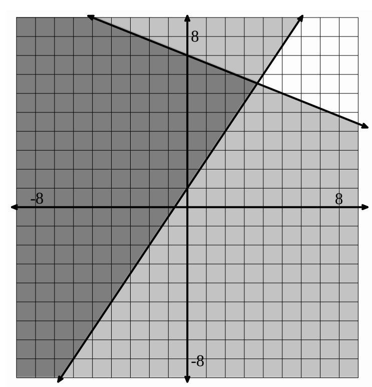
2.



3.



4.

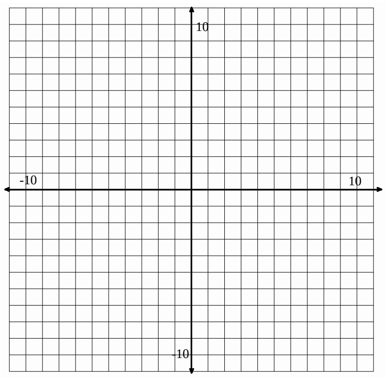


Go

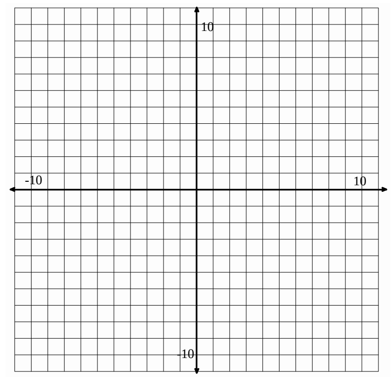
Topic: Graph two variable inequalities

Graph each set of inequalities below. Include the shaded region of both, plus indicate the region that is true for all inequalities.

5. $\begin{cases} x - y < -6 \\ 2y \geq 3x + 18 \end{cases}$



6. $\begin{cases} 5x - y \geq 5 \\ 2y - x \geq -10 \end{cases}$



Solve the following systems of equations.

7. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for \$2.84. Peter also buys three candy bars, but can only afford one additional fruit roll-up. His purchase costs \$1.79. What is the cost of a candy bar and a fruit roll-up individually?

8. $\begin{cases} 5x - 10y = 15 \\ 3x - 2y = 3 \end{cases}$

9. $\begin{cases} 5x - y = 10 \\ 3x - 2y = -1 \end{cases}$

Need help? Check out these related videos.

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/graphing-systems-of-inequalities-2

2.10 Taken Out of Context

A Practice Understanding Task

Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x + 4y = 23 \\ 5x + 3y = 31 \end{cases}$

2. $\begin{cases} 2x + 3y = 14 \\ 4x + 6y = 28 \end{cases}$

3. $\begin{cases} 3x + 2y = 20 \\ 9x + 6y = 35 \end{cases}$

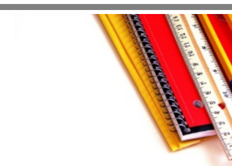
4. $\begin{cases} 4x + 2y = 8 \\ 5x + 3y = 9 \end{cases}$

Three of Carlos' and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, three packages of pencils and two markers for $7.50. Jan buys two notebooks, six packages of pencils and five markers for $15.50. Fran buys a notebook, two packages of pencils and two markers for $6.25. How much do each of these three items cost?

Explain in words or with symbols how you can use your intuitive reasoning about these purchases to find the price of each item.

Name: Systems | 2.10

Ready, Set, Go!



Ready

Topic: Systems of Inequalities

For each of the systems of inequalities, determine if the given coordinates are solutions to the system.

1. $\begin{cases} y \le 3x - 5 \\ y \ge x + 2 \end{cases}$

   a. (6 , 10)

   b. (1 , 4)

   c. (8 , 15)

2. $\begin{cases} y > -2x + 9 \\ y \ge 5x - 6 \end{cases}$

   a. (-2 , -5)

   b. (-1 , 12)

   c. (5 , 0)

3. $\begin{cases} y < -\dfrac{1}{2}x + 9 \\ y > 6x - 10 \end{cases}$

   a. (-2 , -5)

   b. (7 , 3)

   c. (-8 , 10)

Set

Topic: Determine the number of solutions in a system of equations

Express each equation in slope-intercept form. *Without graphing*, state whether the system of equations has zero, one or infinite solutions. How do you know?

4. $\begin{cases} 3x - 4y = 13 \\ y = -3x - 7 \end{cases}$

5. $\begin{cases} 3x - 3y = 3 \\ x - y = 1 \end{cases}$

6. $\begin{cases} 0.5x - y = 30 \\ 0.5x - y = -30 \end{cases}$

7. $\begin{cases} 4x - 2y = -2 \\ 3x + 2y = -12 \end{cases}$

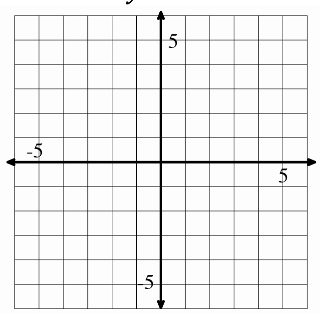
Name: Systems 2.10

Go

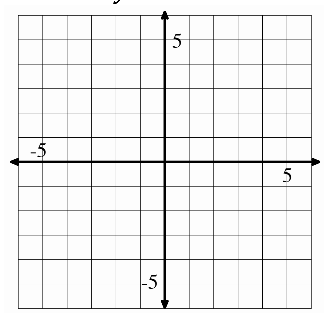
Topic: Graph two variable inequalities

Graph the following inequalities. Be sure to label your axes and scale. Justify the region you shade by showing three points in the region as being solutions to the problem. Show a point you have tested to prove your shaded region is accurate.

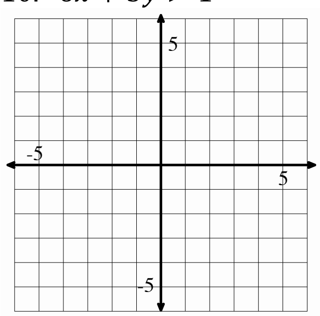
8. $3x - 4y \geq 12$



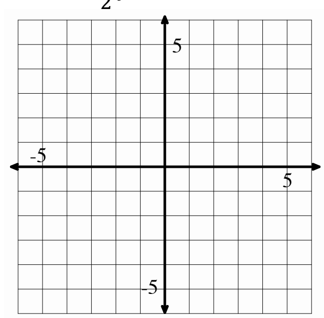
9. $x + 6y < 6$



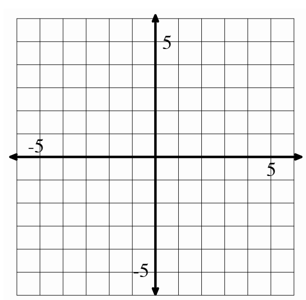
10. $6x + 5y > 1$



11. $x - \frac{1}{2}y \geq 3$



12. On the same set of axes, graph $y < x + 2$ and $y > x + 5$.
What values do these two have in common?



Need help? Check out these related videos

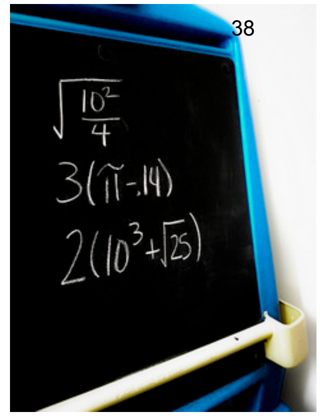
Testing a solution to an equation
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/testing-a-solution-for-a-system-of-equations

Number of solutions
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/special-types-of-linear-systems

Solving inequalities
http://www.khanacademy.org/math/algebra/solving-linear-inequalities/v/solving-inequalities

2.11 More Things Taken Out of Context

A Practice Understanding Task



Solve the following systems of inequalities:

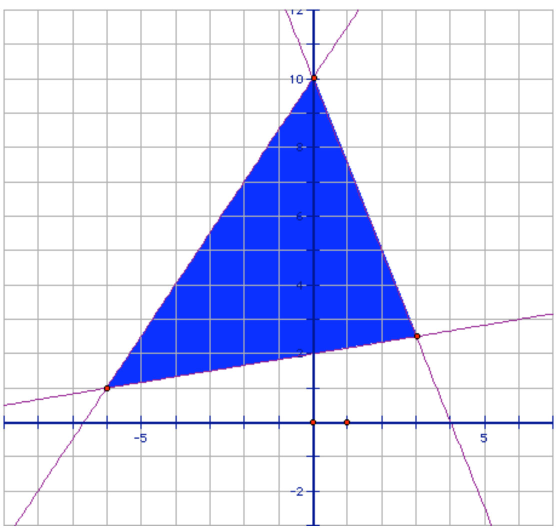
1. $\begin{cases} -5x+3y \le 45 \\ 2x+3y > 24 \end{cases}$

2. $\begin{cases} -10x+6y \le 90 \\ 6x+9y > 36 \end{cases}$

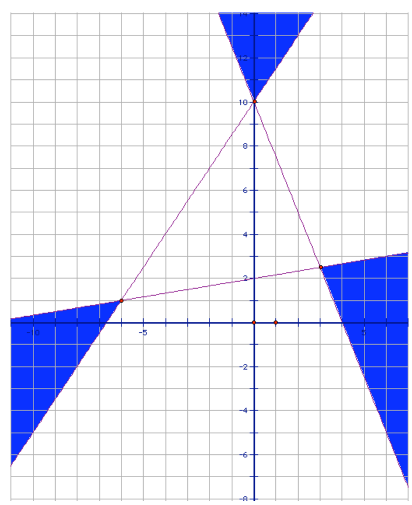
3. Is the point (-3, 10) a solution to the system in problem #1? Why or why not?

4. How are the inequalities representing the boundaries of the solution sets in problems #1 and #2 similar to each other? What accounts for these similarities?

5. Write the system of inequalities whose solution set is shown below:

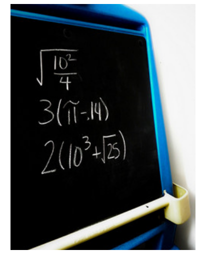


6. Amanda is examining Frank's work on #5, when she exclaims, "You have written all of your inequalities backwards. The solution set to your system would look like this."



What do you think about Amanda's statement?

Name: More Things Taken out of Context **2.11**



Ready, Set, Go!

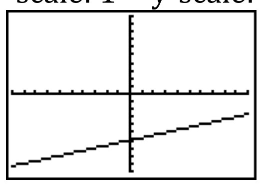
Ready

Topic: Determine a good viewing window for graphs

When sketching a graph of a function, it is important that we see important points. For linear functions, we want a window that shows important information related to the story. Often, this means including both the x- and y-intercepts.

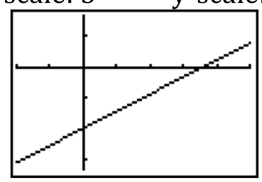
Example: $g(x) = \frac{1}{3}x - 6$

Window: [-10, 10] by [-10,10]
x- scale: 1 y-scale: 1



NOT a good window

Window: [-10, 25] by [-10, 5]
x-scale: 5 y-scale: 5



Good window

For the following equations, state a window that would be satisfactory for the given equation. Then sketch a graph in the boxes provided.

1. $f(x) = 3x - 100$

x: [,] by y: [,]

x-scale: y-scale:

2. $5x + 7y = 15$

x: [,] by y: [,]

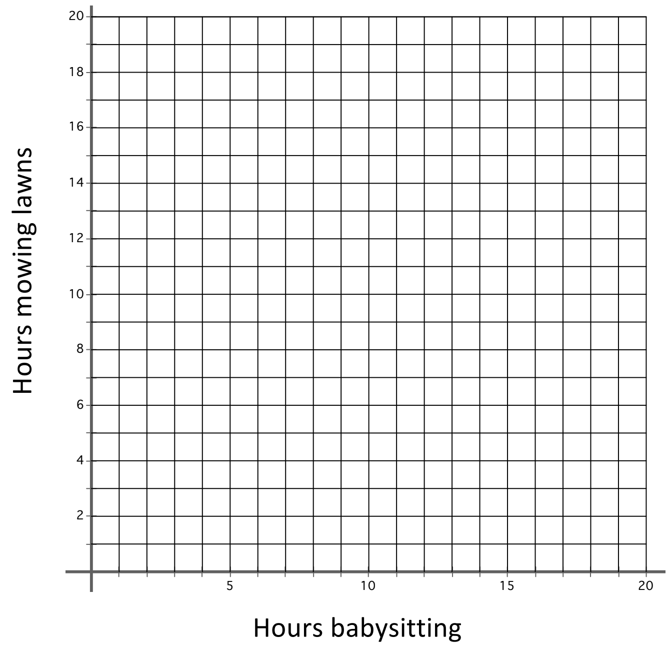
x-scale: y-scale:

Name: More Things Taken out of Context 2.11

Set

Topic: Creating and solving two variable inequalities

3. Patty makes $8 per hour mowing lawns and $12 per hour babysitting. She wants to make at least $100 per week but can work no more than 12 hours a week. Write and graph a system of linear inequalities. Finally, list 2 possible combinations of hours that Patty could work at each job.



Go

Topic: Solve systems of equations

Solve each system of equations using any method you prefer

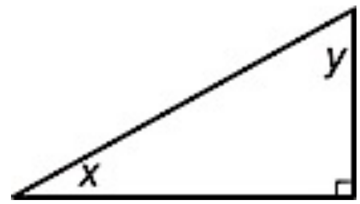
4. $\begin{cases} 3x + 5y = -3 \\ x + 2y = -\frac{4}{3} \end{cases}$

5. $\begin{cases} x - y = -\frac{12}{5} \\ 2x + 5y = -2 \end{cases}$

Name: More Things Taken out of Context | 2.11

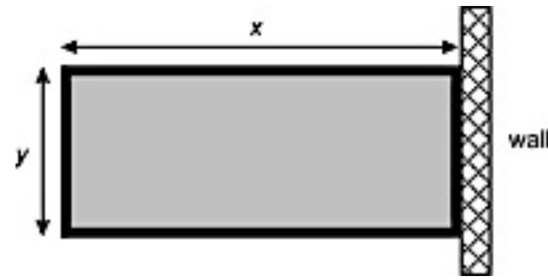
Create a system of equations and use it to solve the following questions.

6. Of the two non-right angles in a right triangle, one measures twice as many degrees as the other. What are the angles?

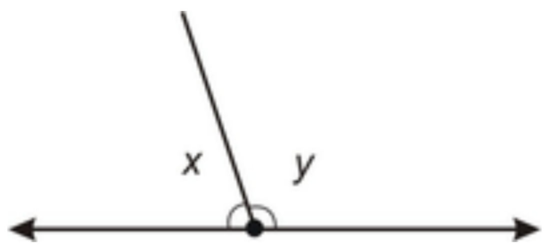


7. The sum of two numbers is 70 and the difference is 11. What are the numbers?

8. A rectangular field is enclosed by a fence on three sides and a wall on the fourth side. The total length of the fence is 320 yards. If the field has a total perimeter of 400 yards, what are the dimensions of the field?



9. A ray cuts a line forming two angles. The difference between the two angles is $18°$. What does each angle measure?



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/system-of-inequalities-application

2.12 Pet Sitters Revisited

A Develop Understanding Task

Carlos and Clarita have successfully found a way to represent *all* of the combinations of cats and dogs that they can board based on *all* of the following constraints.

- *Space*: Cat pens will require 6 ft^2 of space, while dog runs require 24 ft^2. Carlos and Clarita have up to 360 ft^2 available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Feeding Time*: Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.

- *Pampering Time*: The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Now they are trying to determine how many of each type of pet they should plan to accommodate. Of course, Carlos and Clarita want to make as much money as possible from their business, so they need to pay attention to both their daily income as well as their daily costs. They plan to charge $8 per day for boarding each cat and $20 per day for each dog. They estimate that each cat will require $2.00 per day in food and supplies, and that each dog will require $4.00 per day in costs.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for in order to make as much money as possible?

What combination of cats and dogs do you think will make the most money? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

To get started on this task, you might want to look for collections of points where the daily profit is the same. For example, can you find a collection of points where for each point the daily profit is $120? What about $180?

Name:

Ready, Set, Go!

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Ready

Topic: Solve exponential equations

Find the value of *x* for each situation.

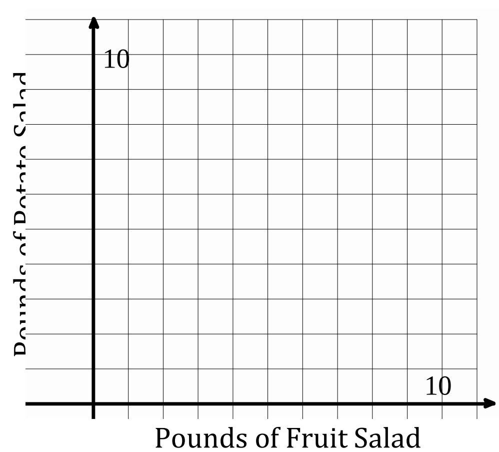
1. $2^x = 8$
2. $3^x = 27$
3. $2^x = 4$
4. $(-2)^x = -8$

Set

Topic: Create and solve two variable inequalities

5. Jane is buying fruit salad and potato salad for a picnic. Fruit salad costs $2.00 per pound and potato salad costs $4.00 per pound. Jane needs to buy at least 6 pounds of salads and she doesn't want to spend more than $20. Write and graph a system of linear inequalities. Also, list 2 possible combinations of salad Jane could buy.

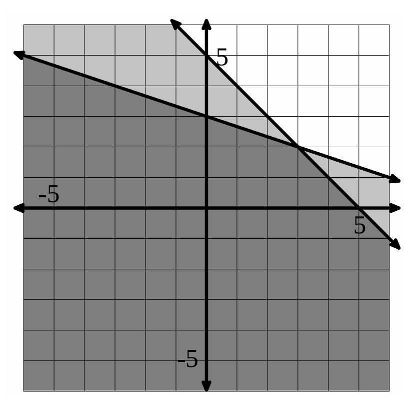
Let x = pounds of fruit salad and
y = pounds of potato salad.



Go

Topic: Find the solution region of the following systems of inequalities.

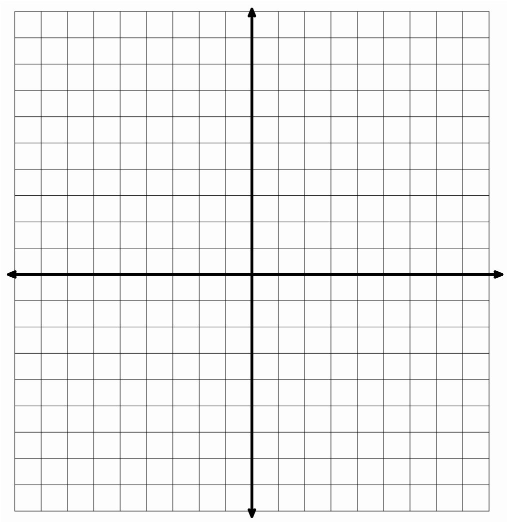
6. Write the system of inequalities that is represented in the graph to the right.



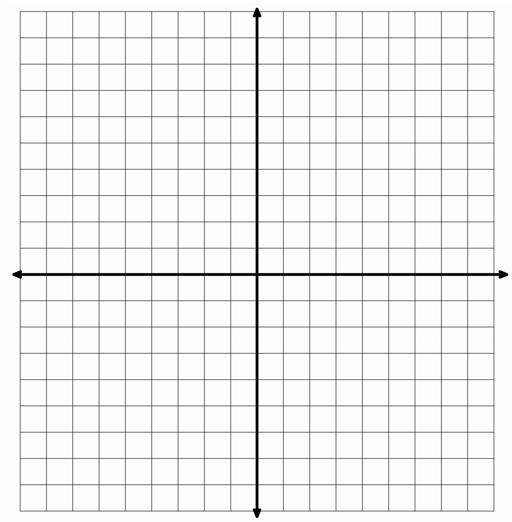
Name: Systems | 2.12

Graph each set of inequalities and determine the solution region.

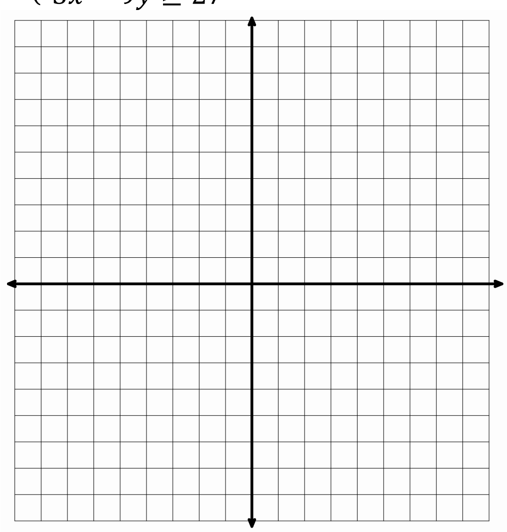
7. $\begin{cases} x - y < -6 \\ -2y \geq 3x - 18 \end{cases}$



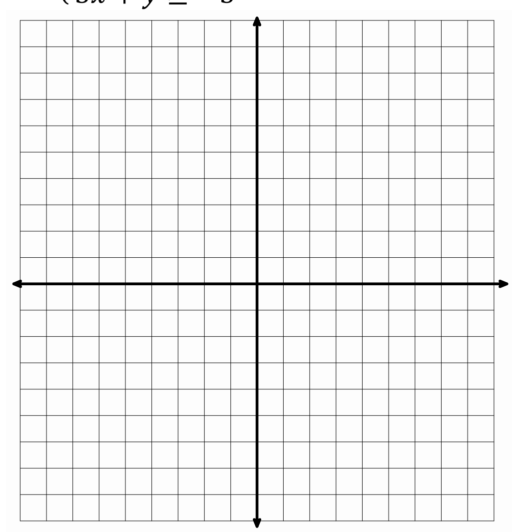
8. $\begin{cases} 5x - y \geq 5 \\ 2y - x \geq 10 \end{cases}$



9. $\begin{cases} 5x + 2y \geq -10 \\ 3x - 2y \leq 18 \\ 3x - 9y \geq 27 \end{cases}$



10. $\begin{cases} 2x - 3y \leq 24 \\ x + 4y \leq 8 \\ 3x + y \geq -3 \end{cases}$



Need help? Check out these related videos.

Exponents http://patrickjmt.com/exponents-intro-to-evaluating-a-few-truefalse-questions/

Rules for exponents http://patrickjmt.com/basic-exponent-properties/

Solving a system of inequalities http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/systems-of-linear-inequalities

Secondary Mathematics I: An Integrated Approach
Module 3
Arithmetic and Geometric Sequences

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 3 – Arithmetic and Geometric Sequences

3.1 Classroom Task: Growing Dots- A Develop Understanding Task
Representing arithmetic sequences with equations, tables, graphs, and story context
Ready, Set, Go Homework: Sequences 3.1

3.2 Classroom Task: Growing, Growing Dots – A Develop Understanding Task
Representing geometric sequences with equations, tables, graphs, and story context
Ready, Set, Go Homework: Sequences 3.2

3.3 Classroom Task: Scott's Workout – A Solidify Understanding Task
Arithmetic sequences: Constant difference between consecutive terms
Ready, Set, Go Homework: Sequences 3.3

3.4 Classroom Task: Don't Break the Chain – A Solidify Understanding Task
Geometric Sequences: Constant ratio between consecutive terms
Ready, Set, Go Homework: Sequences 3.4

3.5 Classroom Task: Something to Chew On – A Solidify Understanding Task
Arithmetic Sequences: Increasing and decreasing at a constant rate
Ready, Set, Go Homework: Sequences 3.5

3.6 Classroom Task: Chew On This – A Solidify Understanding Task
Comparing rates of growth in arithmetic and geometric sequences
Ready, Set, Go Homework: Sequences 3.6

3.7 Classroom Task: What Comes Next? What Comes Later? – A Solidify Understanding Task
Recursive and explicit equations for arithmetic and geometric sequences
Ready, Set, Go Homework: Sequences 3.7

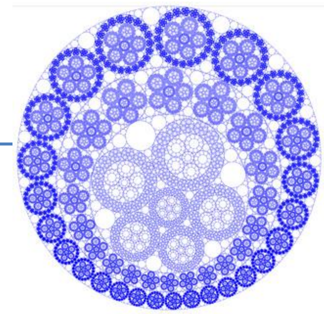
3.8 Classroom Task: What Does It *Mean*? – A Solidify Understanding Task
Using rate of change to find missing terms in an arithmetic sequence
Ready, Set, Go Homework: Sequences 3.8

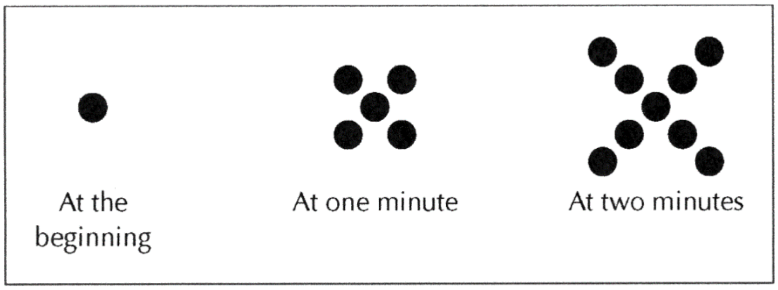
3.9 Classroom Task: Geometric *Mean*ies – A Solidify and Practice Understanding Task
Using a constant ratio to find missing terms in a geometric sequence
Ready, Set, Go Homework: Sequences 3.9

3.10 Classroom Task: I Know . . . What Do You Know? – A Practice Understanding Task
Developing fluency with geometric and arithmetic sequences
Ready, Set, Go Homework: Sequences 3.10

3.1Growing Dots

A Develop Understanding Task





1. Describe the pattern that you see in the sequence of figures above.

2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes?

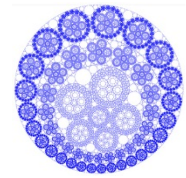
3. How many dots are there at 100 minutes?

4. How many dots are there at t minutes?

Solve the problems by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

Name: Sequences | 3.1

Ready, Set, Go!



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Ready

Topic: Exponents, Substitution, and Function Notation

Find each value.

1. 3^1
2. 3^2
3. 3^3
4. 3^4

For each of the following, find $f(1), f(2)$ and $f(3)$

5. $f(x) = 2^x$
6. $f(x) = 3^x$
7. $f(x) = 2(x-1)+3$

Complete each table.

8.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32			

9.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18				

10.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-3	9	-27	81				

11.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20				

12.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12				

Name: Sequences | 3.1

Set

Topic: Completing a table

Fill in the table. Then write a sentence explaining how you figured out the values to put in each cell. Explain how to figure out what will be in cell #8.

13. You run a business making birdhouses. You spend $600 to start your business, and it costs you $5.00 to make each birdhouse.

# of birdhouses	1	2	3	4	5	6	7
Total cost to build							

Explanation:

14. You borrow $500 from a relative, and you agree to pay back the debt at a rate of $15 per month.

# of months	1	2	3	4	5	6	7
Amount of money owed							

Explanation:

15. You earn $10 per week.

# of weeks	1	2	3	4	5	6	7
Amount of money earned							

Explanation:

16. You are saving for a bike and can save $10 per week. You have $25 already saved.

# of weeks	1	2	3	4	5	6	7
Amount of money saved							

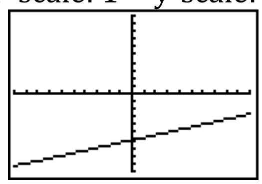
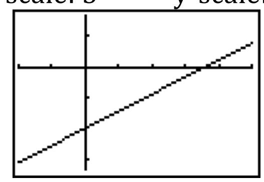
Explanation:

Go

Topic: Good viewing window

When sketching a graph of a function, it is important that we see important points. For linear functions, we want a window that shows important information related to the story. Often, this means including both the x- and y- intercepts.

Example: $g(x) = \frac{1}{3}x - 6$

Window: [-10, 10] by [-10,10] x- scale: 1 y-scale: 1	Window: [-10, 25] by [-10, 5] x-scale: 5 y-scale: 5
	
NOT a good window	Good window

17. $f(x) = -\frac{1}{10}x + 1$

x: [,] by y: [,]

x-scale: y-scale:

18. 7 x – 3 y = 14

x: [,] by y: [,]

x-scale: y-scale:

19. y = 3(x – 5) +12

x: [,] by y: [,]

x-scale: y-scale:

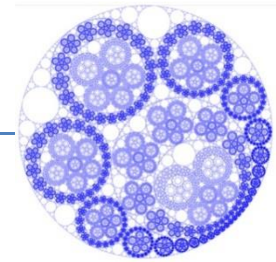
20. $f(x) = -15(x + 10) - 45$

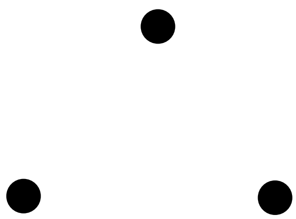
x: [,] by y: [,]

x-scale: y-scale:

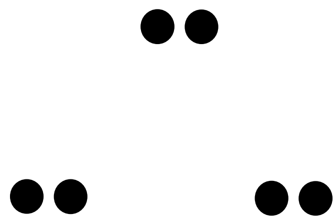
3.2 Growing, Growing Dots

A Develop Understanding Task

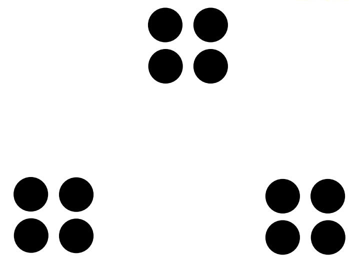




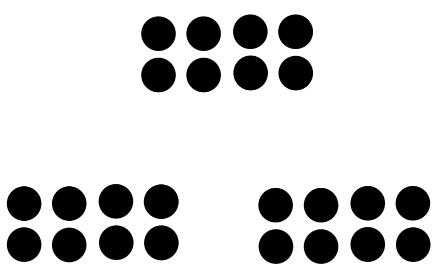
At the beginning



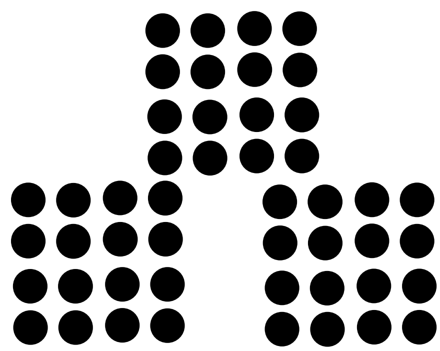
At one minute



At two minutes



At three minutes

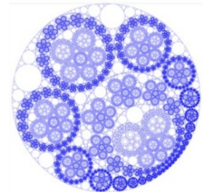


At four minutes

1. Describe and label the pattern of change you see in the above sequence of figures.
2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?
3. Write a recursive formula to describe how many dots there will be after t minutes.
4. Write an explicit formula to describe how many dots there will be after t minutes.

Name: Sequences | 3.2

Ready, Set, Go!



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Ready

Topic: Finding values for a pattern

1. Bob Cooper was born in 1900. By 1930 he had 3 sons, all with the Cooper last name. By 1960 each of Bob's 3 boys had exactly 3 sons of their own. By the end of each 30 year time period, the pattern of each Cooper boy having exactly 3 sons of their own continued. How many Cooper sons were born in the 30 year period between 1960 and 1990?

2. Create a diagram that would show this pattern.

3. Predict how many Cooper sons will be born between 1990 and 2020, if the pattern continues.

4. Try to write an equation that would help you predict the number of Cooper sons that would be born between 2020 and 2050. If you can't find the equation, explain it in words.

Set

Topic: Evaluating Equations

Evaluate the following equations when x = { 1, 2, 3, 4, 5 }. Organize your inputs and outputs into a table of values for each equation. Let x be the input and y be the output.

5. $y = 4^x$

x	y
1	
2	
3	
4	
5	

6. $y = (-3)^x$

x	y
1	
2	
3	
4	
5	

7. $y = -3^x$

x	y
1	
2	
3	
4	
5	

8. $y = 10^x$

x	y
1	
2	
3	
4	
5	

Go

Topic: Solve equations

Solve the following equations for the unknown variable. Check your answer.

9. $3(x-1)=2(x+3)$

10. $7(x+20)=x+5$

11. $9(n-2)=3n+3$

12. $2\left(a-\frac{1}{3}\right)=\frac{2}{5}(a+\frac{2}{3})$

13. $3(t+3)-2(t-1)=0$

14. $6(z+3)-5(3z+2)=2(2z-9)$

Need help? Check out these related videos.

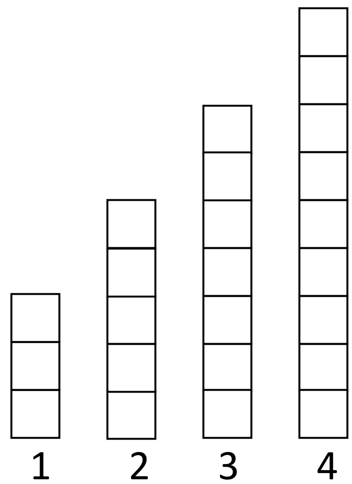
Evaluating with exponents
http://www.khanacademy.org/math/algebra/exponents-radicals/v/level-1-exponents

Solving equations
http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-equations-with-the-distributive-property

3.3 Scott's Workout

A Solidify Understanding Task

Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



1. How many push-ups will Scott do on day 10?
2. How many push-ups will Scott do on day *n*?
3. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.
4. Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.

Name: Sequences 3.3

Ready, Set, Go!

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Ready

Topic: Slopes between two points

Find the slope of the line that goes through each set of points.

1. (3,7) and (5, 10)
2. (-1, 4) and (3,3)
3. (0,0) and (-2, 5)
4. (-1, -5) and (-4, -5)

Set

Topic: Finding terms for a given sequence

Find the next 3 terms in each sequence. Identify the constant difference. Write a recursive function and an explicit function for each sequence. (The first number is the 1st term, not the 0th). Circle the constant difference in both functions.

4. 3 , 8 , 13 , 18 , 23 , _____ , _____ , _____ , … Constant Difference: __________

   Recursive Function: ________________________ Explicit Function:__________________________

5. 11 , 9 , 7 , 5 , 3 , _____ , _____ , _____ , … Constant Difference: __________

   Recursive Function: ________________________ Explicit Function:__________________________

6. 3 , 1.5 , 0 , -1.5 , -3 , _____ , _____ , _____ , … Constant Difference: __________

   Recursive Function: ________________________ Explicit Function:__________________________

Go

Topic: Slope-Intercept Form

Write the equations in slope-intercept form.

7. $y = 12 + (x - 1)(-4)$

8. $\frac{2}{3}(6y + 9) = \frac{3}{5}(15x - 20)$

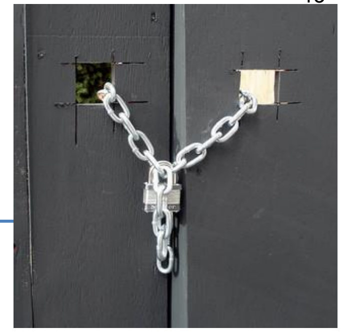
9. $\frac{5}{7}(21y + 7) = \frac{2}{9}(18x + 27)$

Need Help? Check out these related videos:

Finding slope
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/slope-and-rate-of-change

Writing the explicit equation
http://www.khanacademy.org/math/algebra/solving-linear-equations/v/equations-of-sequence-patterns

Writing equations in slope-intercept form
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/converting-to-slope-intercept-form



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3.4 Don't Break the Chain

A Solidify Understanding Task

Maybe you've received an email like this before:

> Hi! My name is Bill Weights, founder of Super Scooper Ice Cream. I am offering you a gift certificate for our signature "Super Bowl" (a $4.95 value) if you forward this letter to 10 people.
>
> When you have finished sending this letter to 10 people, a screen will come up. It will be your Super Bowl gift certificate. Print that screen out and bring it to your local Super Scooper Ice Cream store. The server will bring you the most wonderful ice cream creation in the world—a Super Bowl with three yummy ice cream flavors and three toppings!
>
> This is a sales promotion to get our name out to young people around the country. We believe this project can be a success, but only with your help. Thank you for your support.
>
> Sincerely,
>
> Bill Weights
> Founder of Super Scooper Ice Cream

These chain emails rely on each person that receives the email to forward it on. Have you ever wondered how many people might receive the email if the chain remains unbroken? To figure this out, assume that it takes a day for the email to be opened, forwarded, and then received by the next person. On day 1, Bill Weights starts by sending the email out to his 8 closest friends. They each forward it to 10 people so that on day 2, it is received by 80 people. The chain continues unbroken.

1. How many people will receive the email on day 7?
2. How many people with receive the email on day *n*? Explain your answer with as many representations as possible.
3. If Bill gives away a Super Bowl that costs $4.95 to every person that receives the email during the first week, how much will he have spent?

Name: Sequences 3.4

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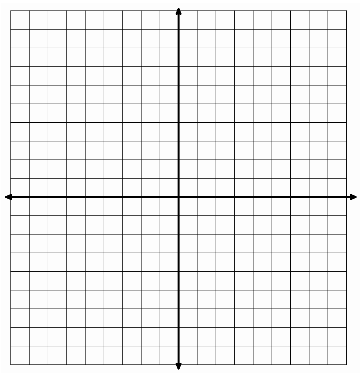
Ready, Set, Go!

Ready

Topic: Write the equation of a line given two points.

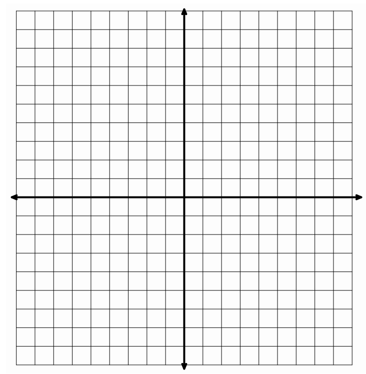
Graph each pair of points, draw a line that goes through both points, and write an equation of that line.

1. (5 , 2) and (7 , 0)



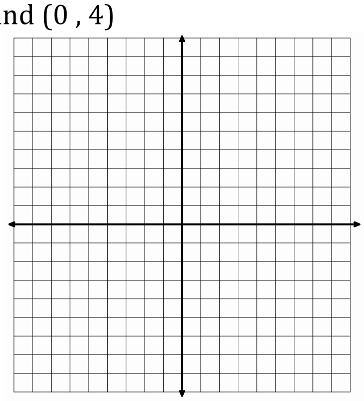
Equation:____________________________

2. (-4 , 2) and (6 , 7)



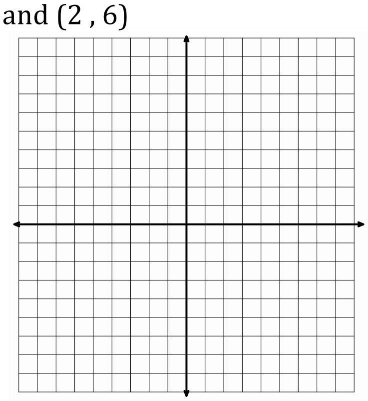
Equation:____________________________

3. (3 , 0) and (0 , 4)



Equation:____________________________

1. (2 , -4) and (2 , 6)



Equation:____________________________

5. Write the equation of the line that passes through the points (2 , 2) and (8 , 8) without the help of a graph.

Set

Topic: Recursive and explicit functions of arithmetic sequences

Below you are given various types of information. Write the recursive and explicit functions for each arithmetic sequence. Finally, graph each sequence, making sure you clearly label your axes.

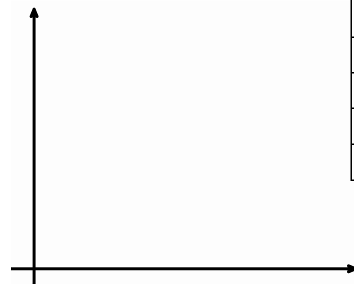
6. 2 , 4 , 6 , 8 , …

Recursive:______________________________

Explicit:________________________________

7.

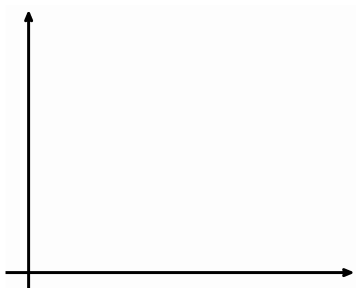
Time (days)	Number of cells
1	3
2	6
3	9
4	12



Recursive:______________________________

Explicit:________________________________

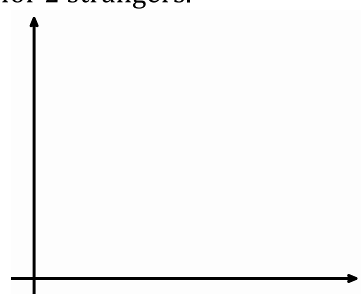
8. Claire has $300 in an account. She decides she is going to take out $25 each month.



Recursive:______________________________

Explicit:________________________________

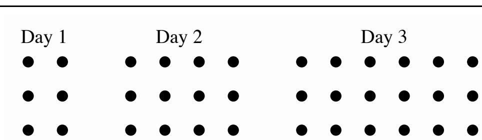
9. Each day Tania decides to do something nice for 2 strangers.



Recursive:______________________________

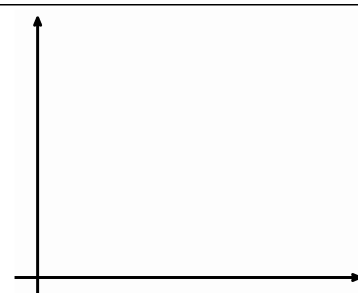
Explicit:________________________________

10.



Recursive:______________________________

Explicit:________________________________

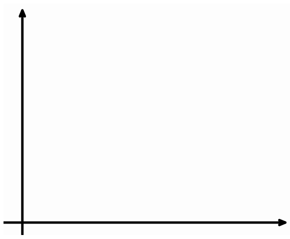


Go

Topic: Recursive and explicit functions of geometric sequences

Below you are given various types of information. Write the recursive and explicit functions for each geometric sequence. Finally, graph each sequence, making sure you clearly label your axes.

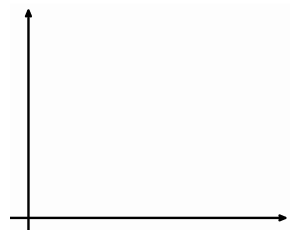
11. 2 , 4 , 8 , 16 , …



Recursive:______________________________

Explicit:________________________________

12.

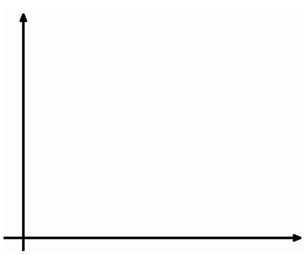


Time (days)	Number of cells
1	3
2	6
3	12
4	24

Recursive:______________________________

Explicit:________________________________

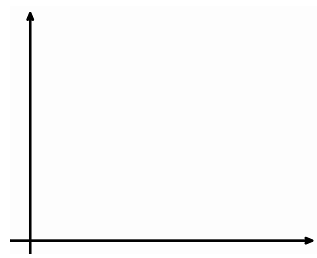
13. Claire has $300 in an account. She decides she is going to take out half of what's left in there at the end of each month.



Recursive:______________________________

Explicit:________________________________

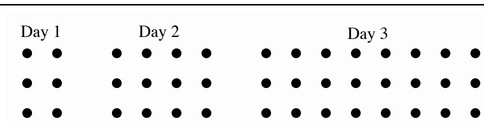
14. Tania creates a chain letter and sends it to four friends. Each day each friend is then instructed to send it to four friends and so forth.



Recursive:______________________________

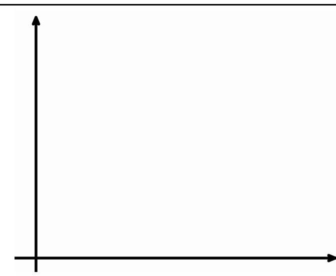
Explicit:________________________________

15.



Recursive:______________________________

Explicit:________________________________



Need Help? Check out these related videos:

Find equation of line http://patrickjmt.com/find-the-equation-of-a-line-using-point-slope-form/

3.5 Something to Chew On

A Solidify Understanding Task

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The Food-Mart grocery store has a candy machine like the one pictured here. Each time a child inserts a quarter, 7 candies come out of the machine. The machine holds 15 pounds of candy. Each pound of candy contains about 180 individual candies.

1. Represent the number of candies in the machine for any given number of customers. About how many customers will there be before the machine is empty?
2. Represent the amount of money in the machine for any given number of customers.
3. To avoid theft, the store owners don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about $25 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?

Name: Sequences | 3.5

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Ready, Set, Go!

Ready

Topic: Finding the constant difference

Find the missing terms for each arithmetic sequence and state the constant difference.

1. 5, 11, ____ , 23, 29, ____...

Constant Difference = ________

2. 7, 3, -1, ____ , ____ , -13...

Constant Difference = ________

3. 8, ____ , ____ , 47, 60...

Constant Difference = ________

4. 0, ____ , ____ , 2, $\frac{8}{3}$ ...

Constant Difference = ________

5. 5, ____ , ____ , ____ , 25...

Constant Difference = ________

6. 3, _____ , _____ , ______ , -13 ...

Constant Difference = ________

Set

Topic: Determine recursive equations

Two consecutive terms in an arithmetic sequence are given. Find the constant difference and the recursive equation.

7. If $f(3) = 5$ *and* $f(4) = 8$. ...

$f(5)$ = ______. $f(6)$ = ______. Recursive Function: ________________________

8. If $f(2) = 20$ *and* $f(3) = 12$.

$f(4)$ = ______. $f(5)$ = ______. Recursive Function: ________________________

9. If $f(5) = 3.7$ *and* $f(6) = 8.7$.

$f(7)$ = ______. $f(8)$ = ______. Recursive Function: ________________________

Go

Topic: Evaluate using function notation

Find each value.

10. $f(n) = 2^n$ Find $f(3)$.

11. $f(n) = 5^n$ Find $f(2)$.

12. $f(n) = (-2)^n$ Find $f(3)$

13. $f(n) = 3 + 4(n-1)$ Find $f(5)$ and $f(6)$.

14. $f(n) = 2(n-1) + 6$ Find $f(1)$ and $f(2)$.

Need Help? Check out these videos:

Arithmetic sequences http://www.khanacademy.org/math/algebra/solving-linear-equations/v/patterns-in-sequences-1

Function notation http://www.youtube.com/watch?v=Kj3Aqov52TY

3.6 Chew on This

A Solidify Understanding Task

Mr. and Mrs. Gloop want their son, Augustus, to do his homework every day. Augustus loves to eat candy, so his parents have decided to motivate him to do his homework by giving him candies for each day that the homework is complete. Mr. Gloop says that on the first day that Augustus turns in his homework, he will give him 10 candies. On the second day he promises to give 20 candies, on the third day he will give 30 candies, and so on.

1. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with his father's plan.
2. Use a formula to find how many candies Augustus will have on day 30 in this plan.

Augustus looks in the mirror and decides that he is gaining weight. He is afraid that all that candy will just make it worse, so he tells his parents that it would be ok if they just give him 1 candy on the first day, 2 on the second day, continuing to double the amount each day as he completes his homework. Mr. and Mrs. Gloop like Augustus' plan and agree to it.

3. Model the amount of candy that Augustus would get each day he reaches his goals with the new plan.
4. Use your model to predict the number of candies that Augustus would earn on the 30^{th} day with this plan.
5. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with this plan.

Augustus is generally selfish and somewhat unpopular at school. He decides that he could improve his image by sharing his candy with everyone at school. When he has a pile of 100,000 candies, he generously plans to give away 60% of the candies that are in the pile each day. Although Augustus may be earning more candies for doing his homework, he is only giving away candies from the pile that started with 100,000. (He's not that generous.)

6. Model the amount of candy that would be left in the pile each day.
7. How many pieces of candy will be left on day 8?
8. When would the candy be gone?

Name: Sequences | 3.6

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Ready, Set, Go!

Ready

Topic: Arithmetic and geometric sequences

Find the missing values for each arithmetic or geometric sequence. Then say if the sequence has a constant difference or a constant ratio, and say what the constant difference/rate is.

1. 5, 10, 15, ___, 25, 30…

Constant difference or a constant ratio?

The constant difference/ratio is ________.

2. 20, 10, ___, 2.5, ___...

Constant difference or a constant ratio?

The constant difference/ratio is ________.

3. 2, 5, 8, ___, 14, ___...

Constant difference or a constant ratio?

The constant difference/ratio is ________.

4. 30, 24, ___, 12, 6…

Constant difference or a constant ratio?

The constant difference/ratio is ________.

Set

Topic: Recursive and explicit equations

Determine whether each situation represents an arithmetic or geometric sequence and then find the recursive and explicit equation for each.

5. 2 , 4 , 6 , 8 , ...

Arithmetic or Geometric?

Recursive: ____________________________

Explicit: ______________________________

6. 2 , 4 , 8 , 16 , ...

Arithmetic or Geometric?

Recursive: ____________________________

Explicit: ______________________________

7.

Time (days)	Number of Dots
1	3
2	7
3	11
4	15

Arithmetic or Geometric?

Recursive: ______________________________

Explicit: ______________________________

8.

Time (days)	Number of cells
1	5
2	8
3	12.8
4	20.48

Arithmetic or Geometric?

Recursive: ______________________________

Explicit: ______________________________

9. Michelle likes chocolate but it causes acne. She chooses to limit herself to three pieces of chocolate every five days.

Arithmetic or Geometric?

Recursive: ______________________________

Explicit: ______________________________

10. Scott decides to add running to his exercise routine and runs a total of one mile his first week. He plans to double the number of miles he runs each week.

Arithmetic or Geometric?

Recursive: ______________________________

Explicit: ______________________________

11. Vanessa has $60 to spend on rides at the State Fair. Each ride cost $4.

Arithmetic or Geometric?

Recursive: ______________________________

Explicit: ______________________________

12. Cami invested $6,000 dollars into an account that earns 10% interest each year.

Arithmetic or Geometric?

Recursive: ______________________________

Explicit: ______________________________

Go

Topic: Solving systems of linear equations

Solve the system of equations.

15. $\begin{cases} y = 2x - 10 \\ x - 4y = 5 \end{cases}$

16. $\begin{cases} x - 7y = 6 \\ -3x + 21y = -18 \end{cases}$

17. $\begin{cases} 5x - 4y = 3 \\ 6x + 4y = 30 \end{cases}$

18. $\begin{cases} 2x - 3y = -12 \\ -x + 2y = 4 \end{cases}$

Need help? Check out these related videos

Arithmetic and geometric sequences http://www.youtube.com/watch?v=THV2Wsf8hro

3.7 What Comes Next? What Comes Later?

A Practice Understanding Task

For each of the following tables,

- describe how to find the next term in the sequence,
- write a recursive rule for the function,
- describe how the features identified in the recursive rule can be used to write an explicit rule for the function, and
- write an explicit rule for the function.
- identify if the function is arithmetic, geometric or neither

Example:

x	y
0	5
1	8
2	11
3	14
4	?
…	…
n	?

- To find the next term: add 3 to the previous term
- Recursive rule: $f(0) = 5, f(n) = f(n-1) + 3$
- To find the n^{th} term: start with 5 and add 3 n times
- Explicit rule: $f(n) = 5 + 3n$
- Arithmetic, geometric, or neither? Arithmetic

x	y
1	5
2	10
3	20
4	40
5	?
…	…
n	?

Function A

1. To find the next term: ________________________________
2. Recursive rule: _____________________________________
3. To find the n^{th} term: _________________________________
4. Explicit rule: _______________________________________
5. Arithmetic, geometric, or neither? ________________________

x	y
0	3
1	4
2	7
3	12
4	19
5	?
…	…
n	?

Function B

6. To find the next term: __________________________________
7. Recursive rule: _______________________________________
8. To find the n^{th} term: __________________________________
9. Explicit rule: ____
10. Arithmetic, geometric, or neither? __________________________

x	y
1	3
2	5
3	9
4	17
5	33
6	?
…	…
n	?

Function C

11. To find the next term: ______________________
12. Recursive rule: ______________________
13. To find the n^{th} term: ______________________
14. Explicit rule: ______________________
15. Arithmetic, geometric, or neither? ______________________

x	y
1	-8
2	-17
3	-26
4	-35
5	-44
6	-53
…	…
n	

Function D

16. To find the next term: ______________________
17. Recursive rule: ______________________
18. To find the n^{th} term: ______________________
19. Explicit rule: ______________________
20. Arithmetic, geometric, or neither? ______________________

x	y
1	2
2	-6
3	18
4	-54
5	162
6	-486
…	…
n	

Function E

21. To find the next term: ______________________
22. Recursive rule: ______________________
23. To find the n^{th} term: ______________________
24. Explicit rule: ______________________
25. Arithmetic, geometric, or neither? ______________________

x	y
0	1
1	$1\frac{3}{5}$
2	$2\frac{1}{5}$
3	$2\frac{4}{5}$
4	$3\frac{2}{5}$
5	4
…	…
n	

Function F

26. To find the next term: ______________________
27. Recursive rule: ______________________
28. To find the n^{th} term: ______________________
29. Explicit rule: ______________________
30. Arithmetic, geometric, or neither? ______________________

x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
…	…
n	

Function G

31. To find the next term: ______________________
32. Recursive rule: ______________________
33. To find the n^{th} term: ______________________
34. Explicit rule: ______________________
35. Arithmetic, geometric, or neither? ______________

x	y
1	-1
2	0.2
3	-0.04
4	0.008
5	-0.0016
6	0.00032
…	…
n	

Function H

36. To find the next term: ______________________
37. Recursive rule: ______________________
38. To find the n^{th} term: ______________________
39. Explicit rule: ______________________
40. Arithmetic, geometric, or neither? ______________

Name: Sequences | 3.7

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Ready, Set, Go!

Ready

Topic: Constant Ratios

Find the constant ratio for each geometric sequence.

1. 2, 4, 8, 16…
2. $\frac{1}{2}$, 1, 2, 4, 8…
3. -5, 10, -20, 40…
4. 10, 5, 2.5, 1.25…

Set

Topic: Recursive and explicit equations

Fill in the blanks for each table, then write the recursive and explicit equation for each sequence.

5. Table 1

x	1	2	3	4	5
y	5	7	9		

Recursive: ______________________ Explicit: ______________________

6. Table 2

x	*y*
1	-2
2	-4
3	-6
4	
5	

Recursive:

Explicit:

7. Table 3

x	*y*
1	3
2	9
3	27
4	
5	

Recursive:

Explicit

8. Table 4

x	*y*
1	27
2	9
3	3
4	
5	

Recursive:

Explicit:

Name: Sequences | 3.7

Go

Topic: Graphing linear equations and labeling windows

Graph the following linear equations. Label your window

13. $y = 4x + 7$

14. $y = \frac{-3}{4}x + 5$

15. $2x + 7y = 10$

16. $x - 3y = 7$

Need Help? Check out these related videos:

Graphing equations

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphs-using-slope-intercept-form

3.8 What Does It *Mean*?

A Solidify Understanding Task

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

x	1	2	3
y	5		11

x	1	2	3	4	5
y	18				-10

x	1	2	3	4	5	6	7
y	12						-6

Describe your method for finding the missing terms. Will the method always work? How do you know?

Here are a few more arithmetic sequences with missing terms. Complete each table, either using the method you developed previously or by finding a new method.

x	1	2	3	4
y	50			86

x	1	2	3	4	5	6
y	40					10

x	1	2	3	4	5	6	7	8
y	-23							5

The missing terms in an arithmetic sequence are called "arithmetic means". For example, in the problem above, you might say, "Find the 6 arithmetic means between -23 and 5". Describe a method that will work to find arithmetic means and explain why this method works.

Name: Sequences | 3.8

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Ready, Set, Go!

Ready

Topic: Comparing arithmetic and geometric sequences

1. How are arithmetic and geometric sequences similar?

2. How are they different?

Set

Topic: Arithmetic sequences

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

3. Table 1

x	1	2	3
y	3		12

4. Table 2

x	y
1	2
2	
3	
4	26

5. Table 3

x	y
1	24
2	
3	6
4	

6. Table 4

x	y
1	16
2	
3	
4	4
5	

Name: Sequences 3.8

Go

Topic: Sequences

Then determine the recursive and explicit equations for each (if the sequence is not arithmetic or geometric, try your best).

7. 5, 9, 13, 17,… This sequence is: Arithmetic , Geometric , Neither

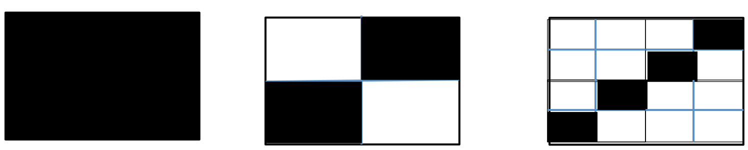
Recursive Equation: ________________________ Explicit Equation: ___________________________

8. 60, 30, 0, -30 ,… This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ________________________ Explicit Equation: ___________________________

9. 60, 30, 15, $\frac{15}{2}$, ,… This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ________________________ Explicit Equation: ___________________________

10. 

(The number of black tiles above) This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ________________________ Explicit Equation: ___________________________

11.. 4, 7, 12, 19, ,… This sequence is: Arithmetic , Geometric , Neither

Recursive Equation: ________________________ Explicit Equation: ___________________________

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3.9 Geometric *Meanies*

A Solidify and Practice Task

Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

Table 1

x	1	2	3
y	3		12

Is the missing term that you identified the only answer? Why or why not?

Table 2

x	1	2	3	4
y	7			875

Are the missing terms that you identified the only answers? Why or why not?

Table 3

x	1	2	3	4	5
y	6				96

Are the missing terms that you identified the only answers? Why or why not?

Table 4

x	1	2	3	4	5	6
y	4					972

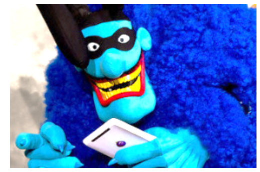
Are the missing terms that you identified the only answers? Why or why not?

A. Describe your method for finding the geometric means.

B. How can you tell if there will be more than one solution for the geometric means?

Name: Sequences | 3.9

Ready, Set, Go!



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Ready

Topic: Arithmetic and geometric sequences

For each set of sequences, find the first five terms. Compare arithmetic sequences and geometric sequences. Which grows faster? When?

1. Arithmetic sequence: $f(1) = 2$, common difference, $d = 3$
Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic:	Geometric:
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

Which value do you think will be more, $f(100)$ or $g(100)$? Why?

2. Arithmetic sequence: $f(1) = 2$, common difference, $d = 10$
Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic:	Geometric:
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

Which value do you think will be more, $f(100)$ or $g(100)$? Why?

3. Arithmetic sequence: $f(1) = 20, d = 10$
Geometric sequence: $g(1) = 2, r = 2$

Arithmetic:	Geometric:
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

Who Which value do you think will be more, $f(100)$ or $g(100)$? Why?

4. Arithmetic sequence: $f(1) = 50, common\ difference, d = 10$
Geometric sequence: $g(1) = 1, common\ ratio, r = 2$

Arithmetic:	Geometric:
$f(1) =$	$g(1) =$
$f(2) =$	$g(2) =$
$f(3) =$	$g(3) =$
$f(4) =$	$g(4) =$
$f(5) =$	$g(5) =$

W Who Which value do you think will be more, $f(100)$ or $g(100)$? Why?

5. Compare arithmetic sequences and geometric sequences growth rates. Which grows faster? When?

Set

Topic: Geometric sequences

Each of the tables below represents a *geometric* sequence. Find the missing terms in the sequence, showing your method.

6. Table 1

x	1	2	3
y	3		12

7. Table 2

x	y
1	2
2	
3	
4	54

8. Table 3

x	y
1	5
2	
3	20
4	

9. Table 4

x	y
1	4
2	
3	
4	
5	324

Name: Sequences 3.9

Go

Topic: Explicit equations of geometric equations

Given the following information, determine the explicit equation for each geometric sequence.

10. $f(1) = 8, common\ ratio,\ r = 2$

11. $f(1) = 4, f(n) = 3f(n-1)$

12. $f(n) = 4f(n-1);\ f(1) = \frac{5}{3}$

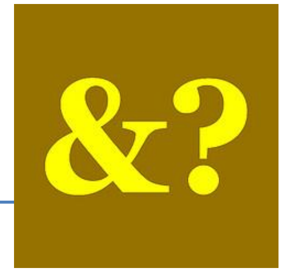
13. Which geometric sequence above has the greatest value at $f(100)$?

Need Help? Check out these videos:

Geometric sequence http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/geometric-sequences--introduction

3.10 I Know . . . What Do You Know?

A Practice Task



In each of the problems below I share some of the information that I know about a sequence. Your job is to add all the things that you know about the sequence from the information that I have given. Depending on the sequence, some of things you may be able to figure out for the sequence are: a table, a graph, an explicit equation, a recursive equation, the constant ratio or constant difference between consecutive terms, any terms that are missing, the type of sequence, or a story context. Try to find as many as you can for each sequence, but you must have at least 4 things for each.

1. I know that: the recursive formula for the sequence is $f(1) = -12,\ f(n) = f(n-1) + 4$
   What do you know?

2. I know that: the first 5 terms of the sequence are 0, -6, -12, -18, -25 . . .
   What do you know?

3. I know that: the explicit formula for the sequence is $f(n) = -10(3)^n$
   What do you know?

4. I know that: The first 4 terms of the sequence are 2, 3, 4.5, 6.75 . . .
   What do you know?

5. I know that: the sequence is arithmetic and $f(3) = 10$ and $f(7) = 26$
   What do you know?

6. I know that: the sequence is a model for the perimeter of the following figures:

Figure 1

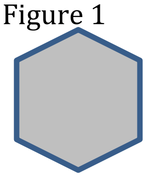


Figure 2

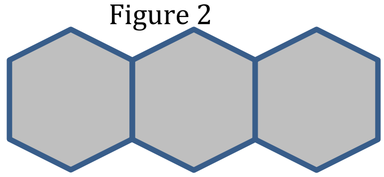
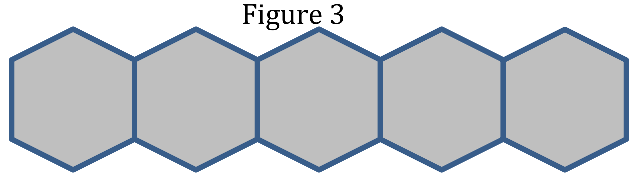


Figure 3



Length of each side = 1

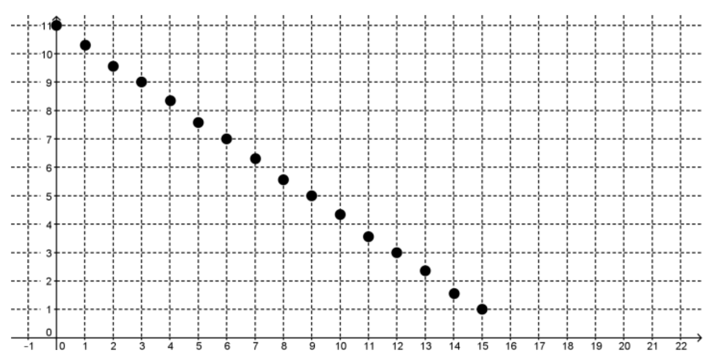
What do you know?

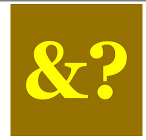
7. I know that: it is a sequence where $a_1 = 5$ and the constant ratio between terms is -2.
What do you know?

8. I know that: the sequence models the value of a car that originally cost $26,500, but loses 10% of its value each year.
What do you know?

9. I know that: the first term of the sequence is -2, and the fifth term is $-\frac{1}{8}$.
What do you know?

10. I know that: a graph of the sequence is:
What do you know?





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Ready, Set, Go!

Ready

Topic: Comparing linear equations and arithmetic sequences

1. Describe similarities and differences between linear equations and arithmetic sequences.

Similarities	Differences

Set

Topic: representations of arithmetic sequences

Use the given information to complete the other representations for each arithmetic sequence.

2. **Recursive Equation:**

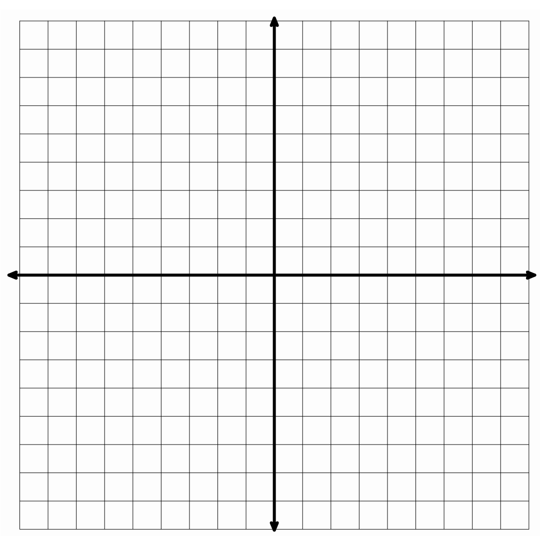
Explicit Equation:

Table

Days	Cost
1	8
2	16
3	24
4	32

Create a context

Graph



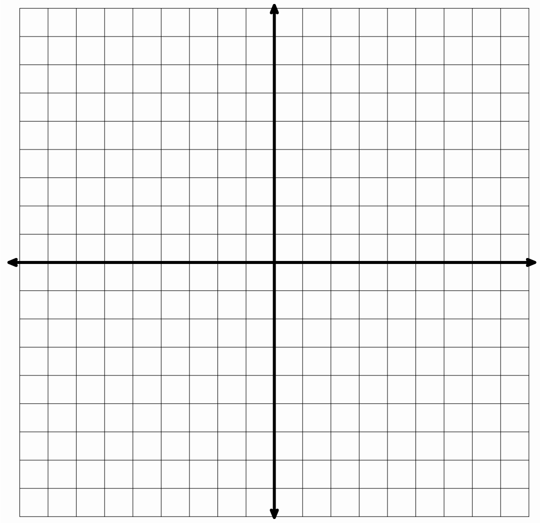
3. **Recursive Equation:** $f(1) = 4, \quad f(n) = f(n-1) + 3$

Explicit Equation:

Table

Create a context

Graph



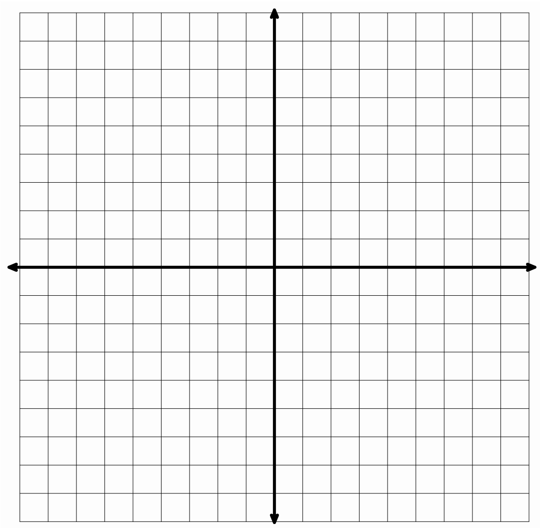
4. **Recursive Equation:**

Explicit Equation: $f(n) = 4 + 5(n-1)$

Table

Create a context

Graph

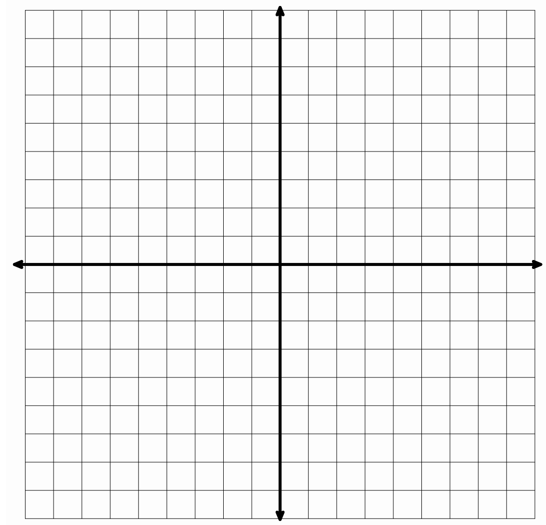


5. **Recursive Equation:**

Explicit Equation:

Table

Graph



Create a context

Janet wants to know how many seats are in each row of the theater. Jamal lets her know that each row has 2 seats more than the row in front of it. The first row has 14 seats.

Go

Topic: Writing explicit equations

Given the recursive equation for each arithmetic sequence, write the explicit equation.

6. $f(n) = f(n-1) - 2; f(1) = 8$

7. $f(n) = 5 + f(n-1); f(1) = 0$

8. $f(n) = f(n-1) + 1;\ f(1) = \frac{5}{3}$

Secondary Mathematics I: An Integrated Approach
Module 4
Linear and Exponential Functions

By

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www.mathematicsvisionproject.org

In partnership with the
Utah State Office of Education

Module 4 – Linear and Exponential Functions

4.1 Classroom Task: Connecting the Dots: Piggies and Pools – A Develop Understanding Task
Introducing continuous linear and exponential functions **(F.IF.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.1

4.2 Classroom Task: Sorting Out the Change – A Solidify Understanding Task
Defining linear and exponential functions based upon the pattern of change (**F.LE.1, F.LE.2**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.2

4.3 Classroom Task: Where's My Change – A Practice Understanding Task
Identifying rates of change in linear and exponential functions (**F.LE.1, F.LE.2**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.3

4.4 Classroom Task: Linear, Exponential or Neither – A Practice Understanding Task
Distinguishing between linear and exponential functions using various representations (**F.LE.3, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.4

4.5 Classroom Task: Getting Down to Business – A Solidify Understanding Task
Comparing the growth of linear and exponential functions (**F.LE.2, F.LE.3, F.LE.5, F.IF.7**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.5

4.6 Classroom Task: Growing, Growing, Gone – A Solidify Understanding Task
Comparing linear and exponential models of population **(F.BF.1, F.BF.2, F.LE.1, F.LE.2, F.LE.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.6

4.7 Classroom Task: Making My Point – A Solidify Understanding Task
Interpreting equations that model linear and exponential functions (**A.SSE.1, A.CED.2, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.7

4.8 Classroom Task: Efficiency Experts – A Solidify Understanding Task
Evaluating the use of various forms of linear and exponential equations (**A.SSE.1, A.SSE.3, A.CED.2, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.8

4.9 Classroom Task: Up a Little, Down a Little – A Solidify Understanding Task
*Understanding and interpreting formulas for exponential growth and decay (**A.SSE.1, A.CED.2, F.LE.5, F.IF.7**)*
Ready, Set, Go Homework: Linear and Exponential Functions 4.9

4.10 Classroom Task: X Marks the Spot – A Practice Understanding Task
Solving exponential and linear equations (**A.REI.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.10

4.1 Connecting the Dots: Piggies and Pools

A Develop Understanding Task

www.flickr.com/photos/teegardin

1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Create a mathematical model for the number of pennies in the piggy bank on day *n*.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute. Create a mathematical model for the number of gallons of water in the pool at *t* minutes.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. Create a mathematical model of the amount of money I will have in the account after *m* months.

4. At the end of the summer, I decide to drain the swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that it was draining at a rate of 3% every minute. Create a mathematical model of the gallons of water in the pool at *t* minutes.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?

Name: Linear and Exponential Functions | 4.1

Ready, Set, Go!

www.flickr.com/photos/teegardin

Ready

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1. 4 , -20 , 100 , -500 , . . .

2. 3 , 5 , 8 , 12 , . . .

3. 64 , 48 , 36 , 27 , . . .

4. 1.5 , 0.75 , 0 , -0.75 , . . .

5. 40 , 10 , $\frac{5}{2}$, $\frac{5}{8}$, …

6. 1, 11, 111, 1111, . . .

7. -3.6, -5.4, -8.1, -12.15, . . .

8. -64, -47, -30, -13, . . .

9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

Name: Linear and Exponential Functions 4.1

Set

Topic: Discrete and continuous relationships

Identify whether the following statements represent a *discrete* or a *continuous* relationship.

10. The hair on your head grows ½ inch per month.

11. For every ton of paper that is recycled, 17 trees are saved.

12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily.

13. The average person laughs 15 times per day.

14. The city of Buenos Aires adds 6,000 tons of trash to its landfills every day.

15. During the Great Depression, stock market prices fell 75%.

Go

Topic: Slopes of lines

Determine the slope of the line that passes through the following points.

16. (-15 , 9) , (-10 , 4)

17. (0.5 , 4) , (3 , 3.5)

18. (50 , 85) , (60 , 80)

19.

x	y
-5	-20
-4	-17
-3	-14

20.

x	y
-1	-1
0	½
1	2

21.

x	y
-5	33
0	30
5	27

Need Help? Check out these related videos and internet sites:

Discrete vs. continuous: http://www.mathsisfun.com/data/data-discrete-continuous.html

Arithmetic and geometric sequences: http://home.windstream.net/okrebs/page131.html

Slope: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/slope-of-a-line

Linear relationships: http://www.mathsteacher.com.au/year7/ch15_linear/04_modelling/linear.htm

4.2 Sorting Out the Change

A Solidify Understanding Task

A. Identify the pattern of change in each of the relations and sort each relation into the following categories:
   - Equal differences over equal intervals
   - Equal factors over equal intervals
   - Neither

B. Be prepared to describe the pattern of change and to tell how you found it.

1.

x	$f(x)$
-30	-57
-25	-47
-20	-37
-15	-27
-10	-17
-5	-7
0	3

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

2.

$f(0) = -3, \; f(n+1) = \frac{5}{3} f(n)$

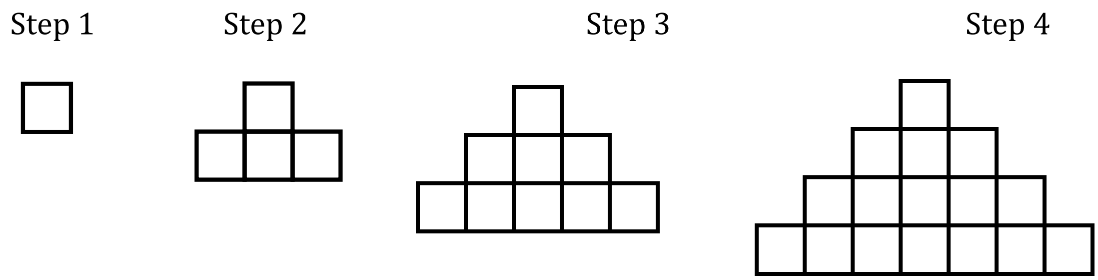
Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____



3. The pattern of change in the perimeter of the figures from one step to the next.

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

4. The pattern of change in the area of the figures from one step to the next.

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

---

5. $y = ax - 3$

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

6.

x	$f(x)$
-10	7
-5	7
-0	7
5	7
10	7
15	7
20	7

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

---

7. The height from the ground of a person on a ferris wheel that is rotating at a constant speed.

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

---

8. $y = x$

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

9.

x	$f(x)$
0	-5
4	-2
-4	-8
8	1
-8	-11

Type of pattern of change ______________________

How I found the pattern of change:

____
____
____

---

10. The algae population in a pond increases by 3% each year until it depletes its food supply and then maintains a constant population.

Type of pattern of change ______________________

How I found the pattern of change:

____
____
____

11.

x	$f(x)$
-5	238
-4	76
-3	22
-2	4
-1	-2
0	-4

Type of pattern of change ______________________

How I found the pattern of change:

____
____
____

12. The change in the height of the ball from one bounce to the next if the ball is dropped from a height of 8 feet and the ball bounces to 80% of its previous height with each bounce.

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

13.

x	y
3	-5
3	-10
3	-20
3	15
3	35
3	50

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

Ready, Set, Go!

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Ready

Topic: Rates of change in linear models

Say which situation has the greatest rate of change

1. The amount of stretch in a short bungee cord stretches 6 inches when stretched by a 3 pound weight. A slinky stretches 3 feet when stretched by a 1 pound weight.

2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.

3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.

4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

Set

Topic: linear rates of change

Determine the rate of change in each table below.

5.

x	y
-3	-13
-1	-5
0	-1
3	11

6.

x	y
-4	-4
0	4
2	8
6	16

7.

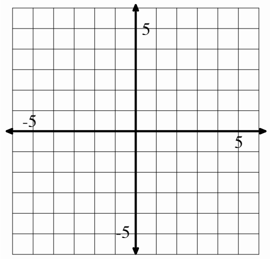
x	y
-10	-14
5	-8
25	0
50	10

Go

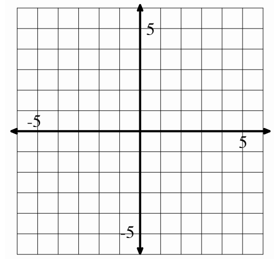
Topic: Graphing linear equations in slope-intercept form.

Graph the following equations

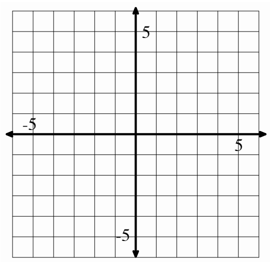
8. $y = 3x - 1$



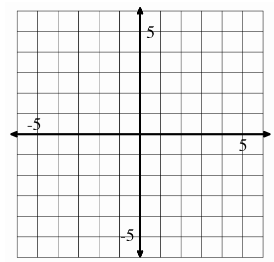
9. $y = -5x + 4$



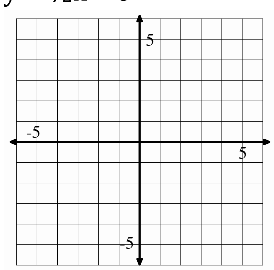
10. $y = x$



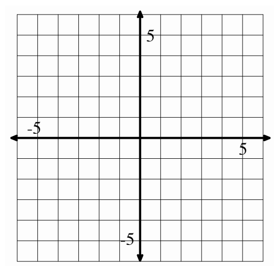
11. $y = -4$



12. $y = \frac{1}{2}x - 6$



13. $x = 3$



Need Help? Check out these related videos:

http://www.algebra-class.com/rate-of-change.html

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphs-using-slope-intercept-form

4.3 Where's My Change?
A Practice Understanding Task

Look through the problems that you worked with in the "Sorting Out the Change" task.

Choose two problems from your linear category (equal differences over equal intervals) and two problems from your exponential category (equal factors over equal intervals).

Add as many representations as you can to the problem you selected. For instance, if you choose problem #1 which is a table, you should try to represent the function with a graph, an explicit equation, a recursive equation, and a story context.

Identify the rate of change in the function. If the function is linear, identify the constant rate of change. If the function is exponential, identify the factor of change.

How does the rate of change appear in each of your representations?

Name: Linear and Exponential Functions | 4.3

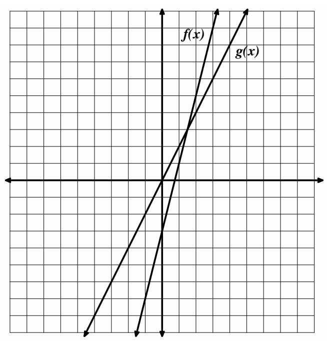
Ready, Set, Go!

Ready

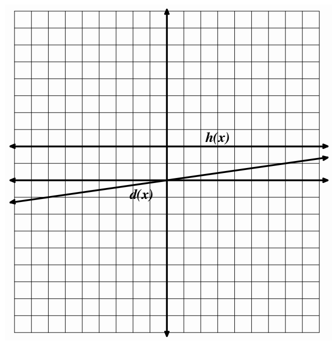
Topic: Recognizing the greater rate of change when comparing 2 linear functions or 2 exponential functions.

Decide which function is growing faster

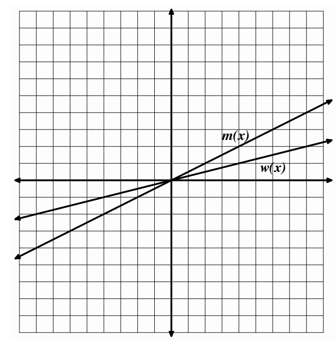
1.



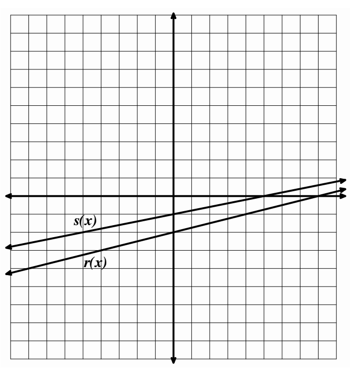
2.



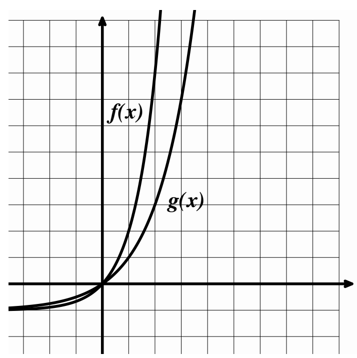
3.



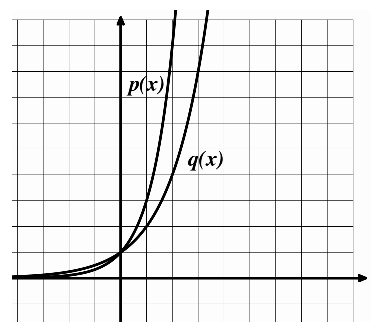
4.

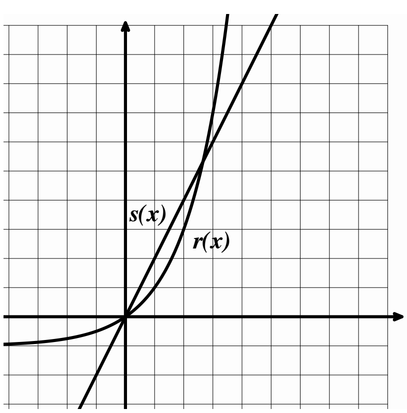


5.



6.





7a. Examine the graph at the left from 0 to 1.
Which graph do you think is growing faster?

b. Now look at the graph from 2 to 3.
Which graph is growing faster in this interval?

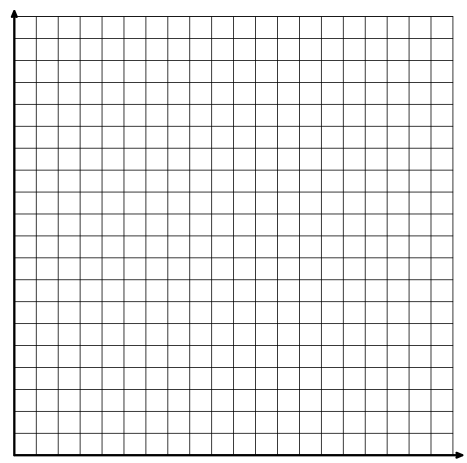
Set

Topic: Representations of linear and exponential functions.

In each of the following problems, you are given one of the representations of a function. Complete the remaining 3 representations. Identify the rate of change for the relation.

9. **Equation**:

Graph



Table

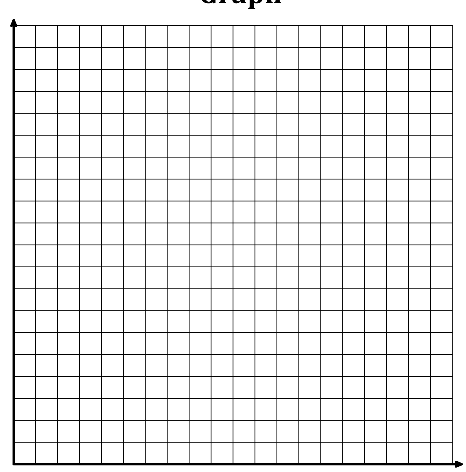
Rides	Cost

Create a context

You and your friends go to the state fair. It costs $5 to get into the fair and $3 each time you go on a ride.

10. **Equation**:

Graph



Table

Time	Amount
1	18
2	53
3	162
4	486
5	1458
6	4374

Create a context

Go

Topic: Recursive and explicit equations of geometric sequences.

Write the recursive and explicit equations for each geometric sequence.

10. Marissa has saved $1000 in a jar. She plans to withdraw half of what's remaining in the jar at the end of each month.

11.

Time (Days)	Number of Bacteria
1	10
2	100
3	1000
4	10000

12.

Folds in paper	Number of rectangles
0	1
1	2
2	4
3	8

13. 1024, 256, 64, 16, . . .

14. 3, 9, 27, 81, . . .

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/identifying-exponential-models

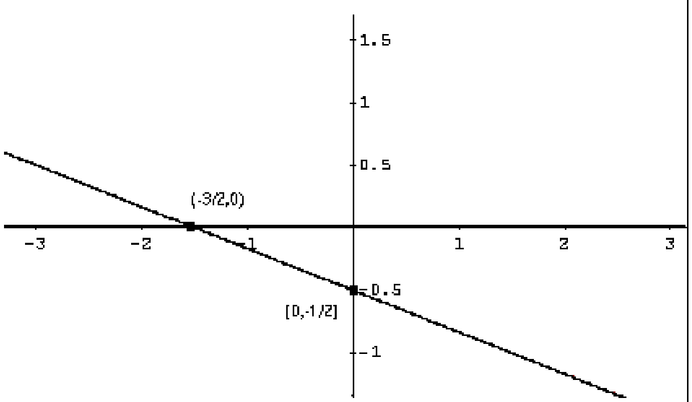
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/linear--quadratic--and-exponential-models

4.4 Linear, Exponential or Neither?

A Solidify Understanding Task

For each representation of a function, decide if the function is linear, exponential, or neither. **Give at least 2 reasons for your answer.**

1.



2.

Tennis Tournament

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

There are 4 players remaining after 5 rounds.

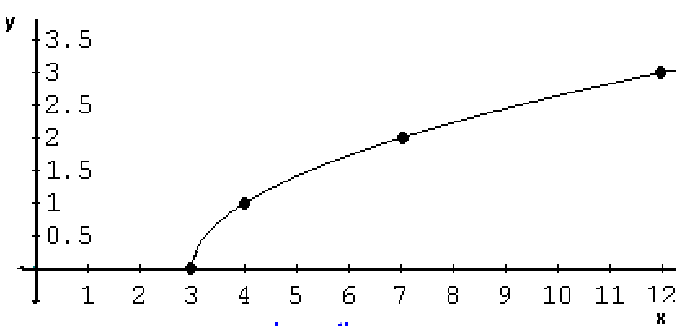
3.

$$y = 4x$$

4.

This function is decreasing at a constant rate

5.



6.

A person's height as a function of a person's age (from age 0 to 100)

7.

$$-3x = 4y + 7$$

8.

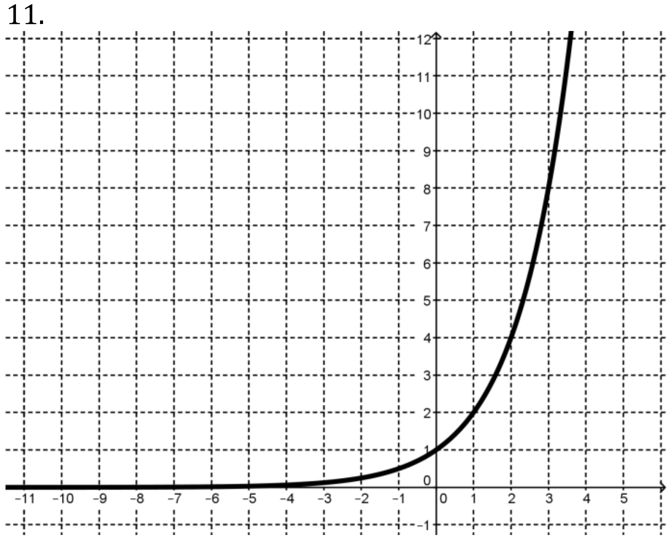
x	*y*
-2	23
0	5
2	-13
4	-31
6	-49

9.

Height in Inches	Shoe Size
62	6
74	13
70	9
67	11
53	4
58	7

10.
The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

11.



12.
The time it takes you to get to work as a function the speed at which you drive

13. $y = 7x^2$	14. Each point on the graph is exactly 1/3 of the previous point.
15. $f(1) = 7, f(2) = 7, f(n) = f(n-1) + f(n-2)$	16. $f(0) = 1, f(n+1) = \frac{2}{3}f(n)$

Ready, Set, Go!

Ready

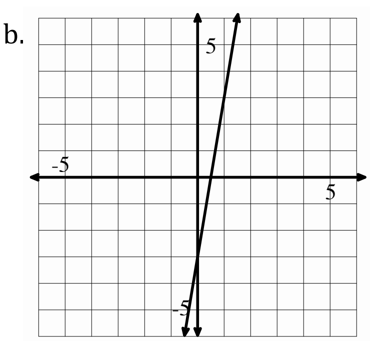
Topic: Comparing rates of change in both linear and exponential situations.

Identify whether situation "a" or situation "b" has the greater rate of change.

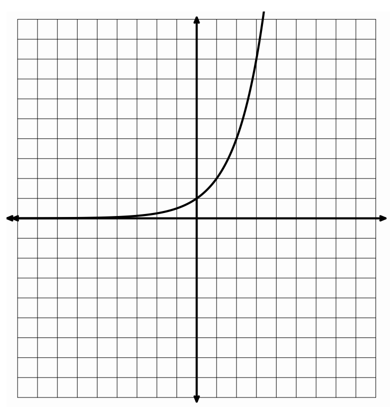
1. a.

x	y
-10	-48
-9	-43
-8	-38
-7	-33

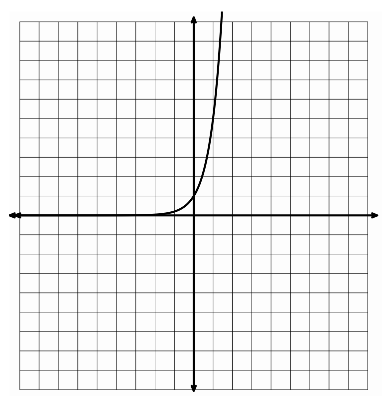
b.



2. a.



b.



3. a. Lee has $25 withheld each week from his salary to pay for his subway pass.

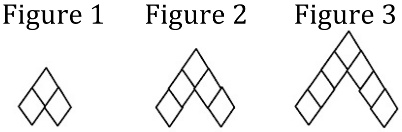
b. Jose owes his brother $50. He has promised to pay half of what he owes each week until the debt is paid.

4. a.

x	6	10	14	18
y	13	15	17	19

b. The number of rhombi in each shape.

Figure 1 Figure 2 Figure 3



5. a. $y = 2(5)^x$

b. In the children's book, *The Magic Pot,* every time you put one object into the pot, two of the same object come out. Imagine that you have 5 magic pots.

Set

Topic: Recognizing linear and exponential functions.

For each representation of a function, decide if the function is linear, exponential, or neither.

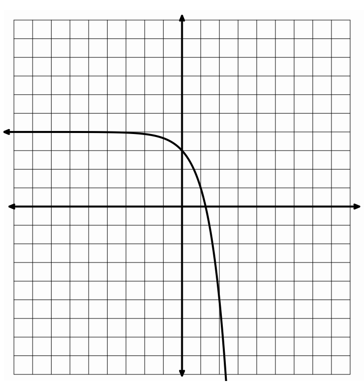
6. The population of a town is decreasing at a rate of 1.5% per year.

7. Joan earns a salary of $30,000 per year plus a 4.25% commission on sales.

8. 3x +4y = -3

9. The number of gifts received each day of "The 12 Days of Christmas" as a function of the day. ("On the 4th day of Christmas my true love gave to me, 4 calling birds, 3 French hens, 2 turtledoves, and a partridge in a pear tree.")

10.



11.

Side of a square	Area of a square
1 inch	1 in^2
2 inches	4 in^2
3 inches	9 in^2
4 inches	16 in^2

Go

Topic: Geometric means

For each geometric sequence below, find the missing terms in the sequence.

12.

x	1	2	3	4	5
y	2				162

13.

x	1	2	3	4	5
y	1/9			-3	

14.

x	1	2	3	4	5
y	10				0.625

15.

x	1	2	3	4	5
y	g				gz^4

16.

x	1	2	3	4	5
y	-3				-243

Need Help? Check out these related videos and internet sites:

Sequences http://www.youtube.com/watch?v=THV2Wsf8hro

4.5 Getting Down to Business

A Solidify Understanding Task

Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

a. Express the projected net incomes in these two business plans as recursive formulas.

b. Write an explicit equation for the net income as a function of time for each company's business plan.

c. Compare your answers in a and b. How are the two representations similar? How do they differ? What relationships are highlighted in each representation?

d. Explain why if both companies are able to meet their net income growth goals, the net income of Computafest will eventually be larger than that of Calcu-rama. In what year will the net income of Computafest be larger than that of Calcu-rama?

Name: Linear and Exponential Functions | 4.5

Ready, Set, Go!

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Ready

Topic: Comparing arithmetic and geometric sequences

The first and 5th terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

1.

Arithmetic	4				324
Geometric	4				324

2.

Arithmetic	3				48
Geometric	3				48

3.

Arithmetic	-6250				-10
Geometric	-6250				-10

4.

Arithmetic	-12				-0.75
Geometric	-12				-0.75

5.

Arithmetic	-1377				-17
Geometric	-1377				-17

Name: Linear and Exponential Functions | 4.5

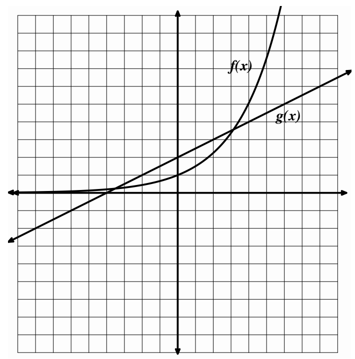
Set

Topic: comparing the rates of change of linear and exponential functions.

Compare the rates of change of each pair of functions by identifying the interval where it appears that *f(x)* is changing faster and the interval where it appears that *g(x)* is changing faster. Verify your conclusions by making a table of values for each equation and exploring the rates of change in your tables.

6. $f(x) = (1.5)^x$

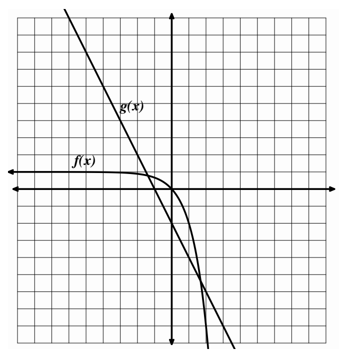
   $g(x) = ½\, x + 2$



x	$f(x)$	$g(x)$

7. $f(x) = -3^x + 1$

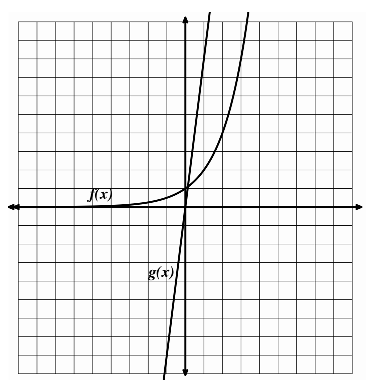
   $g(x) = -2x - 2$



x	$f(x)$	$g(x)$

8. $f(x) = 2^x$

   $g(x) = 8x$



x	$f(x)$	$g(x)$

Name: Linear and Exponential Functions | 4.5

Go

Topic: Writing explicit equations for linear and exponential models.

Write the explicit equation for the tables and graphs below.

9.

x	$f(x)$
2	-4
3	-11
4	-18
5	-25

10.

x	$f(x)$
-1	2/5
0	2
1	10
2	50

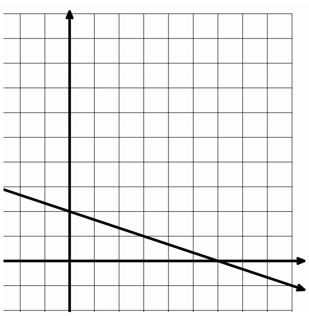
11.

x	$f(x)$
2	-24
3	-48
4	-96
5	-192

12.

x	$f(x)$
-4	81
-3	27
-2	9
-1	3

13.



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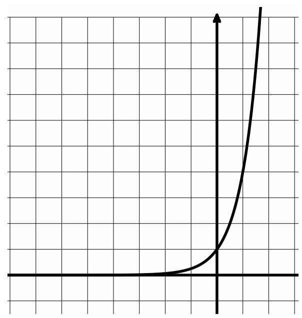

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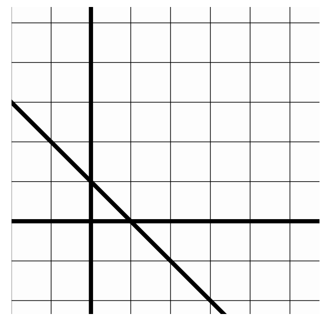

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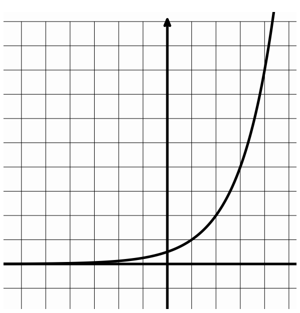

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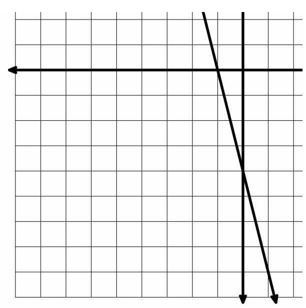
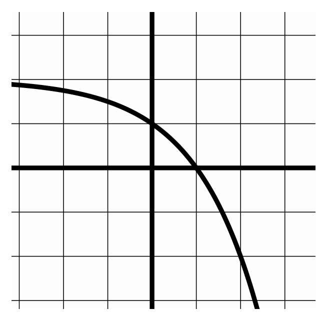


18.



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/equations-of-sequence-patterns

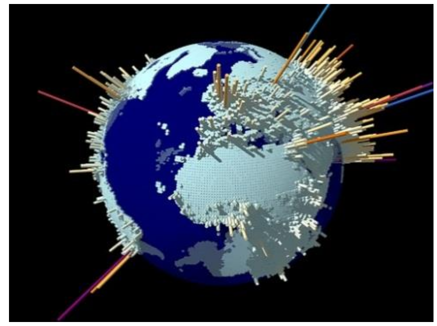
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/geometric-sequences--introduction

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/exponential-growth-functions

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/exponential-decay-functions?v=AXAMVxaxjDg

4.6 Growing, Growing, Gone

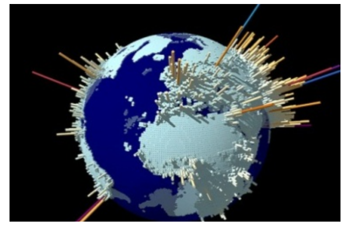
A Solidify Understanding Task



1. The U.S. population in 1910 was 92 million people. In 1990 the population was 250 million. Create both a linear and an exponential model of the population from 1910 to 2030, with projected data points at least every 20 years, starting in 1910.

2. The actual U.S. population data (in millions) was:
   1930: 122.8
   1950: 152.3
   1970: 204.9
   Which model provides a better forecast of the U.S. population for the year 2030? Explain your answer.

Ready, Set, Go!



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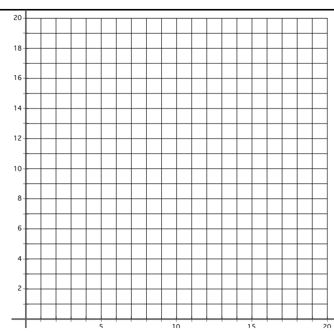
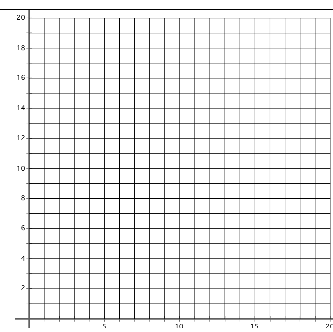
Ready

Topic: Comparing Linear and Exponential Models

Compare different characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 4 + 3x$	$y = 4(3^x)$
1. Type of growth		
2. What kind of sequence corresponds to each model?		
3. Make a table of values	x | y	x | y
4. Find the rate of change		
5. Graph each equation. Compare the graphs. What is the same? What is different?		
6. Find the y-intercept for each function.		

7. Find the y-intercepts for the following equations

a) $y = 3x$

b) $y = 3^x$

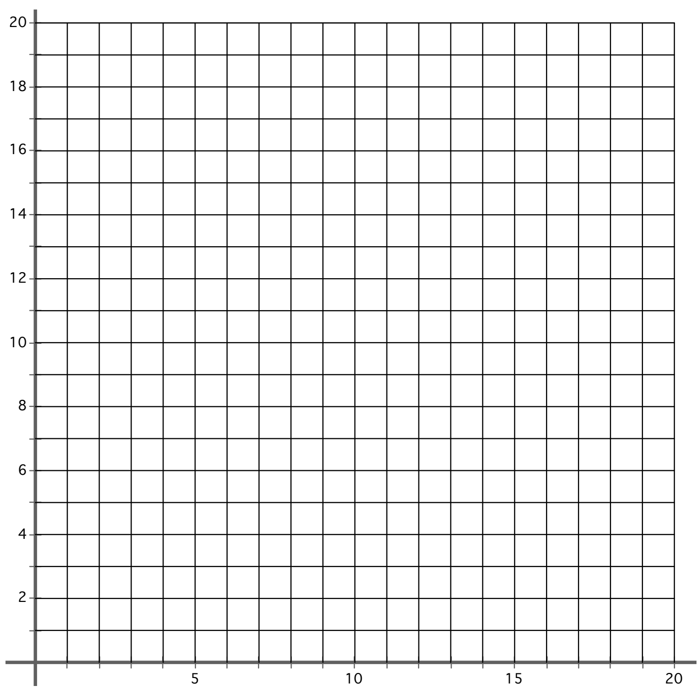
8. Explain how you can find the y-intercept of a linear equation and how that is different from finding the y-intercept of a geometric equation.

Set

Topic: Finding patterns

Use the picture below to answer questions 9-12

9. Graph.



10. Table

Stage	# of small triangles
1	
2	
3	
4	
5	
⋮	
10	

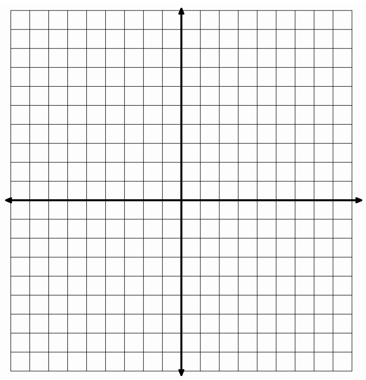
11. Write an explicit function to describe the pattern

Go

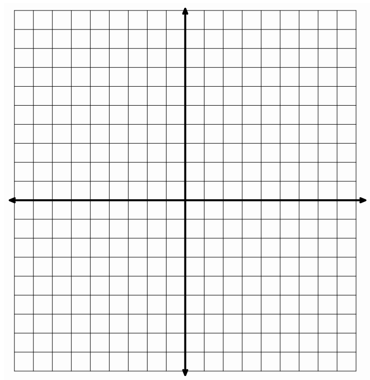
Topic: Solving systems through graphing.

Find the solution of the systems of equations by graphing.

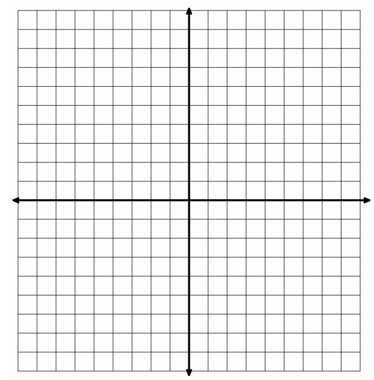
12. $\begin{cases} y = -x \\ y = 3x - 4 \end{cases}$



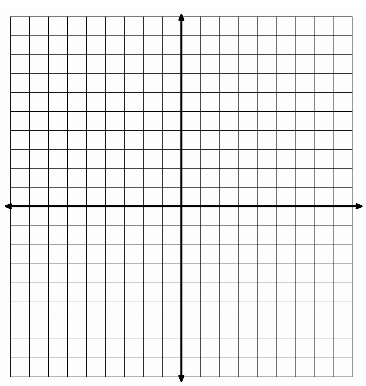
13. $\begin{cases} 2x + y = -6 \\ y = 6 \end{cases}$



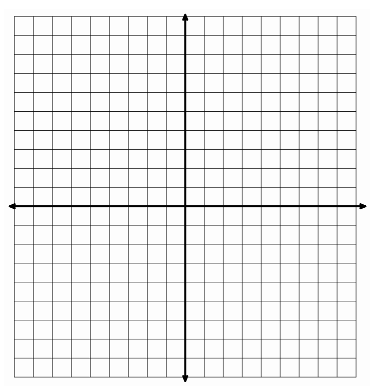
14. $\begin{cases} y = 2x - 2 \\ x + 3y = 15 \end{cases}$



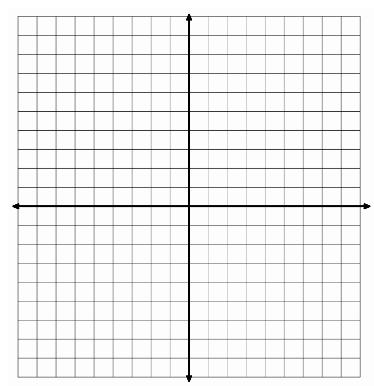
15. $\begin{cases} y + 3 = 6x - 2 \\ y - 2x + 1 = 4(x - 1) \end{cases}$



16. $\begin{cases} y = -(x - 4) \\ y - 2x - 1 = 0 \end{cases}$



17. $\begin{cases} y = 3(x - 2) \\ y + x - 2 = 4(x - 1) \end{cases}$



Need Help? Check out these related videos:

Comparing Linear and exponential functions:

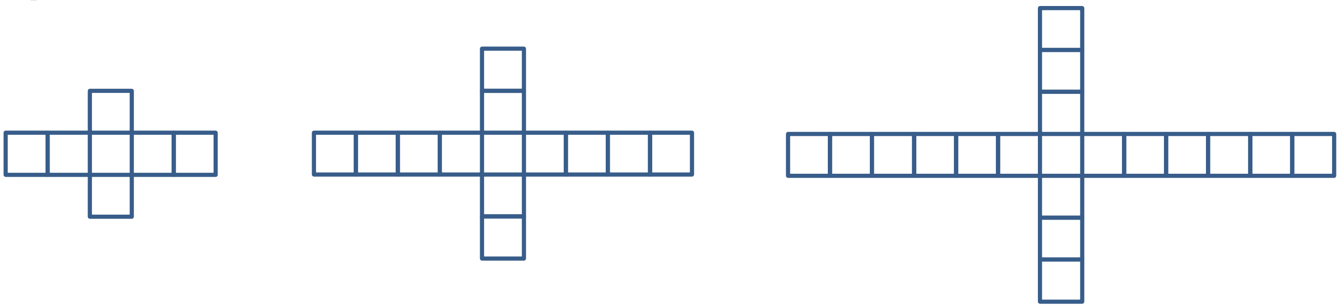
http://www.khanacademy.org/math/algebra/algebra-functions/v/recognizing-linear-functions

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/identifying-exponential-models

4.7 Making My *Point*

A Solidify Understanding Task

Zac and Sione were working on predicting the number of quilt blocks in this pattern:



When they compared their results, they had an interesting discussion:

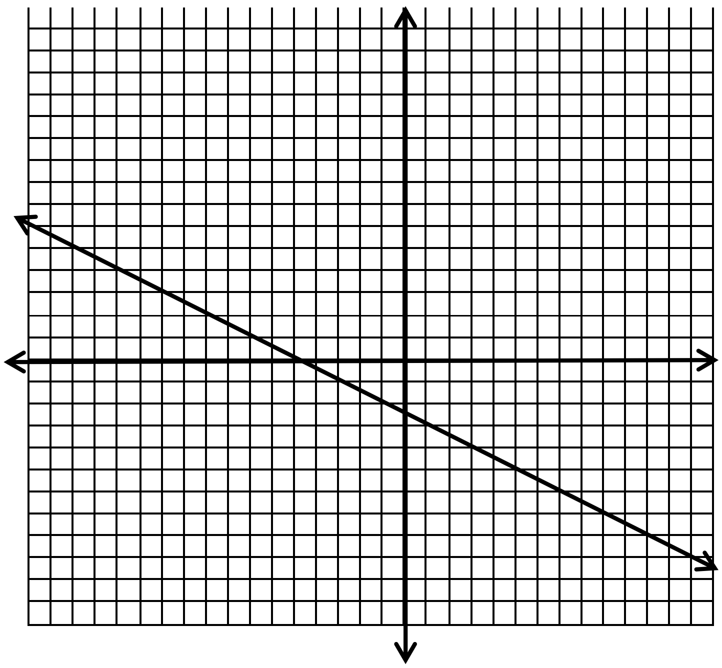
Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at $n = 0$.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at $n = 0$ there were 7 blocks and at n = 1 there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

0	1	2	n
7	13	19	$6(n\text{-}1)$ +7

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.

The next problem Zac and Sione worked on was to write the equation of the line shown on the graph below.



When they were finished, here is the conversation they had about how they got their equations:

Sione: It was hard for me to tell where the graph crossed the y axis, so I found two points that I could read easily, (-9, 2) and (-15, 5). I figured out that the slope was -$\frac{1}{2}$ and made a table and checked it against the graph. Here's my table:

x	-15	-13	-11	-9	n
$f(x)$	5	4	3	2	$-\frac{1}{2}(n+9)+2$

I was surprised to notice that the pattern was to start with the n, add 9, multiply by the slope and then add 2.

I got the equation: $f(x) = -\frac{1}{2}(x+9)+2$.

Zac: Hey—I think I did something similar, but I used the points, (7,-6) and (9,-7).

I ended up with the equation: $f(x) = -\frac{1}{2}(x-9)-7$. One of us must be wrong because yours says that you add 9 to the *n* and mine says that you subtract 9. How can we both be right?

2. What do you say? Can they both be right?

Zac: My equation made me wonder if there was something special about the point (9,- 7) since it seemed to appear in my equation $f(x) = -\frac{1}{2}(x-9)-7$ when I looked at the number pattern. Now I'm noticing something interesting—the same thing seems to happen with your equation, $f(x) = -\frac{1}{2}(x+9)+2$ and the point (-9, 2)

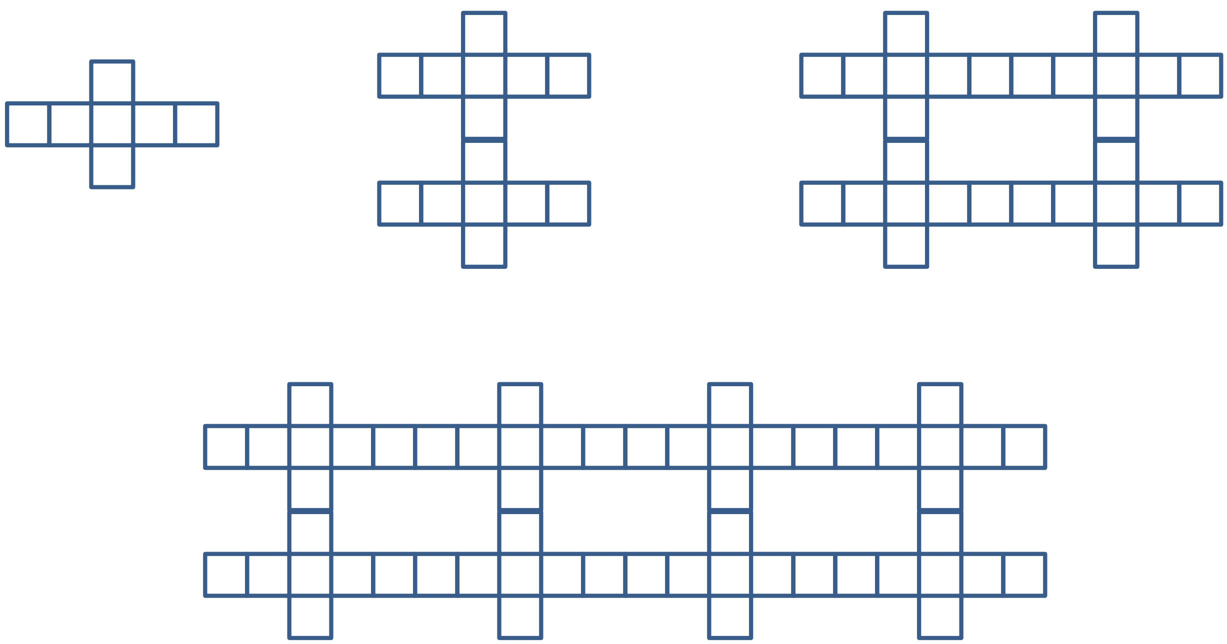
3. Describe the pattern that Zac is noticing.

4. Find another point on the line given above and write the equation that would come from Zac's pattern.

5. What would the pattern look like with the point (*a, b*) if you knew that the slope of the line was *m*?

6. Could you use this pattern to write the equation of any linear function? Why or why not?

Zac and Sione went back to work on an extension of the quilt problem they were working on before. Now they have this pattern:



Zac: This one works a lot like the last quilt pattern to me. The only difference is that the pattern is doubling, so I knew it was exponential. I thought that it starts with 7 blocks and doubles, so the equation must be $f(x) = 7(2)^x$.

Sione: I don't know about that. I agree that it is an exponential function—just look at that growth pattern. But, I made this table:

x	1	2	3	*n*
f(x)	7	14	28	$7(2)^{n-1}$

I used the numbers in the table and got this equation: $f(x) = 7(2)^{x-1}$.

This seems just like all that stuff we were doing with the lines, but I think that the graphs of these two equations would be different. There is something definitely wrong here.

7. What is different about the thinking that Zac and Sione used to come to different equations?

8. How are their results similar to their results on the linear quilt pattern above? How are they different?

Zac: I know! Let's try doing the same thing with your exponential function as the linear function. What if we took the point (1, 7) and wrote the equation this way:

$$f(x) = 2^{(x-1)} + 7$$

See what I did? I did the subtract 1 thing with the *x* and then added on the 7 from the *y* value of the point. I'll bet this is a really good shortcut trick.

9. What do you think of Zac's equation and his strategy? Does it always work? Why or why not?

Ready, Set, Go!

Ready

Topic: Writing equations of lines.

Write the equation of a line in slope-intercept form: y = mx + b, using the given information.

1. m = -7, b = 4
2. m = 3/8, b = -3
3. m = 16, b = -1/5

Write the equation of the line in point-slope form: $y - y_1 = m(x - x_1)$, using the given information.

4. m = 9, (0. -7)
5. m = 2/3, (-6, 1)
6. m = -5, (4, 11)

7. (2,-5) (-3, 10)
8. (0, -9) (3, 0)
9. (-4, 8) (3, 1)

Set

Topic: Graphing linear and exponential functions

Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.

10. The beginning value of the function is 5 and its value is 3 units smaller at each stage. Equation:	11. The beginning value is 16 and its value is ¼ smaller at each stage. Equation:
12. The beginning value is 1 and its value is 10 times as big at each stage. Equation:	13. The beginning value is -8 and its value is 2 units larger at each stage. Equation:

Name: Linear and Exponential Functions 4.7

Go

Topic: Slope-Intercept Form

Rewrite the equations in slope-intercept form.

14. $2y + 10 = 6x + 12$

15. $5x + y = 7x + 4$

16. $(y - 13) = \frac{1}{2}(8x - 14)$

17. $(y + 11) = -7(x - 2)$

18. $(y - 5) = 3(x + 2)$

19. $3(2x - y) = 9x + 12$

20. $y - 2 = 1/5\,(10x - 25)$

21. $y + 13 = -1(x + 3)$

22. $y + 1 = \frac{3}{4}(x + 3)$

Need Help? Check out these related videos:

Equations in slope-intercept form:
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/linear-equations-in-slope-intercept-form

Equations in point-slope form:
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/linear-equations-in-point-slope-form

4.8 Efficiency Experts

A Solidify Understanding Task

In our work so far, we have worked with linear and exponential equations in many forms. Some of the forms of equations and their names are:

Equation	**Name**
$y = \frac{1}{2}x + 1$	Slope Intercept Form $y = mx + b$, where m is the slope and b is the y-intercept
$y - 3 = \frac{1}{2}(x - 4)$	Point Slope Form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) the coordinates of a point on the line
$x - 2y = -2$	Standard Form $ax + by = c$
$f(0) = 1, f(n) = f(n-1) + \frac{1}{2}$	Recursion Formula $f(n) = f(n-1) + D$, Given an initial value *f(a)* D = constant difference in consecutive terms

1. Verify that the four equations above are equivalent.

2. Explain how you know that the four equations are linear.

You have been appointed as a mathematics efficiency expert. Your job is to compare these four forms of equations for different uses and decide which form is most efficient and effective for each use. The investigation will be conducted in four parts with a report to be written at the end.

Linear Investigation Part A: Which form best tells the story?

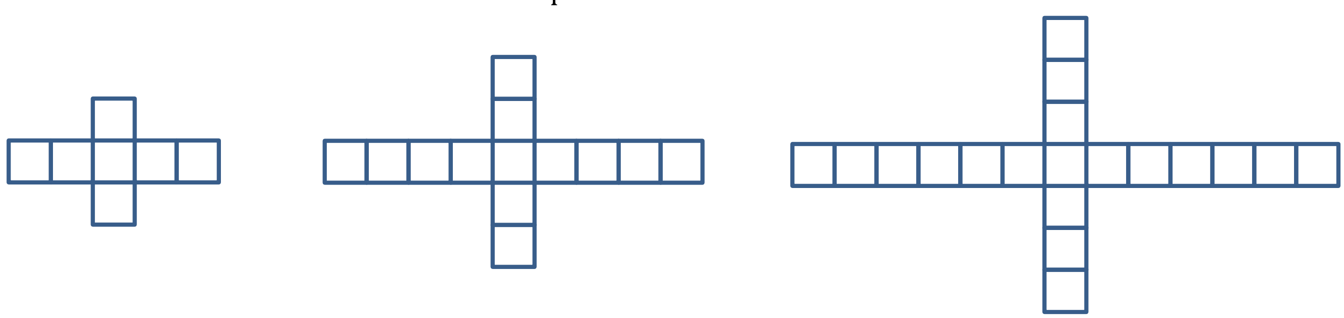
1. In his job selling vacuums, Joe makes \$500 each month plus \$20 for each vacuum he sells. Which equation best describes Joe's monthly income?

   $20x - y = 500$ $\quad\quad\quad\quad$ $y = 20x + 500$

2. The Tree Hugger Granola Company makes trail mix with candies and nuts. The cost of candies for a trail mix is \$2 per pound and the cost of the nuts is \$1.50 per pound. The total cost of a batch of trail mix is \$540. Which equation best models the quantities of candies and nuts in the mix?

   $2x + 1.5\,y = 540$ $\quad\quad\quad\quad$ $y = \frac{4}{3}x + 360$

3. Grandma Billings is working on a quilt with blocks in the following pattern. Which equation best models the number of blocks in each pattern?



   $f(n) = 6n + 1$ $\quad\quad\quad\quad$ $f(1) = 7, f(n) = f(n-1) + 6$

4. What is the important difference between the type of situations that can be modeled with a recursion formula and the other equation forms?

Linear Investigation Part B: Which is the best form for writing equations?

1. Write the equation of the line with a slope of -2 through the point (-2, 5)

2. Write the equation of the line through the points (1, -2) and (4, 1)

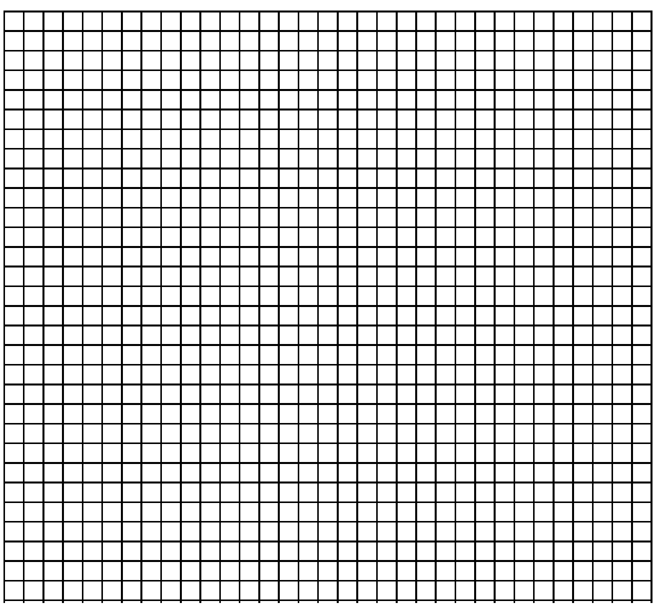
3. Write the equation of the arithmetic sequence that starts with -7 and each term decreases by 3.

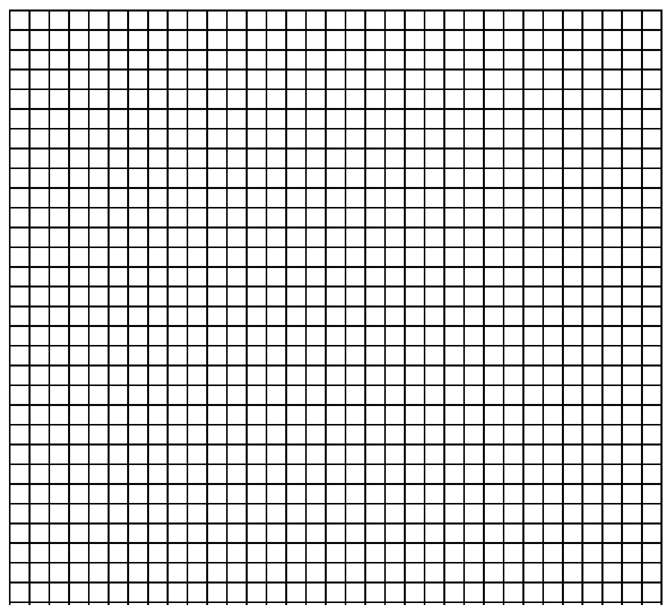
Linear Investigation Part C: Which is the best form for graphing?

Graph the following equations:

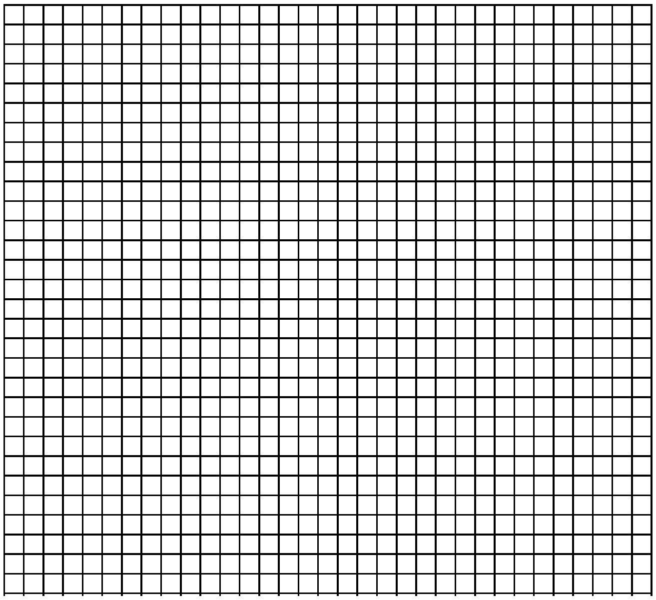
1. $y = \frac{3}{4}x + 5$

2. $3x - 5y = 15$

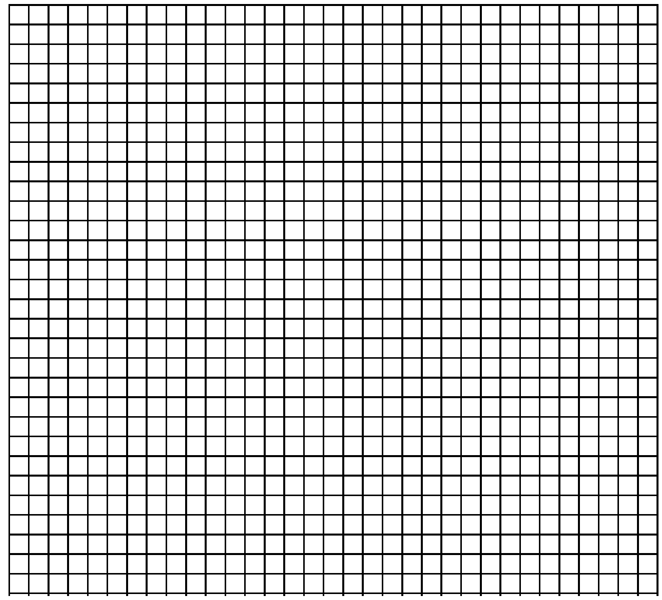




3. $f(n) = -4(n+6)+5$



4. $f(0) = -2, f(n) = f(n-1) - 5$



Linear Investigation Part D: What about tables?

1. Create a table for each equation.

   a. $-x + 2y = 6$

   b. $y = -\frac{1}{2}x - 4$

2. Write an equation for the relation described in this table.

x	***y***
12	-3
9	-8
6	-13
3	-18
0	-23

<u>Your Efficiency Analysis Report, Part 1</u>

Using the results of your investigation, describe the best uses for each form of an equation of a line, with sections for standard form, slope intercept form, point slope form and recursion formulas. Be sure to include a discussion of tables, graphs, and story contexts as part of your report.

Investigating Exponential Forms

During the course of the year, we have also worked with forms of exponential equations, with a few more coming before the end of the module. The forms of exponential equations that we have seen so far:

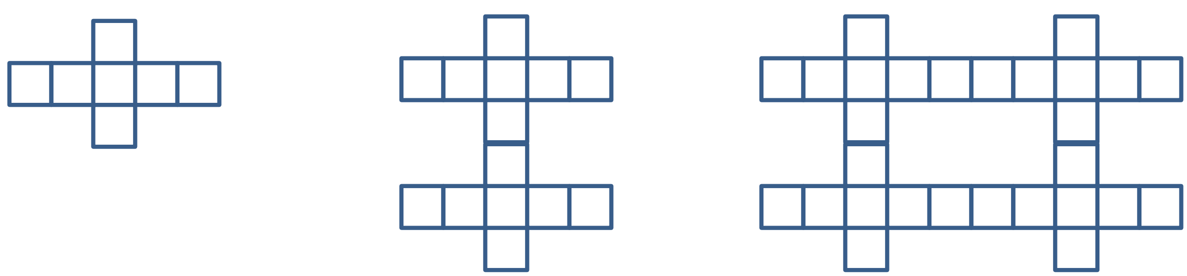
Equation	Name
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n+1) = 3f(n)$	Recursion Formula $f(n+1) = Rf(n)$ Given an initial value f(a) R = constant ratio between consecutive terms

Test out the efficiency of these two exponential equation types for these tasks.

Exponential Investigation Part A: Which form tells the story best?

1. Grandma Billings has started piecing her quilt together and has created the following growth pattern:

Block 1 Block 2 Block 3



Which equation best models the number of squares in each block?

$f(n) = 7(2)^{n-1}$ $f(1) = 7, f(n) = 2f(n-1)$

2. The population of the resort town of Java Hot Springs in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Which equation best models the number of people in Java Hot Springs, with t = the number of years from 2003?

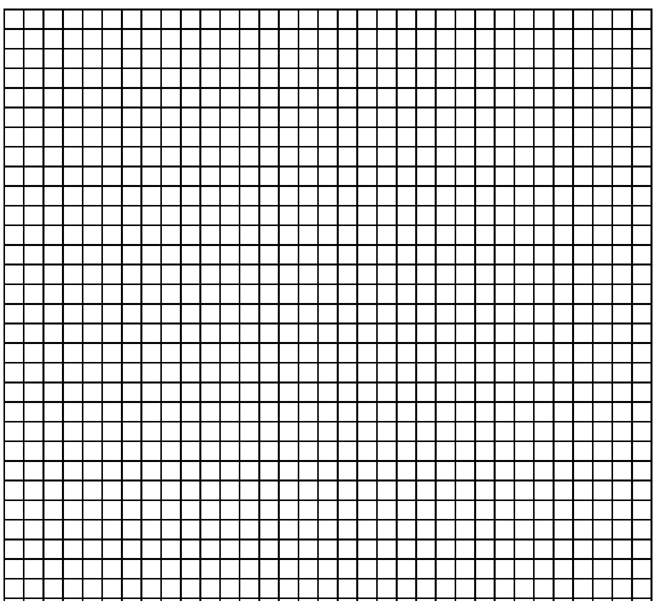
$f(t) = 35{,}000(1.024)^t$ $\qquad$ $f(0) = 35{,}000, f(t) = 1.024 \cdot f(t-1)$

3. How would you have to change the definition of t in the recursive formula to model the situation?

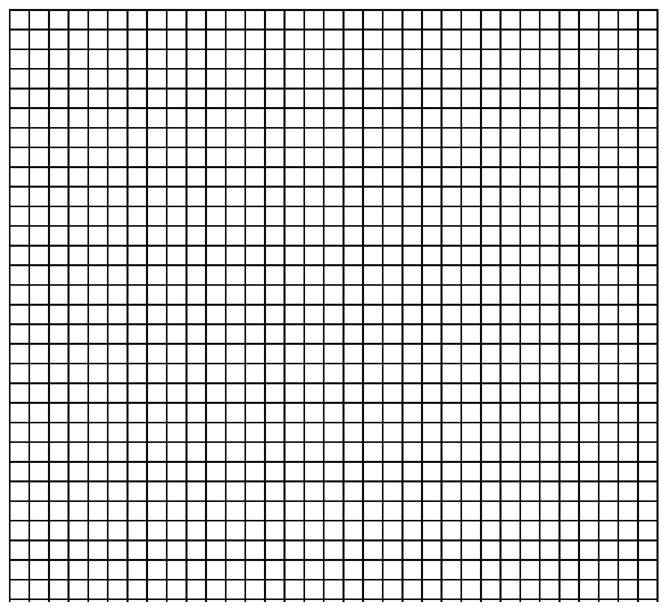
Exponential Investigation Part B: Which is the best form for graphing?

Graph each equation:

1. $y = 2(\,1.8)^x$

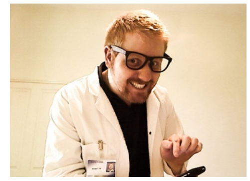


2. $f(0) = 5, f(n) = \ 0.6 \cdot f(n-1)$



<u>Your Efficiency Analysis Report, Part 2</u>

Using the results of your investigation, describe the best uses for each form of an exponential equation, with sections for standard form and recursion formulas. Be sure to include a discussion of tables, graphs, and story contexts as part of your report.



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Ready, Set, Go!

Ready

Topic: Simple interest

*When a person borrows money, the lender usually charges "rent" on the money. This "rent" is called interest. Simple interest is a percent "**r**" of the original amount borrowed "**p**" multiplied by the time "**t**", usually in years. The formula for calculating the interest is* **i = prt**.

Calculate the simple interest owed on the following loans.

1. *p = $1000* *r = 11%* *t = 2 years* *i =* ______________
2. *p = $6500* *r = 12.5%* *t = 5 years* *i =* ______________
3. *p = $20,000* *r = 8.5%* *t = 6 years* *i =* ______________
4. *p = $700* *r = 20%* *t = 6 months* *i =* ______________

*Juanita borrowed $1,000 and agreed to pay 15% interest for 5 years. Juanita did not have to make any payments until the end of the 5 years, but then she had to pay back the amount borrowed "**P**" plus all of the interest "**i**" for the 5 years "**t**." Below is a chart that shows how much money Juanita owed the lender at the end of each year of the loan.*

End of year	Interest owed for the year	Total Amount owed to the lender to pay back the loan.
1	$1000 X .15 = $150	A = Principal + interest = $1150
2	$1000 X .15 = $150	A = P + i + i = $1300
3	$1000 X .15 = $150	A = P + i + i + i = $1450
4	$1000 X .15 = $150	A = P + i + i + i + i = $1600
5	$1000 X .15 = $150	A = P + i + i + i + i + i = $1750

5. Look for the pattern you see in the chart above for the amount (A) owed to the lender. Write an function that best describes A with respect to time (in years).

6. At the end of year 5, the interest was calculated at 15% of the original loan of $1000. But by that time Juanita owed $1600 (before the interest was added.) What percent of $1600 is $150?

7. Consider if the lender charged 15% of the amount owed instead of 15% of the amount of the original loan. Make a fourth column on the chart and calculate the interest owed each year if the lender required 15% of the amount owed at the end of each year. Note that the interest owed at the end of the first year would still be $150. Fill in the 4th column.

Name: Linear and Exponential Functions | 4.8

Set

Topic: The 4 forms of a linear equation

8. Below are the 4 forms of the same linear equation. For each equation, do the following
   (a) Circle the rate of change
   (b) Name the point that describes the y-intercept
   (c) Name the point that describes the x-intercept

Slope-intercept	**Point-slope**	**Standard**	**Recursive formula**	**(b)**	**(c)**
8. $y = 3x - 2$	$y - 13 = 3(x - 5)$	$3x - y = 2$	$f(0) = -2$ $f(n) = f(n-1) + 3$		
9. $y = \frac{1}{4}x + 7$	$y - 5 = \frac{1}{4}(x + 8)$	$x - 4y = -28$	$f(0) = 7$ $f(n) = f(n-1) + \frac{1}{4}$		
10. $y = -\frac{2}{3}x + 3$	$y + 1 = -\frac{2}{3}(x - 6)$	$2x + 3y = 9$	$f(0) = 3$ $f(n) = f(n-1) - \frac{2}{3}$		

Go

Topic: Solving multi-step equations

Solve the following equations

11. $12 + 6x - 4 = 5 + 2(3x - 1)$

12. $5(2x + 4) = 3(x + 5) - 19$

13. $7 - 3(4x + 2) = 6(2x + 3) - 17$

14. $2(x + 1) = 6(x - 3)$

15. What does it mean when you have solved an equation?

16. Explain how a linear equation can have more than one solution.

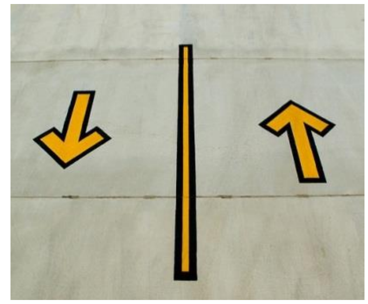
Need Help? Check out these related videos:

Solving equations: http://www.purplemath.com/modules/solvelin4.htm

Interest: http://www.khanacademy.org/finance-economics/core-finance/v/introduction-to-interest

4.9 Up a Little, Down a Little

A Solidify Understanding Task



One of the most common applications of exponential growth is compound interest. For example, Mama Bigbucks puts $20,000 in a bank savings account that pays 3% interest compounded annually. "Compounded annually" means that at the end of the first year, the bank pays Mama 3% of $20,000, so they add $600 to the account. Mama leaves her original money ($20000) and the interest ($600) in the account for a year. At the end of the second year the bank will pay interest on the entire amount, $20600. Since the bank is paying interest on a previous interest amount, this is called "compound interest".

Model the amount of money in Mama Bigbucks' bank account after *t* years.

Use your model to find the amount of money that Mama has in her account after 20 years.

A formula that is often used for calculating the amount of money in an account that is compounded annually is:

$$A = P(1 + r)^t$$

Where:
A = amount of money in the account after t years
P = principal, the original amount of the investment
r = the annual interest rate
t = the time in years

Apply this formula to Mama's bank account and compare the result to the model that you created.

Based upon the work that you did in creating your model, explain the (1 + r) part of the formula.

Another common application of exponential functions is depreciation. When the value of something you buy goes down a certain percent each year, it is called depreciation. For example, Mama Bigbucks buys a car for $20,000 and it depreciates at a rate of 3% per year. At the end of the first year, the car loses 3% of its original value, so it is now worth $19,400.

Model the value of Mama's car after *t* years.

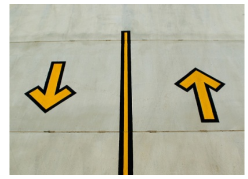
Use your model to find how many years will it take for Mama's car to be worth less than $500?

How is the situation of Mama's car similar to Mama's bank account?

What differences do you see in the two situations?

Consider your model for the value of Mama's car and develop a general formula for depreciation.

Ready, Set, Go!



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Ready

Topic: Evaluating equations

Fill out the table of values for the given equations.

1. $y = 17x - 28$

x	y
-3	
1	
4	
5	

2. $y = -8x - 3$

x	y
-10	
-6	
2	
9	

3. $y = \frac{1}{2}x + 15$

x	y
-26	
-14	
-1	
9	

4. $y = 6^x$

x	y
-3	
-1	
1	
2	
5	

5. $y = 10^x$

x	y
-3	
-1	
0	
2	
6	

6. $y = \left(\frac{1}{5}\right)^x$

x	y
-4	
-2	
0	
3	
5	

Name: Linear and Exponential Functions | 4.9

Set

Topic: Evaluate using the formulas for simple interest or compound interest.

Given the formula for simple interest: $i = Prt$**, calculate the simple interest paid.**

(Remember, i = interest, P = the principal, r = the interest rate per year as a decimal, t = time in years)

7. Find the simple interest you will pay on a 5 year loan of $7,000 at 11% per year.

8. How much interest will you pay in 2 years on a loan of $1500 at 4.5% per year?

Use $i = Prt$ **to complete the table. All interest rates are annual.**

	i	= P	× r	× t
9.		$11,275	12%	3 years
10.	$1428	$5100	4%	
11.	$93.75	$1250		6 months
12.	$54		8%	9 months

Given the formula for compound interest: $A = P(1+r)^t$, write a compound interest function to model each situation. Then calculate the balance after the given number of years.

(Remember: *A = the balance after t years, P = the principal, t =the time in years, r = the annual interest rate expressed as a decimal)*

13. $22,000 invested at a rate of 3.5% compounded annually for 6 years.

14. $4300 invested at a rate of 2.8% compounded annually for 15 years.

15. Suppose that when you are 15 years old, a magic genie gives you the choice of investing $10,000 at a rate of 7% or $5,000 at a rate of 12%. Either choice will be compounded annually. The money will be yours when you are 65 years old. Which investment would be the best? Justify your answer.

Name: Linear and Exponential Functions | 4.9

Go

Topic: Using order of operations when evaluating equations

Evaluate the equations for the given values of the variables.

16. $pq \div 6 + 10$; when $p = 7$ and $q = -3$

17. $m + n(m - n)$; when $m = 2$, and $n = 6$

18. $(b - 1)^2 + ba^2$; when $a = 5$, and $b = 3$

19. $y(x - (9 - 4y))$; when $x = 4$, and $y = -5$

20. $x - (x - (x - y^3))$; when $x = 7$, and $y = 2$

21. $an^4 + a(n - 7)^2 + 2n$; when $a = -2$, and $n = 4$

Need Help? Check out these related videos:

http://www.basic-mathematics.com/simple-vs-compound-interest.html

http://www.khanacademy.org/finance-economics/core-finance/v/introduction-to-interest

4.10 *X* Marks the Spot

A Practice Understanding Task

Table Puzzles

1. Use the tables to find the missing values of x:

a.

x	$y = 0.7x - 3$
-2	-4.4
-10	10
	-8.6
4	-0.2
	1.2

b.

x	$y = -\frac{2}{3}x + 4$
10	$-10\frac{2}{3}$
-3	6
5	$\frac{2}{3}$
	0
	10

c. What equations could be written, in terms of *x* only, for each of the rows that are missing the *x* in the two tables above?

d.

x	$y = 3^x$
5	243
	81
-3	$\frac{1}{27}$
	$\frac{1}{3}$
2	9

e.

x	$y = \left(\frac{1}{2}\right)^x$
-5	32
	8
	1
2	$\frac{1}{4}$
	$\frac{1}{16}$

f. What equations could be written, in terms of *x* only, for each of the rows that are missing the *x* in the two tables above?

2. What strategy did you use to find the solutions to equations generated by the tables that contained linear functions?

3. What strategy did you use to find the solutions to equations generated by the tables that contained exponential functions?

Graph Puzzles

4. The graph of y= $-\frac{1}{2}$x+3 is given below. Use the graph to solve the equations for *x* and label the solutions.

a. $5 = -\frac{1}{2}x + 3$

x = _____

Label the solution with an A on the graph.

b. $-\frac{1}{2}x + 3 = 1$

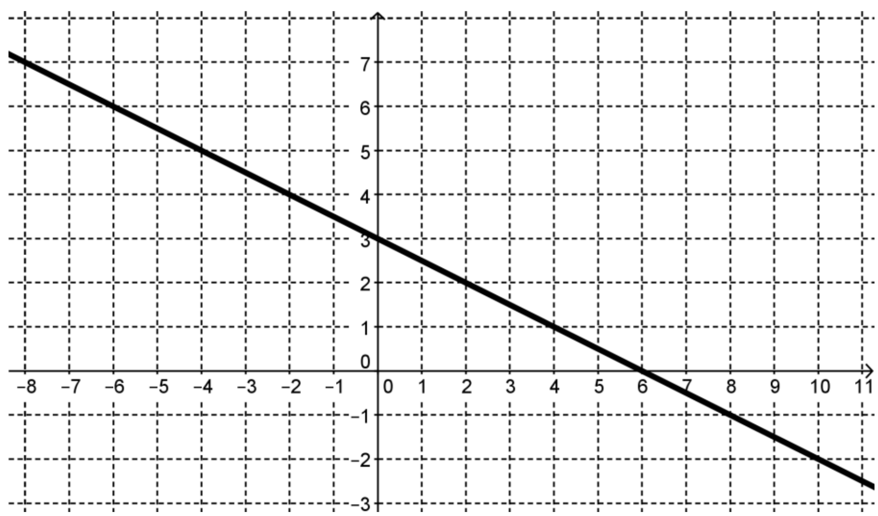
x = _____

Label the solution with a B on the graph.

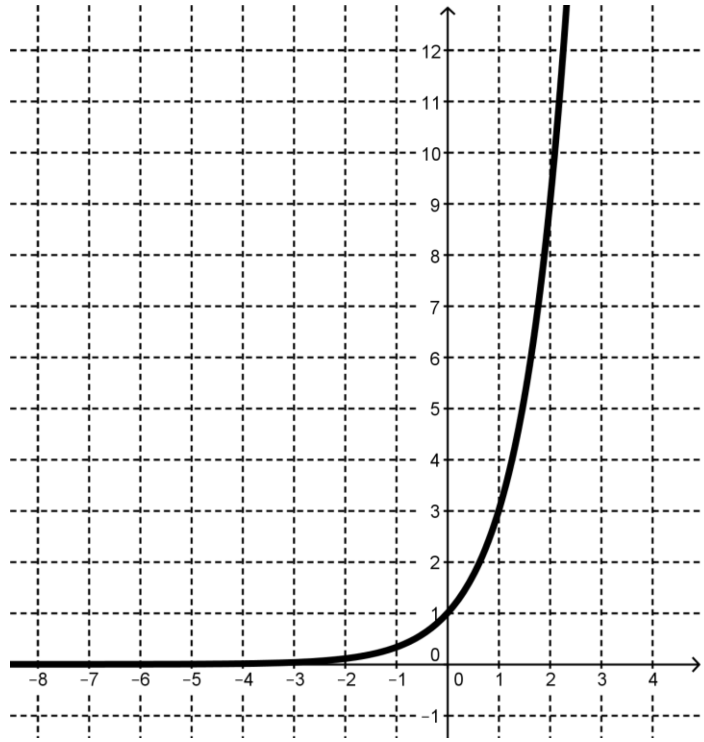
c. $-0.5x + 3 = -1$

x = _____

Label the solution with a C on the graph.



5. The graph of y= 3^x is given below. Use the graph to solve the equations for *x* and label the solutions.



a. $3^x = \frac{1}{9}$

x = _____

Label the solution with an A on the graph.

b. $3^x = 9$

x = _____

Label the solution with a B on the graph.

c. $3\sqrt{3} = 3^x$

x = _____

Label the solution with a C on the graph.

d. $1 = 3^x$

x = _____

Label the solution with a D on the graph.

e. $6 = 3^x$

x = _____

Label the solution with an E on the graph.

6. How does the graph help to find solutions for *x*?

Equation Puzzles:

Solve each equation for *x:*

7. 5^x=125

8. $7 = -6x + 9$

9. $10^x = 10{,}000$

10. $2.5 - 0.9x = 1.3$

11. $6^x = \frac{1}{36}$

12. $\left(\frac{1}{4}\right)^x = 16$

Name: Linear and Exponential Functions 4.10

Ready, Set, Go!

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Ready

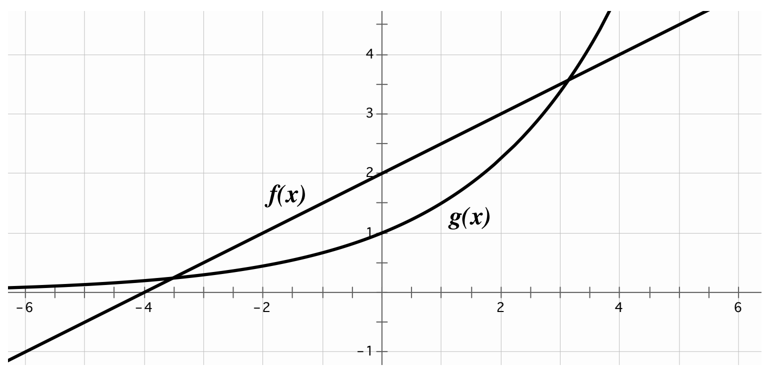
1. Give an example of a discrete function.

2. Give an example of a continuous function.

3. The first and 5th terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

Arithmetic	-6250				-10
Geometric	-6250				-10

4. Compare the rate of change in the pair of functions in the graph by identifying the interval where it appears that *f (x)* is changing faster and the interval where it appears that *g (x)* is changing faster. Verify your conclusions by making a table of values for each function and exploring the rates of change in your tables.



5. Identify the following sequences as linear, exponential, or neither.

a. -23, -6. 11, 28, . . .

b. 49, 36, 25, 16, . . .

c. 5125, 1025, 205, 41, . . .

d. 2, 6, 24, 120, . . .

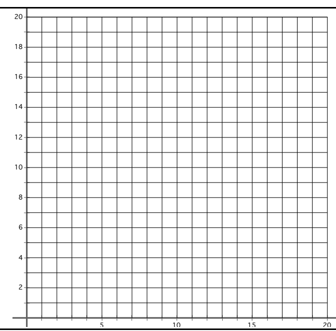
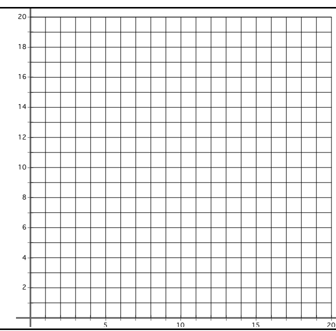
e. 0.12, 0.36, 1.08, 3.24, . . .

f. 21, 24.5, 28, 31.5, . . .

Name: Linear and Exponential Functions 4.10

Set

Describe the defining characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 6 + 5x$	$y = 6(5^x)$
6. Type of growth		
7. What kind of sequence corresponds to each model?		
8. Make a table of values	x | y	x | y
9. Find the rate of change		
10. Graph each equation. Compare the graphs. What is the same? What is different?		
11. Find the y-intercept for each function.		
12. Write the recursive form of each equation.		

There were 2 girls in my grandmother's family, my mother and my aunt. They each had 3 daughters. My two sisters, 3 cousins, and I each had 3 daughters. Each one of our 3 daughters have had 3 daughters...

13. If the pattern of each girl having 3 daughters continues for 2 more generations (my mom and aunt being the 1st generation, I want to know about the 5th generation), how many daughters will be born then?

14. Write the explicit equation for this pattern.

15. Create a table and a graph describing this pattern. Is this situation discrete or continuous?

Go

Solve the following equations.

16. $5x + 3 = 2(x - 6)$

17. $6x - 12x + 10 = 2(-3x - 6)$

18. $13x - 12x + \frac{1}{2} = x + \frac{3}{6}$

Write the equation of the line in slope-intercept form given the following information. (P and Q are points on the line)

19. $f(0) = 6,\ f(n) = f(n-1) + \frac{1}{4}$

20. m = -3, P : (-5, 8)

21. $14x - 2y + 9 = 0$

22. P : (17, -4) , Q : (-5, -26)

23. $y - 9 = \frac{1}{2}(x + 6)$

24. P : (11, 8) , Q : (-1, 8)

Name: Linear and Exponential Functions 4.10

Recall the following formulas: Simple interest **$i = prt$** Compound interest **$A = P(1+r)^t$**

Using the formulas for simple interest or compound interest, calculate the following.

25. The simple interest on a loan of $12,000 at an interest rate of 17% for 6 years.

26. The simple interest on a loan of $20,000 at an interest rate of 11% for 5 years.

27. The amount owed on a loan of $20,000, at 11%, compounded annually for 5 years.

28. Compare the interest paid in #26 to the interest paid in #27. Which kind of interest do you want if you have to take out a loan?

29. The amount in your savings account at the end of 30 years, if you began with $2500 and earned an interest rate of 7% compounded annually.

Secondary One Mathematics: An Integrated Approach
Module 5
Features of Functions

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 5 – Features of Functions

Classroom Task: 5.1 Getting Ready for a Pool Party- A Develop Understanding Task
Using a story context to graph and describe key features of functions t **(F.IF. 4)**
Ready, Set, Go Homework: Features of Functions 5.1

Classroom Task: 5.2 Floating Down the River – A Solidify Understanding Task *Using tables and graphs to interpret key features of functions* **(F.IF. 4, F.IF. 5)**
Ready, Set, Go Homework: Features of Functions 5.2

Classroom Task: 5.3 Features of Functions – A Practice Understanding Task
Features of functions using various representations **(F.IF. 4, F.IF. 5)**
Ready, Set, Go Homework: Features of Functions 5.3

Classroom Task: 5.4 The Water Park – A Solidify Understanding Task
Interpreting functions using notation **(F.IF.2, F.IF.4, F.IF. 5, F.IF.7, A.REI.11, A.CED.3)**
Ready, Set, Go Homework: Features of Functions 5.4

Classroom Task: 5.5 Pooling it Together – A Solidify Understanding Task
Combining functions and analyzing contexts using functions **(F.BF.1b, F.IF.2, F.IF.4, F.IF. 5, F.IF.7, A.REI.11, A.CED.3)**
Ready, Set, Go Homework: Features of Functions 5.5

Classroom Task: 5.6 Interpreting Functions – A Practice Understanding Task
Using graphs to solve problems given in function notation **(F.BF.1b, F.IF.2, F.IF.4, F.IF. 5, F.IF.7, A.REI.11, A.CED.3)**
Ready, Set, Go Homework: Features of Functions 5.6

Classroom Task: 5.7 A Water Function – A Solidify Understanding Task
Defining Function **(F.IF.1)**
Ready, Set, Go Homework: Features of Functions 5.7

Classroom Task: 5.8 To Function or Not to Function – A Practice Understanding Task
Identifying whether or not a relation is a function given various representations **(F.IF.1, F.IF.3)**
Ready, Set, Go Homework: Features of Functions 5.8

Classroom Task: 5.9 Match that Function – A Practice Understanding Task
Matching features and representations of a specific function **(F.IF.2, F.IF.4, F.IF. 5, F.IF.7, A.REI.11, A.CED.3)**
Ready, Set, Go Homework: Features of Functions 5.9

5.1 Getting Ready for a Pool Party

A Develop Understanding Task

Sylvia has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Sylvia did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Sylvia and her two friends removed water with three buckets	Took a break

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of activities Sylvia did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

2. Create a story connecting Sylvia's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate math vocabulary.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function?

Ready, Set, Go!

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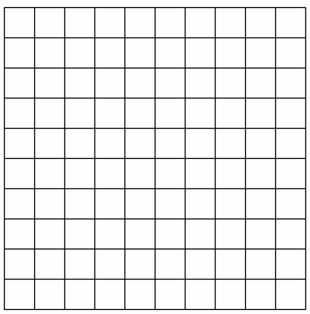
Ready

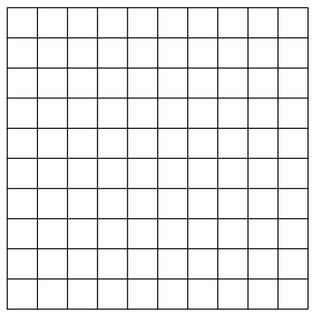
Topic: Graphing linear and exponential functions

Graph each of the functions.

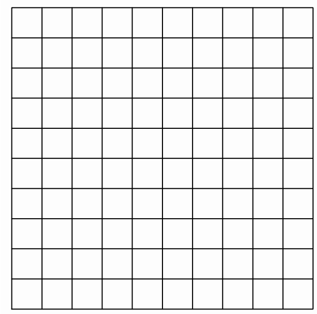
1. $f(x) = -2x + 5$



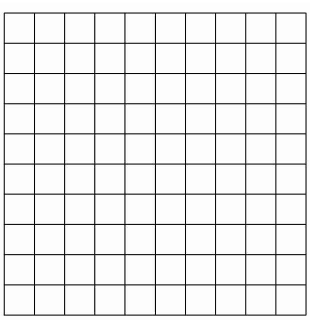
2. $g(x) = 4 - 3x$



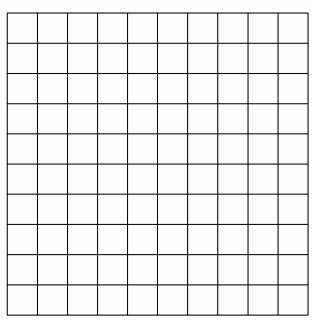
3. $h(x) = 5(3)^x$



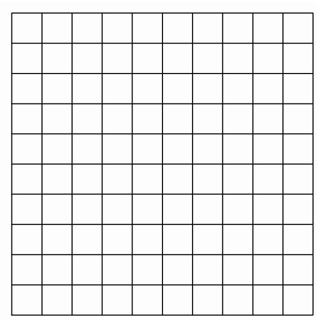
4. $k(x) = 4(2)^x$



5. $v(t) = 2.5t - 4$



6. $f(x) = 8(3)^x$



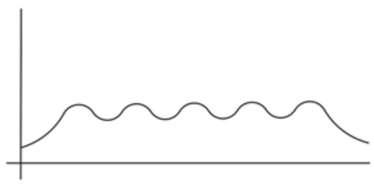
Set

Topic: Describing attributes of a function based on the graphical representation.

For each graph given match it to the contextual description that fits best. Then label the independent and dependent axis with the proper variables.

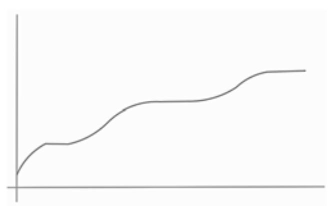
Graphs | Contextual Descriptions

____ 7.



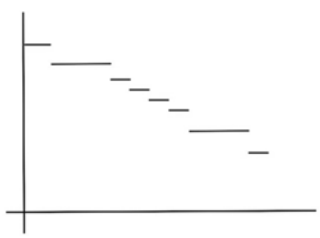
a. The amount of money in a savings account where regular deposits and some withdrawals are made.

____ 8.



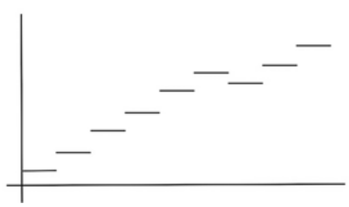
b. The temperature of the oven on a day that mom bakes several batches of cookies.

____ 9.



c. The amount of gasoline on hand at the gas station before a tanker truck delivers more.

____ 10.



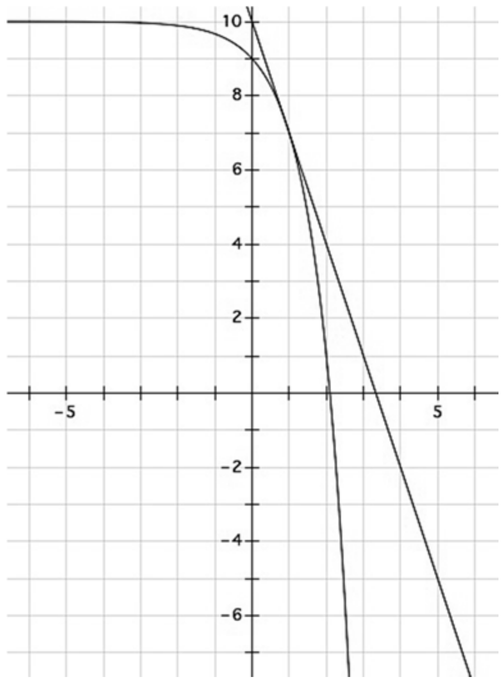
d. The number of watermelons available for sale at the farmer's market on Thursday.

____ 11.

e. The amount of mileage recorded on the odometer of a delivery truck over a time period.

Given the pair of graphs on each coordinate grid, create a list of similarities the two graphs share and a list of differences. (Consider attributes like, continuous, discrete, increasing, decreasing, linear, exponential, restrictions on domain or range, etc.)

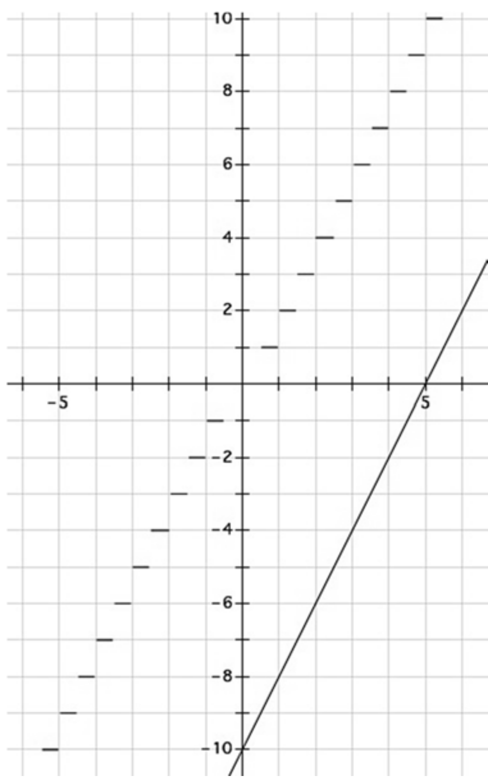
12.



Similarities:

Differences:

13.



Similarities:

Differences:

Go

Topic: Solving Equations

Find the value of *x* in each equation.

14. $10^x = 100{,}000$

15. $3x + 7 = 5x - 21$

16. $-6x - 15 = 4x + 35$

17. $5x - 8 = 37$

18. $3^x = 81$

19. $3x - 12 = -4x + 23$

20. $10 = 2^x - 22$

21. $243 = 8x + 3$

22. $5^x - 7 = 118$

5.2 Floating Down the River

A Solidify Understanding Task

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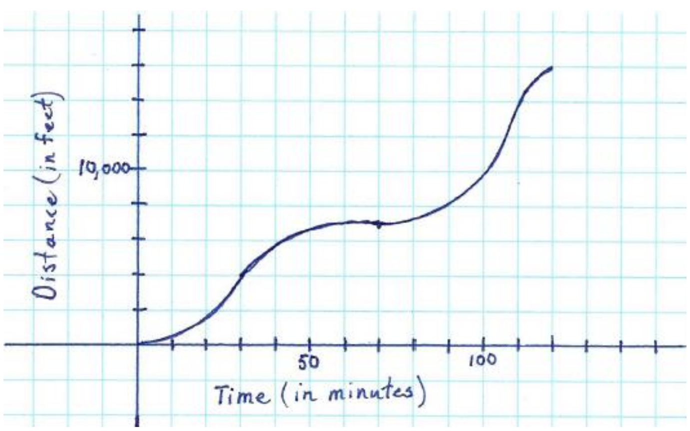
Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said “Math is everywhere!” To learn more about the river, Alonzo and Maria collected data throughout the trip.

Alonzo created a table of values by measuring the depth of the water every ten minutes.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

1. Use the data collected by Alonzo to interpret the key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.



2. Using the graph created by Maria, describe the key features of this relationship.

3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.

- The depth of the water increases and decreases throughout the 120 minutes of floating down the river.
- The distance traveled is always increasing.
- The distance traveled is a function of time.
- The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.
- The domain of the distance/time graph is all real numbers.
- The y-intercept of the depth of water over time function is (0,0).
- The distance traveled increases and decreases over time.
- The water level is a function of time.
- The range of the distance/time graph is from [0, 15000].
- The domain of the depth of water with respect to time is from [0,120]
- The range of the depth of water over time is from [4,5].
- The distance/ time graph has no maximum value.
- The depth of water reached a maximum at 30 minutes.

Ready, Set, Go!

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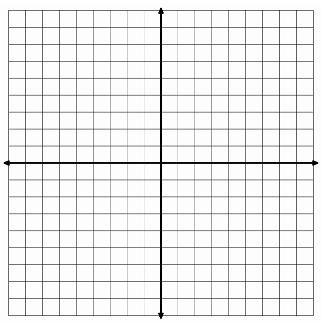
Ready

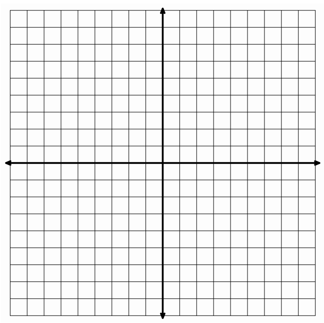
Topic: Solve systems by graphing

Graph each system of linear equations and find where $f(x) = g(x)$

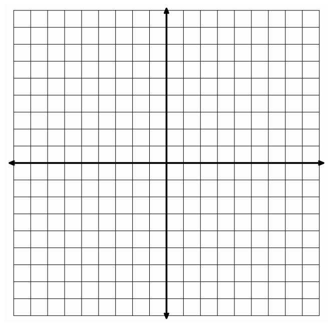
1. $\begin{cases} f(x) = 2x - 7 \\ g(x) = -4x + 5 \end{cases}$



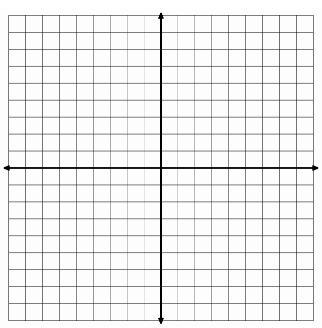
2. $\begin{cases} f(x) = -5x - 2 \\ g(x) = -2x + 1 \end{cases}$



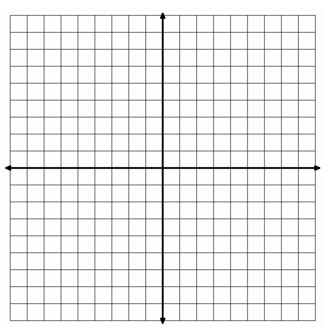
3. $\begin{cases} f(x) = -\frac{1}{2}x - 2 \\ g(x) = 2x + 8 \end{cases}$



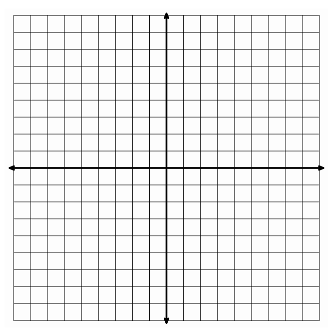
4. $\begin{cases} f(x) = \frac{2}{3}x - 5 \\ g(x) = -x \end{cases}$



5. $\begin{cases} f(x) = \frac{2}{3}x + 4 \\ g(x) = -\frac{1}{3}x + 1 \end{cases}$



6. $\begin{cases} f(x) = x \\ g(x) = -x - 3 \end{cases}$

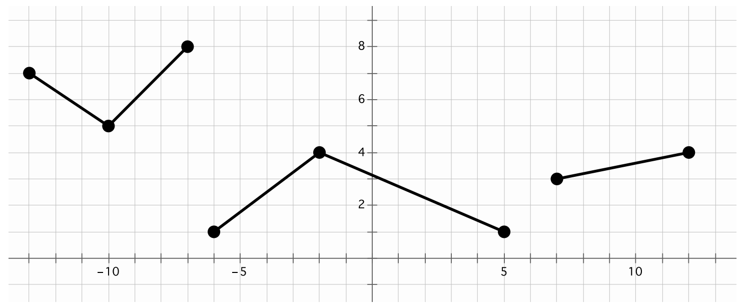


Set

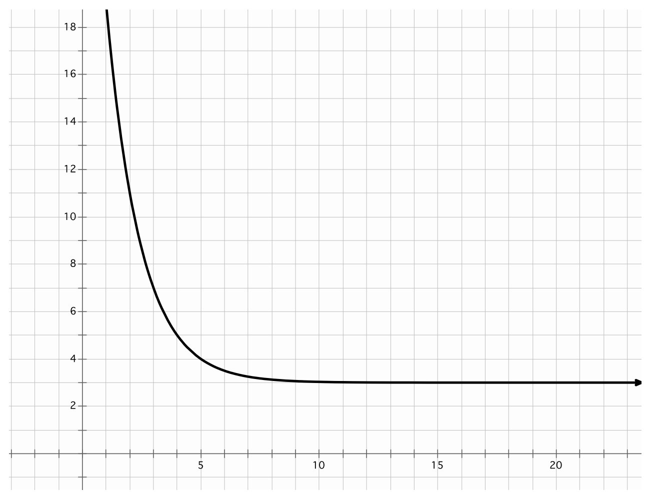
Topic: Describe features of a function from its graphical representation.

For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.

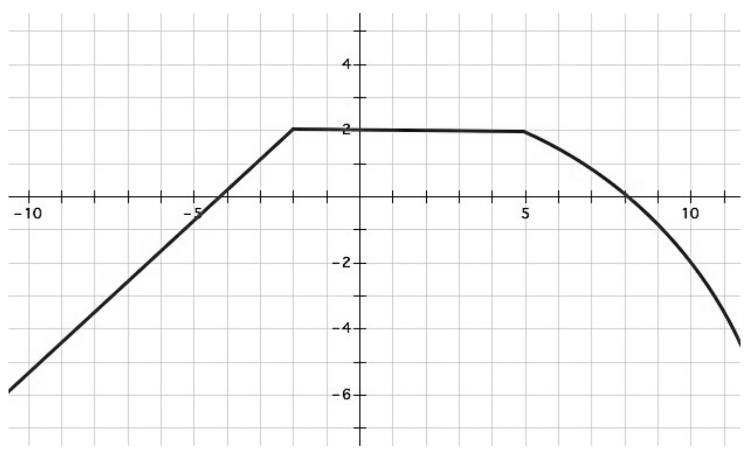
7. Description of function



8. Description of function



9. Description of function



Go

Topic: Create equations using both explicit and recursive notation.

Write equations for the given tables in both recursive and explicit form.

10.

n	$f(n)$
1	5
2	2
3	-1

Explicit:

Recursive:

11.

n	$f(n)$
1	6
2	12
3	24

Explicit:

Recursive:

12.

n	$f(n)$
0	-13
2	-5
3	-1

Explicit:

Recursive:

13.

n	$f(n)$
1	5
4	11
5	13

Explicit:

Recursive:

14.

n	$f(n)$
2	5
7	15,625
9	390,625

Explicit:

Recursive:

15.

n	$f(n)$
0	-4
1	-16
2	-64

Explicit:

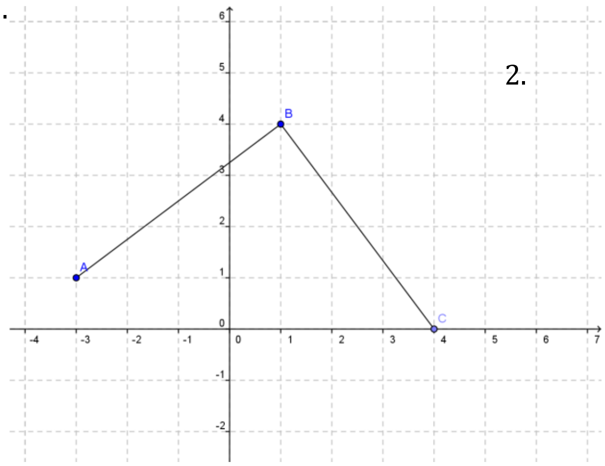
Recursive:

5.3 Features of Functions

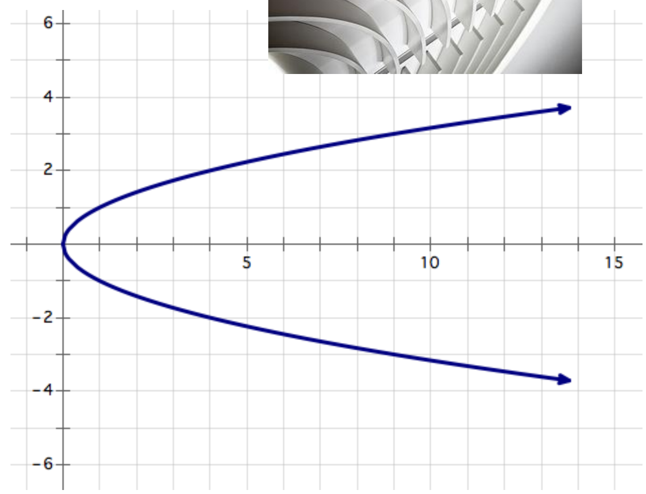
A Practice Understanding Task

For each graph, determine if the relationship represents a function, and if so, state the key features of the function (intervals where the function is increasing or decreasing, the maximum or minimum value of the function, domain and range, x and y intercepts, etc.)

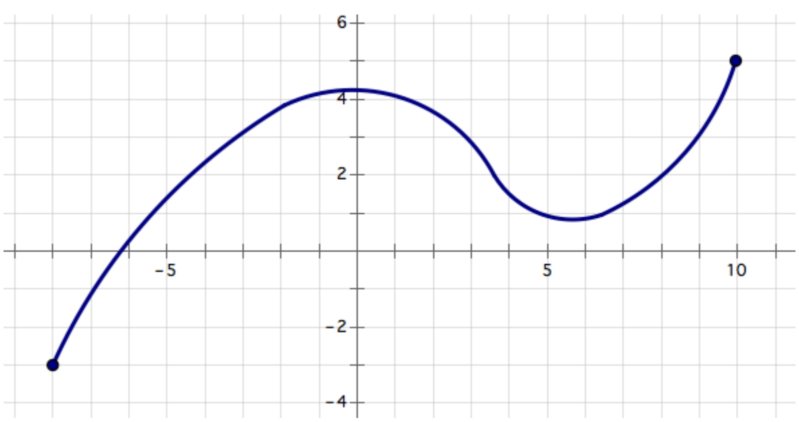
1.



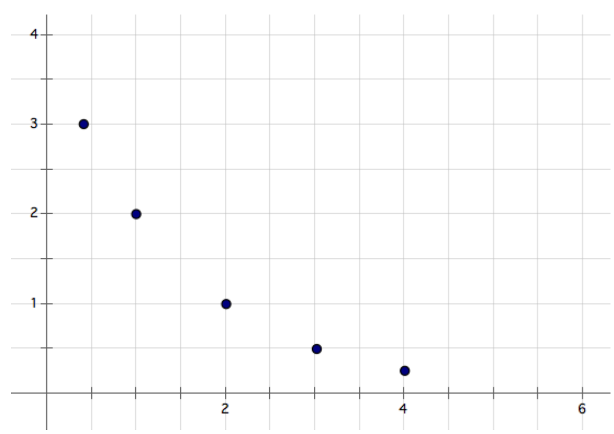
2.



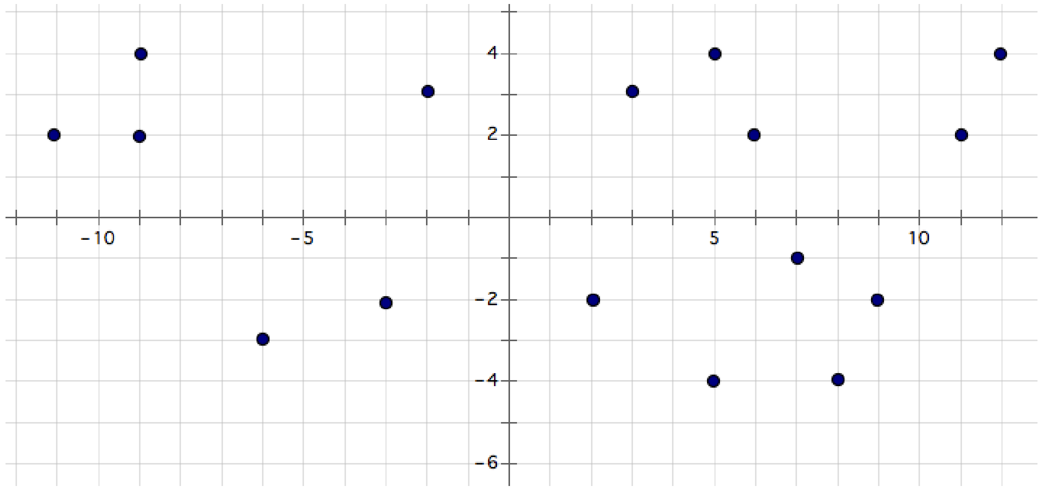
3.



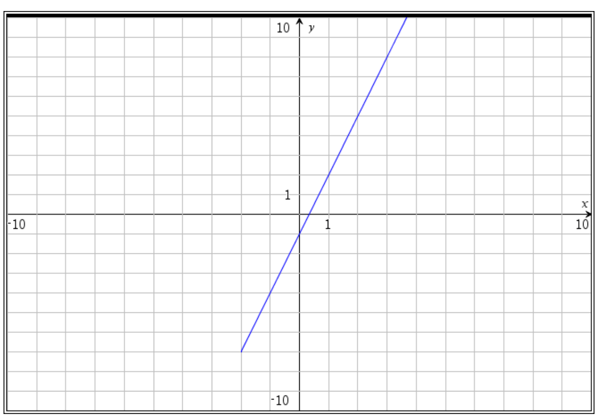
4.



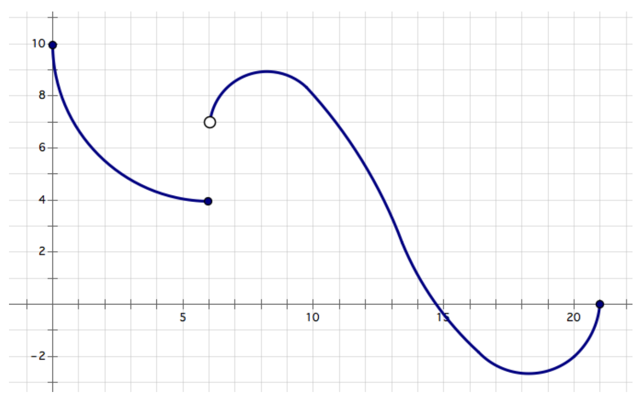
5.



6.



7.



The following represents a continuous function defined on the interval from [0, 6].

x	*f(x)*
0	2
1	-3
2	0
3	2
4	6
5	12
6	20

8. Determine the domain, range, x and y intercepts.
9. Based on the table, identify the minimum value and where it is located

The following represents a discrete function defined on the interval from [1,5].

x	*f(x)*
1	4
2	10
3	5
4	8
5	3

10. Determine the domain, range, x and y intercepts.
11. Based on the table, identify the minimum value and where it is located.

Describe the key features for each situation.

12. The amount of daylight dependent on the time of year.

13. The first term in a sequence is 36. Each consecutive term is exactly 1/2 of the previous term.

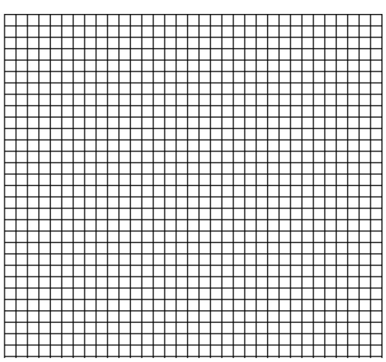
14. Marcus bought a $900 couch on a six months, interest free payment plan. He makes $50 payments to the loan each week.

15. The first term in a sequence is 36. Each consecutive term is 1/2 less than the previous term.

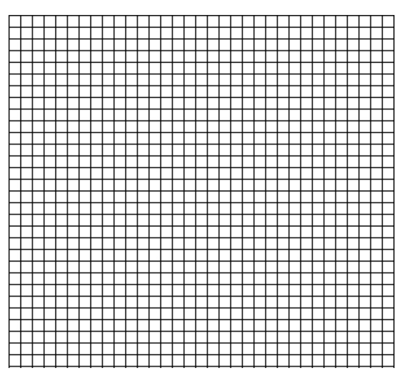
16. An empty 15 gallon tank is being filled with gasoline at a rate of 2 gallons per minute.

For each equation, sketch a graph and show key features of the graph.

17. $f(x) = -2x + 4, when\ \ x \geq 0$



18. $g(x) = 3^x$



Ready, Set, Go!

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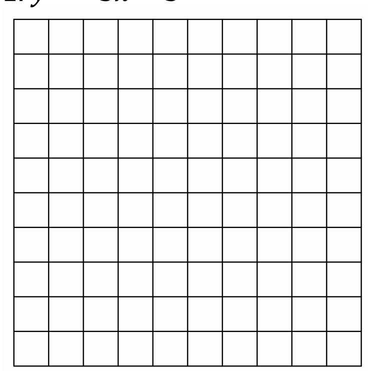
Ready

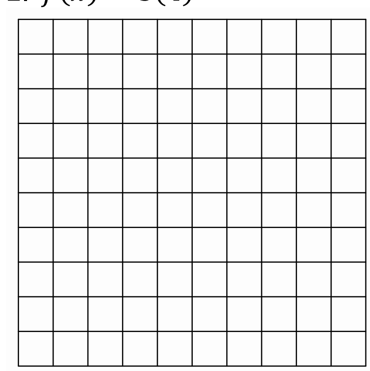
Topic: Creating graphical representations and naming the domain.

Sketch a graph to represent each function, then state the domain of the function.

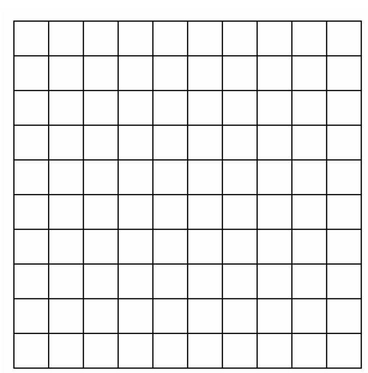
1. $y = 3x - 5$



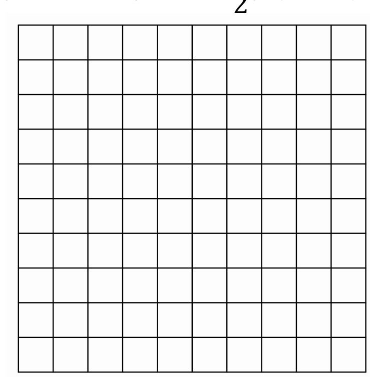
2. $f(x) = 3(4)^x$



3. A sequence of terms such that $f(0) = 1, f(n) = f(n-1) - 7$



4. A sequence of terms such that $f(1) = 8, f(n) = \frac{1}{2}f(n-1)$



Set

Topic: Attributes of linear and exponential functions.

Determine if the statement is true or false, then justify why.

5. All linear functions are increasing.

6. Arithmetic sequences are an example of linear functions.

7. Exponential functions have a domain that includes all real numbers.

8. Geometric sequences have a domain that includes all integers.

9. The range for an exponential function includes all real numbers.

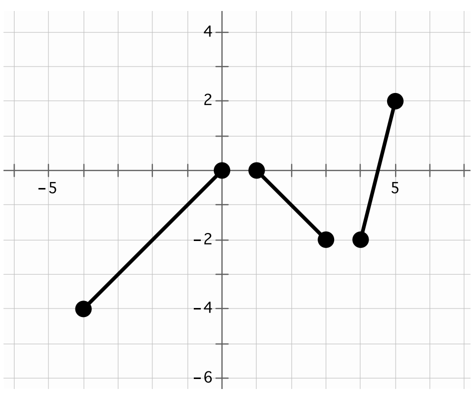
10. All linear relationships are functions with a domain and range containing all real numbers.

Go

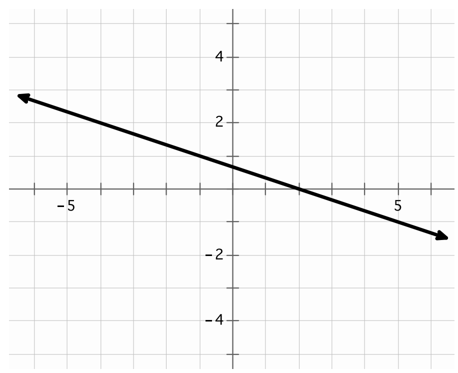
Topic: Determine the domain of a function from the graphical representation.

For each graph determine the domain of the function.

11.



12.



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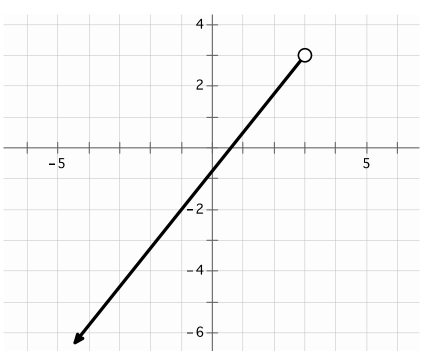
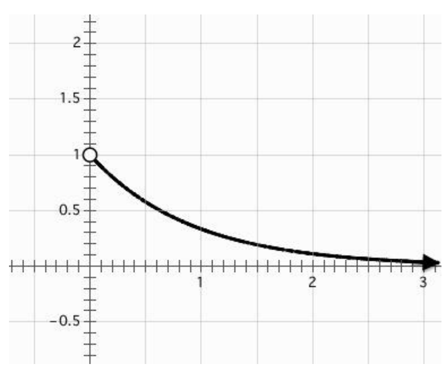


14.



5.4 The Water Park
A Solidify Understanding Task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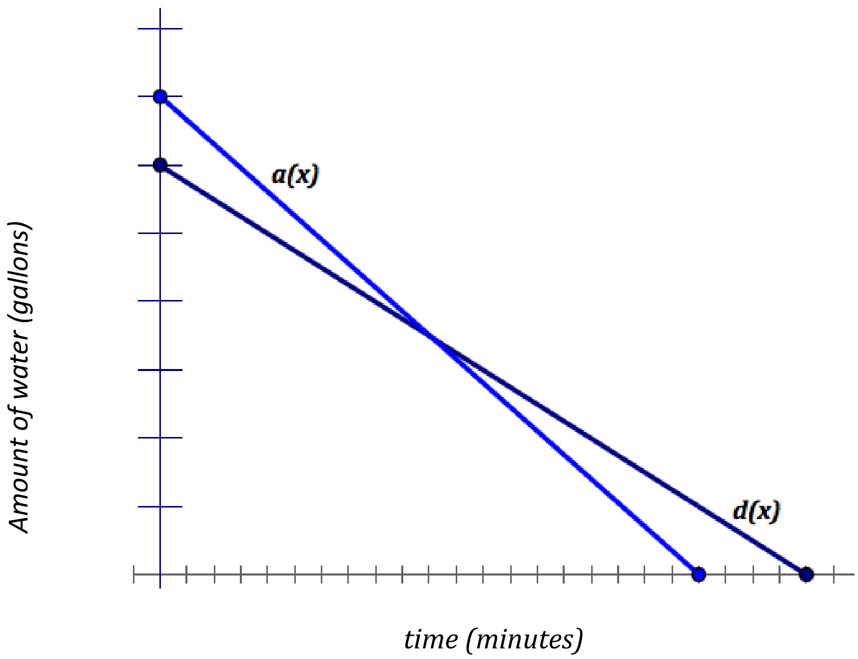
Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, *a(x),* and Dayne's pool, *d(x)*, over time.



Part I

1. Make as many observations as possible with the information given in the graph above.

Part II

Dayne figured out that the pump he uses drains water at a rate of 1000 gallons per minute and takes 24 minutes to drain.

2. Write the equation to represent the draining of Dayne's pool, *d(x)*. What does each part of the equation mean?

3. Based on this new information, correctly label the graph above.
4. For what values of x make sense in this situation? Use interval notation to write the domain of the situation.
5. Determine the range, or output values, that make sense in this situation.
6. Write the equation used to represent the draining of Aly's pool, $a(x)$. Using interval notation, state the domain and range for the function, $a(x)$ as well as the domain and range of the situation. Compare the two domains by describing the constraints made by the situation.

Part III

Based on the graph and corresponding equations for each pool, answer the following questions.

7. When is $a(x) = d(x)$? What does this mean?

8. Find $a(10)$. What does this mean?

9. If $d(x)$=2000, then x= ____. What does this mean?

10. When is $a(x) > d(x)$? What does this mean?

Ready, Set, Go!

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Ready

Topic: Attributes of linear and exponential functions.

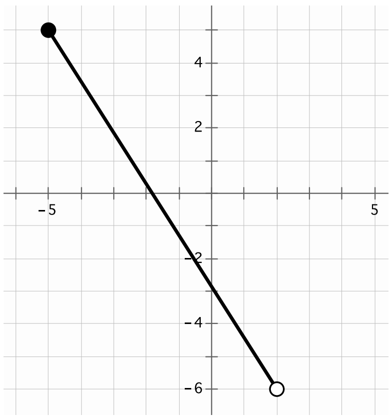
1. Write a well-developed paragraph comparing and contrasting linear and exponential functions. Be sure to include as many characteristics of each function as possible and be clear about the similarities and differences these functions have.

Set

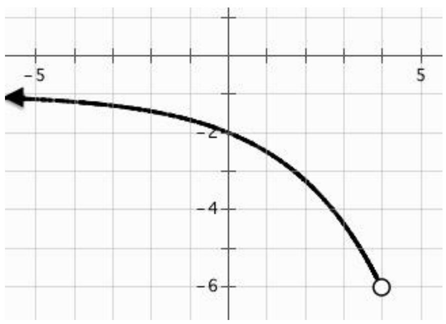
Topic: Identifying attributes of a function from its graphical representation.

Based on the graph given in each problem below, identify attributes of the function such as the domain, range and whether or not the function is increasing or decreasing, etc.

2.



3.



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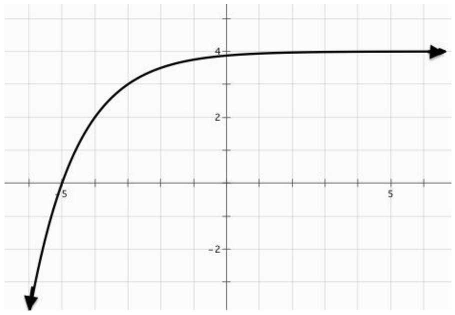

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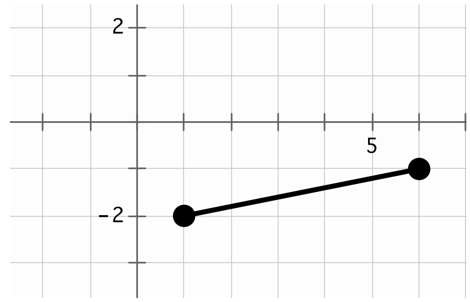

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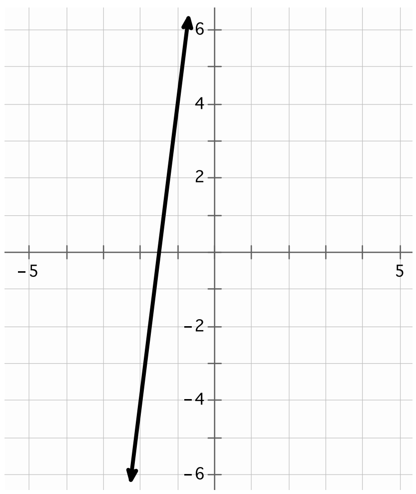

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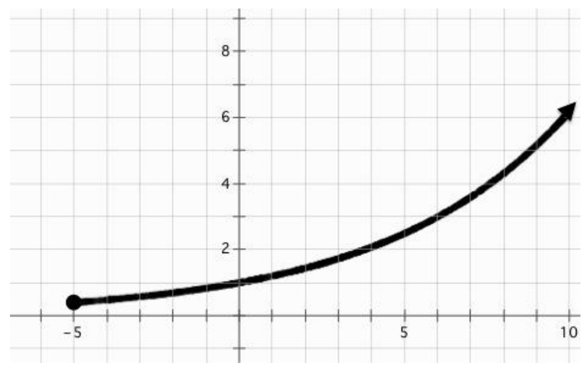

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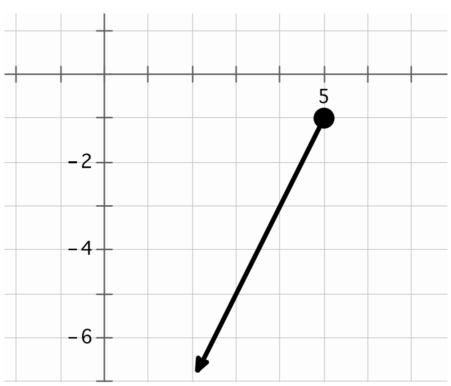

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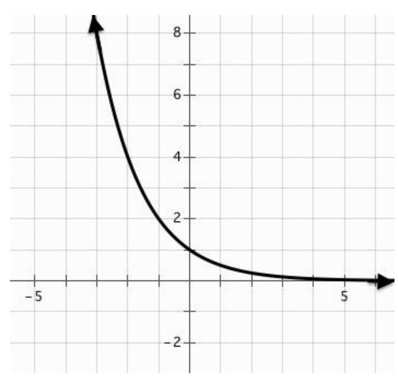

Go

Topic: Finding equations and rules for functions

Find both the explicit and the recursive equations for each table of values below.

10.

n	$f(n)$
1	3
2	5
3	7
4	9

Explicit:

Recursive:

11.

n	$f(n)$
2	4
3	8
4	16
5	32

Explicit:

Recursive:

12.

n	$f(n)$
6	23
7	19
8	15
9	11

Explicit:

Recursive:

13.

n	$f(n)$
1	1
2	3
3	9

Explicit:

Recursive:

14.

n	$f(n)$
3	8
4	4
5	2

Explicit:

Recursive:

15.

n	$f(n)$
6	7
9	13
12	19

Explicit:

Recursive:

16.

n	$f(n)$
2	40
4	32
8	16

Explicit:

Recursive:

17.

n	$f(n)$
2	16
3	4
4	1

Explicit:

Recursive:

18.

n	$f(n)$
17	5
20	10
26	20

Explicit:

Recursive:

5.5 Pooling It Together
A Solidify Understanding Task

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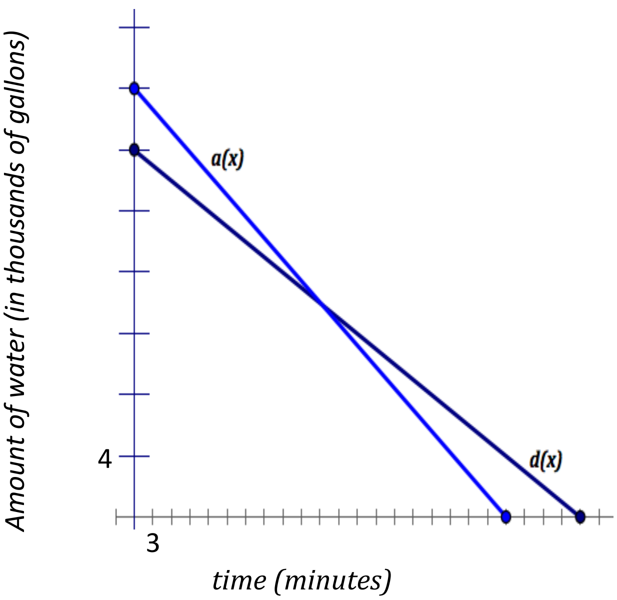
Aly and Dayne work at a water park and have to drain the water at the end of each month for the ride they supervise. Each uses a pump to remove the water from the small pool at the bottom of their ride. The graph below represents the amount of water in Aly's pool, *a(x),* and Dayne's pool, *d(x)*, over time. In this scenario, they decided to work together to drain their pools and created the equation *g(x) = a(x) +d(x).* Using the graph below showing *a(x)* and *d(x),* create a new set of axes and graph *g(x).* Identify *g(x)* and label (scale, axes).



Answer the following questions about *g(x)*.

1. What does *g(x)* represent?

2. Name the features of *g(x)* and explain what each means (each intercept, domain and range for this situation and for the equation, maxima and minima, whether or not *g(x)* is a function, etc.)

3. Write the equation for *g(x)* using the intercepts from the graph. Compare this equation to the sum of the equations created for *a(x)* and *d(x)* from "The Water Park" task. Should be they be equivalent? Why or why not?

When combining functions, a lot of connections can be made. Make at least three connections showing how the equations *a(x), d(x),* and *g(x)* relate to the graphs of *a(x), d(x),* and *g(x).*(hint: think about the key features of these functions).

For A Twist:

If Aly and Dayne's boss started to drain the water before they arrived and when they got there, there was already 5,000 less gallons of water to be drained, how would this impact the equation?

Write the new equation representing how long it will take them to drain the two pools.

Ready, Set, Go!

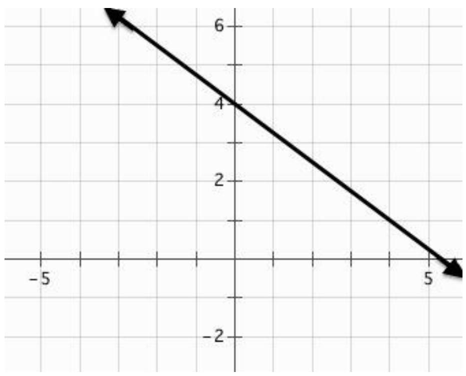
© eilt

Ready

Topic: Use a graphical representation to find solutions.

Use the graph of each function provided to find the values indicated.

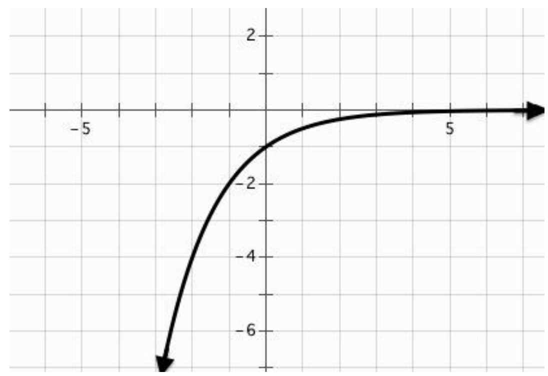
1. . $f(x)$



a. $f(4)$ = _______ b. $f(-4)$ = _________

c. $f(x) = 4$, x = ________ d. $f(x) = 7$, x = _______

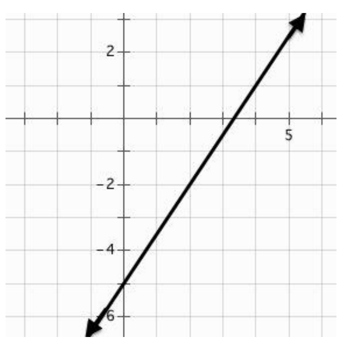
2. $g(x)$



a. $g(-1)$= _________ b. $g(-3)$ = __________

c. $g(x) = -4$ x = ______ d. $g(x) = -1$, x = ________

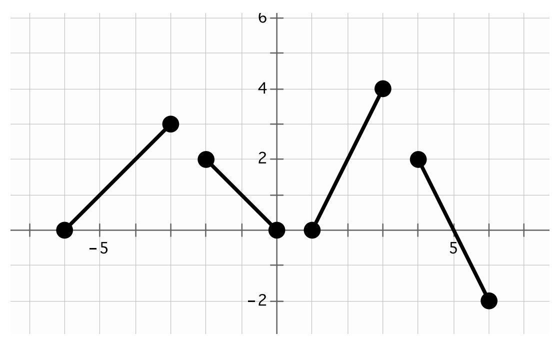
3. $h(x)$



a. $h(0)$ = _______ b. $h(3)$ = _________

c. $h(x) = 1$, x = ________ d. $h(x) = -2$, x = _____

4. $d(x)$



a. $d(-5)$ = _______ b. $d(4)$ = _________

c. $d(x) = 4$, x = ________ d. $d(x) = 0$, x = _____

Set

Topic: Given context of a function find solutions.

For each situation either create a function or use the given function to find and interpret solutions.

5. Fran collected data on the number of feet she could walk each second and wrote the following rule to model her walking rate $d(t) = 4t$.

a. What is Fran looking for if she writes "$d(12) =$" ?

b. In this situation what does $d(t) = 100$ tell you?

c. How can the function rule be used to indicate a time of 16 seconds was walked?

d. How can the function rule be used to indicated that a distance of 200 feet was walked?

6. Ms. Callahan works hard to budget and predict her costs for each month. She is currently attempting to determine how much her cell phone company will likely charge her for the month. She is paying a flat fee of $80 a month for a plan that allows for unlimited calling but costs her an additional twenty cents per text message.

a. Write a function, c(t), for Ms. Callahan's current cell plan that will calculate the cost for the month based on the number of text messages she makes.

b. Find c(20)

d. Find c(t) = 100

c. Find c(45)

e. Find c(t) = 90

f. At what number of texts would $20 unlimited texting be less expensive then her current plan?

7. Mr. Multbank has developed a population growth model for the rodents in the field by his house. He believes that starting each spring the population can be modeled based on the number of weeks with the function $p(t) = 8(2^t)$.

a. Find $p(t) = 128$ b. Find $p(4)$ c. Find $p(10)$

d. Find the number of weeks it will take for the population to be over 20,000.

e. In a year with 16 weeks of summer, how many rodents would he expect by the end of the summer using Mr. Multbank's model? What are some factors that could change the actual result from your estimate?

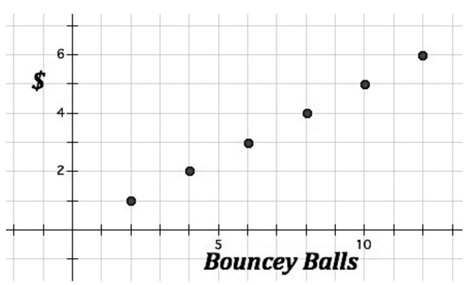
Go

Topic: Discrete and continuous

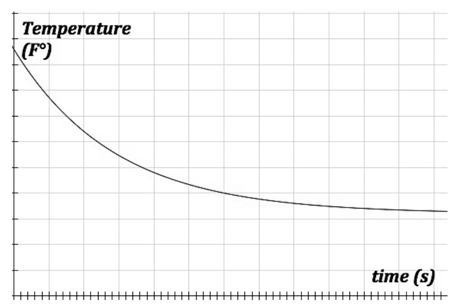
For each context or representation determine whether it is discrete or continuous or could be modeled best in a discrete or continuous way and state why.

8. Susan has a savings plan where she places \$5 a week in her piggy bank.

9.



10.



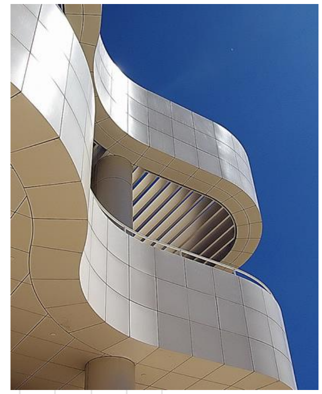
11. Marshal tracks the number of hits he gets each baseball game and is recording his total number of hits for the season in a table.

12. The distance you have traveled since the day began.

13.

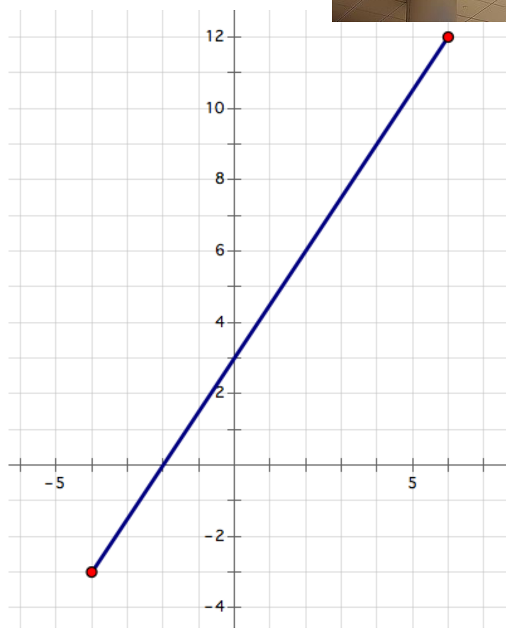
Number of Gum Balls	Cost
5	1
10	2
15	3
20	4

5.6 Interpreting Functions
A Practice Understanding Task



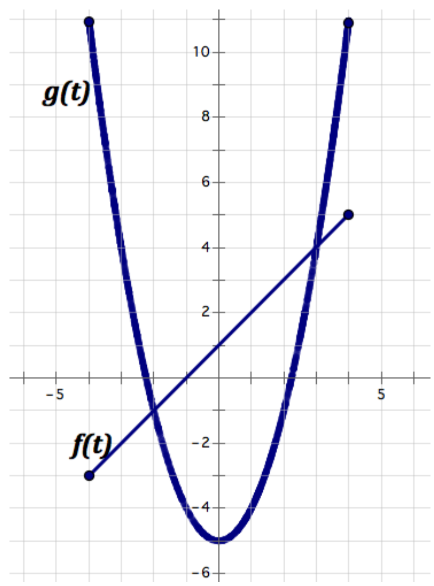
Given the graph of *f(x),* answer the following questions. Unless otherwise specified, restrict the domain of the function to what you see in the graph below. Approximations are appropriate answers.

1. What is *f*(2)?
2. For what values, if any, does *f(x)* = 3?
3. What is the x-intercept?
4. What is the domain of *f(x)*?
5. On what intervals is *f(x)* > 0?
6. On what intervals is *f(x)* increasing?
7. On what intervals is *f(x)* decreasing?
8. For what values, if any, is *f(x)* > 3?



Consider the linear graph of *f(t)* and the nonlinear graph of *g*(*t*) to answer questions 9-14. Approximations are appropriate answers.

9. Where is *f(t)* = *g*(*t*)?
10. Where is *f(t)* > *g(t)* ?
11. What is *f(0) + g(0)* ?
12. What is *f(-1) + g(-1)* ?
13. Which is greater: *f(0) or g(-3)*?
14. Graph: *f(t)* + *g(t)* from [-1, 3]



The following table of values represents two continuous functions, *f(x) and g(x)*. Use the table to answer the following questions:

x	*f(x)*	*g(x)*
-5	42	-13
-4	30	-9
-3	20	-5
-2	12	-1
-1	6	3
0	2	7
1	0	11
2	0	15
3	2	19
4	6	23
5	12	27
6	20	31

15. What is $g(-3)$?

16. For what value(s) is $f(x) = 0$?

17. For what values is $f(x)$ increasing?

18. On what interval is $g(x) > f(x)$

19. Which function is changing faster in the interval [-5, 0]? Why?

Use the following relationships to answer the questions below.

$h(x) = 2^x$ $\quad f(x) = 3x - 2$ $\quad g(x) = 5$ $\quad x = 4$ $\quad y = 5x + 1$

20. Which of the above relations are functions? Explain.

21. Find *f(2), g(2), and h(2).*

22. Write the equation for $g(x) + h(x)$.

23. Where is $g(x) < h(x)$?

24. Where is $f(x)$ increasing?

25. Which of the above functions has the fastest growth rate?

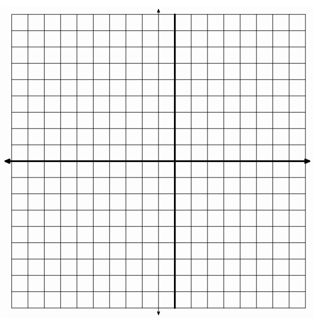
Ready, Set, Go!

Ready

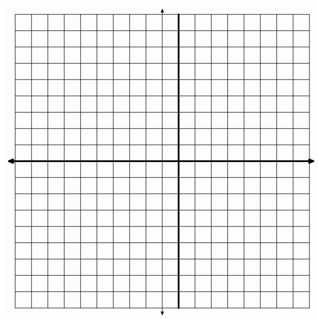
Topic: Solve systems of equations

Solve each system of equations either by graphing, substitution, elimination, or matrix row reduction. *Use each method at least once.*

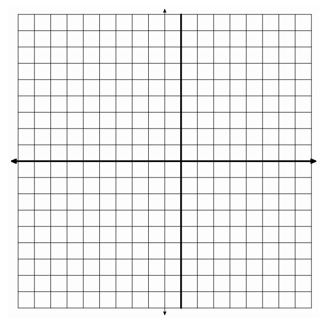
1. $\begin{cases} 3x + 4 \\ 4x + 1 \end{cases}$



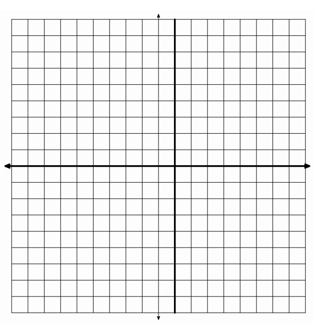
2. $\begin{cases} -5x + 12 \\ -2x - 3 \end{cases}$



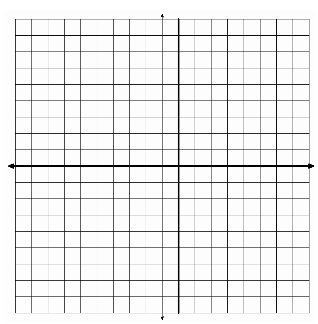
3. $\begin{cases} \frac{1}{2}\ x + 2 \\ 2x - 7 \end{cases}$



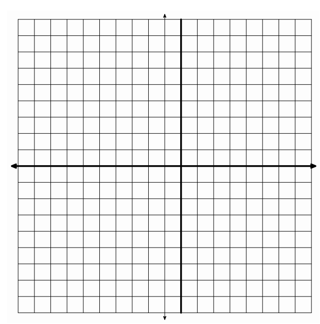
4. $\begin{cases} -\frac{2}{3}\ x + 5 \\ -x + 7 \end{cases}$



5. $\begin{cases} x + 5 \\ -x - 3 \end{cases}$



6. $\begin{cases} x - 6 \\ -x - 6 \end{cases}$

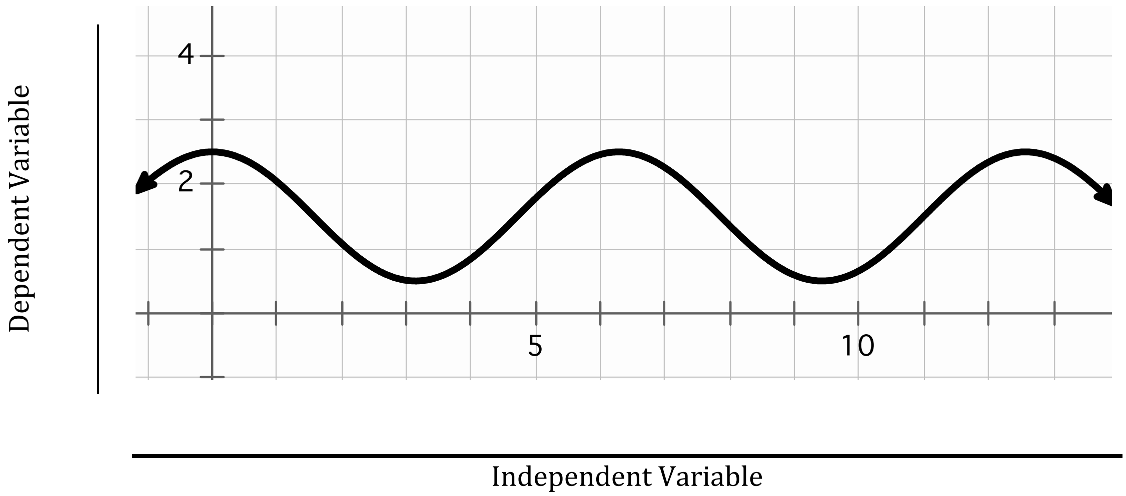


Set

Topic: Connecting context to graphical representations

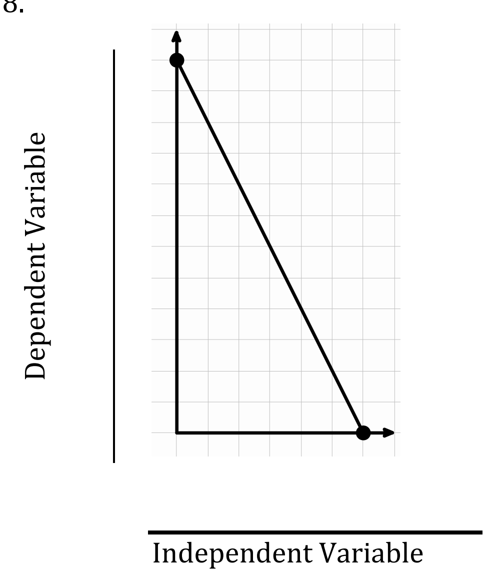
For each graph create a context, provide independent and dependent variables that will fit the context you choose. Then create a story that describes what is happening on the graph.

7.



Description of context and a story for the graph:

8.



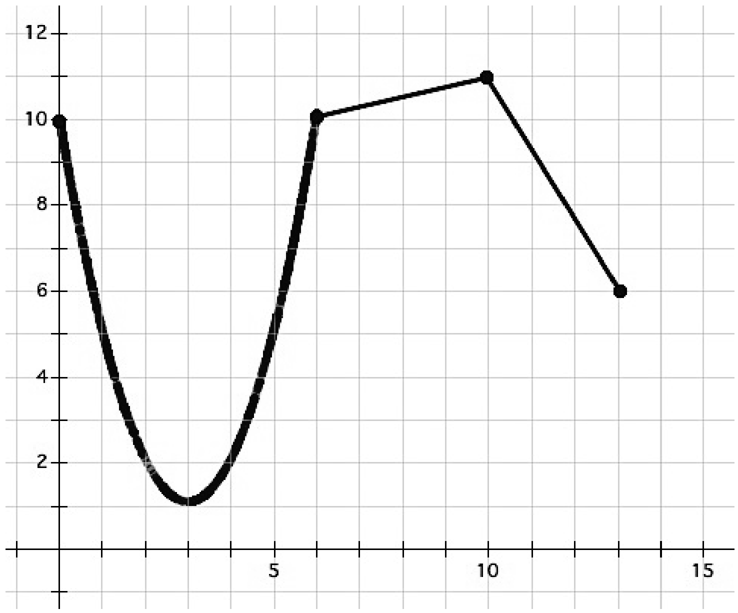
Description of context and a story for the graph:

Go

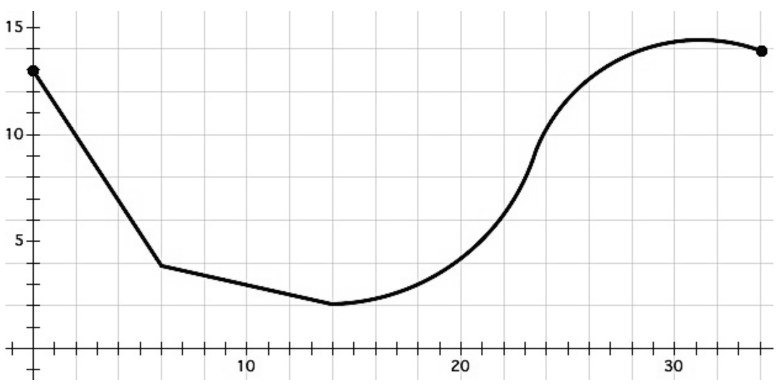
Topic: Describe features of a function from its graphical representation.

For each graph given provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.

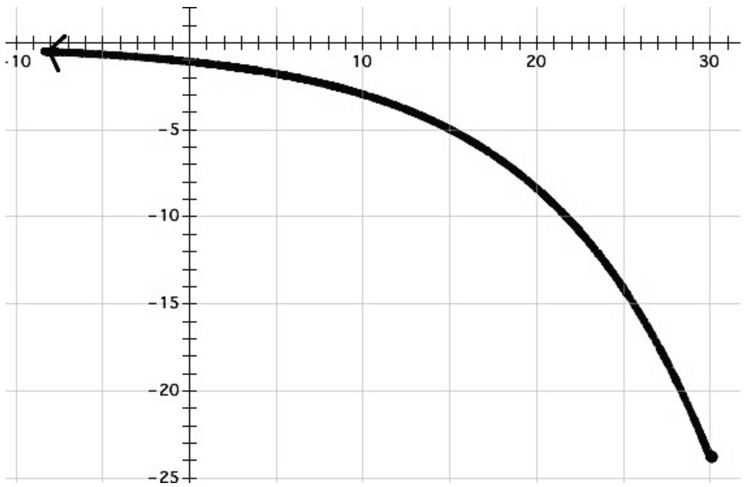
9. Description of function:



10. Description of function:

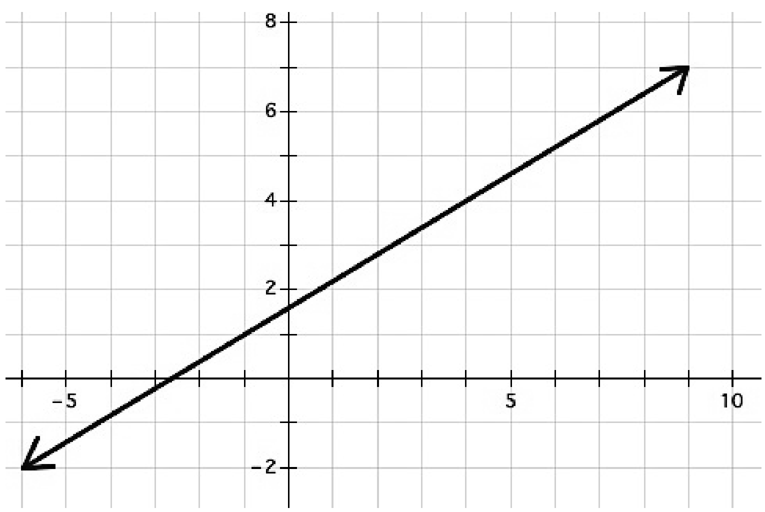


11.



Description of function:

12.



Description of function:

5.7 A Water Function

A Develop Understanding Task

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Andrew walked around the water park taking photos of his family with his phone. Later, he discovered his phone was missing. So that others could help him look for his lost phone, he drew a picture that 'retraced his steps' showing where he had walked.

If we wanted to determine Andrew's location in the park with respect to time, would his location be a function of time?

Why or why not? Explain.

1. Situation A: Sketch a graph of the total distance Andrew walked if he walked at a constant rate the entire time.

2. Situation B: Sketch a graph of Andrew's distance from the entrance (his starting point) as a function of time.

3. How would the graph of each situation change if Andrew stopped at the slide for a period of time? Would this change whether or not this situation is a function?

Ready, Set, Go!

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Ready

Topic: Mathematical comparisons

Use the given comparison statements to answer the questions.

1. 3 out of 5 students prefer playing football to playing basketball.

   a. What percent of students prefer playing football?

   b. What percent of students prefer playing basketball?

2. The ratio of student wearing yellow to students not wearing yellow is 3 to 7.

   a. What fraction of students have on yellow?

   b. What percent of students don't have on yellow?

3. Of the students at school, 40% attended the basketball game.

   a. What fraction of the students attended the basketball game?

   b. How many times more students did not attend the basketball game?

4. 1000 students ride buses to school while 600 walk or carpool.

   a. What fraction of students ride the bus?

   b. How many more students ride the bus than walk or carpool?

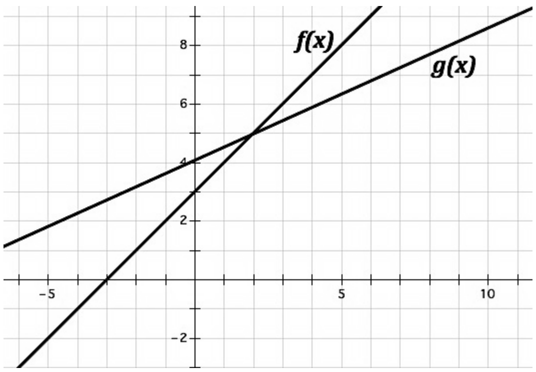
   c. What percent of students walk or carpool?

Set

Topic: Comparing functions from different representations

Use the given representation of the functions to answer the questions.

5.



a. Where does $f(x) = g(x)$?

b. What is $f(4) + g(4)$?

c. What is g(-2) – f(-2) ?

d. On what interval is $g(x) > f(x)$?

e. Sketch f(x) + g(x) on the graph provided.

6. The functions $a(x)$ and $b(x)$ are defined in the table below. Each function is a set of exactly five ordered pairs.

x	$a(x)$	$b(x)$
-3	1	-1
-1	7	-5
0	3	-10
2	8	2
7	3	3

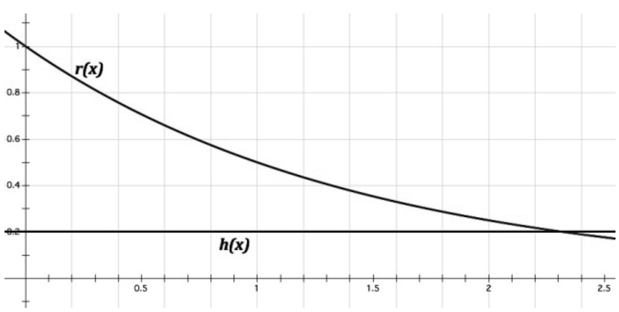
a. What is $a(-3)+b(-3)$?

b. What is $a(-1) - b(-1)$?

c. What is $a(0) + b(0)$?

d. Add two columns to the table and provided a(x) + b(x) in one and a(x) – b(x) in the other.

7.



a. Where is $r(x) > h(x)$?

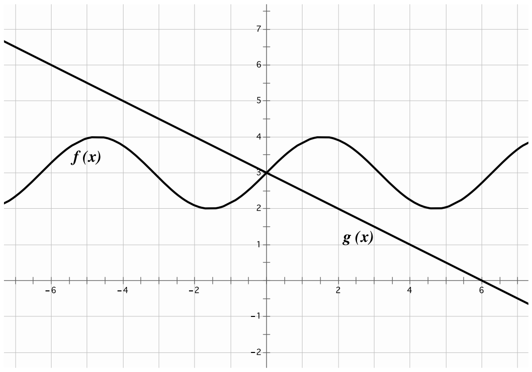
b. What is $r(1) - h(1)$?

c. What is $r(0) + h(0)$?

d. Create an explicit rule for $r(x)$ and for $h(x)$.

e. Sketch $r(x) - h(x)$ on the graph.

8.



a. Where does f(x) = g(x)?

b. What is f(4) + g(4) ?

c. What is g(-2) – f(-2) ?

d. On what interval is g(x) > f(x) ?

e. Sketch f(x) - g(x) on the graph provided.

Go

Topic: Solving equations for a specified variable. Literal equations.

Rewrite each equation in slope-intercept form (y = mx+b).

9. $12x + 3y = 6$

10. $8x + y = 5$

11. $y - 5 = -3(x + 2)$

12. $9x - y = 7$

13. $y - 9x = 4(x - 2)$

14. $16x = 20 + 8y$

Write an explicit function for the linear function that goes through the given point with the given slope simplified into slope-intercept form.

15. m = 3, (-1, 2)

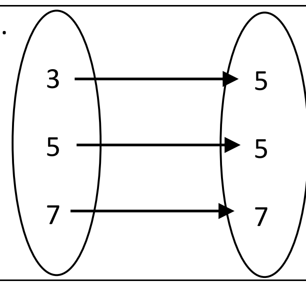
16. m = -5, (3, 4)

17. m = $\frac{3}{4}$, (-4, 2)

5.8 To Function or Not To Function

A Practice Understanding Task

Determine if the following relationships are functions and then justify your reasoning.

<table>
<tr>
<td>1.A person's name versus their social security number.</td>
<td>2. A person's social security number versus their name.</td>
<td>3. The cost of gas versus the amount of gas pumped.</td>
</tr>
<tr>
<td>4.
{ (3,6), (4, 10), (8,12), (4, 10)}</td>
<td>5. The temperature in degrees Fahrenheit with respect to the time of day.</td>
<td>6.
<table>
<tr><td>distance</td><td>days</td></tr>
<tr><td>6</td><td>2</td></tr>
<tr><td>10</td><td>4</td></tr>
<tr><td>6</td><td>5</td></tr>
<tr><td>9</td><td>8</td></tr>
</table>
</td>
</tr>
<tr>
<td>7. The area of a circle as it relates to the radius.</td>
<td>8.

</td>
<td>9. The radius of a cylinder is dependent on the volume.</td>
</tr>
<tr>
<td>10. The size of the radius of a circle dependent on the area.</td>
<td>11. Students letter grade dependent on the percent earned.</td>
<td>12. The length of fence needed with respect to the amount of area to be enclosed.</td>
</tr>
<tr>
<td>13. The explicit formula for the recursive situation below:
f(1) = 3 and
f(n + 1) = f(n) + 4</td>
<td>14.
If x is a rational number, then f(x) = 1
If x is an irrational number, then f(x) = 0</td>
<td>15. The national debt with respect to time.</td>
</tr>
</table>

Ready, Set, Go!

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Ready

Topic: Determine domain and range, and whether a relation is a function or not a function.

Determine if each set of ordered pairs is a function or not then state the domain and range.

1. { (-7, 2), (3, 5), (8, 4), (-6, 5), (-2, 3)} Function: Yes / No

Domain: Range:

2. { (9, 2), (0, 4), (4, 0), (5, 3), (2, 7) (0, -3), (3, -1)} Function: Yes / No

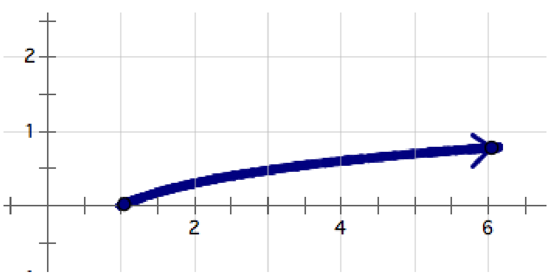
Domain: Range:

3. { (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9)} Function: Yes / No

Domain: Range:

For the representation of the function given determine the domain and range.

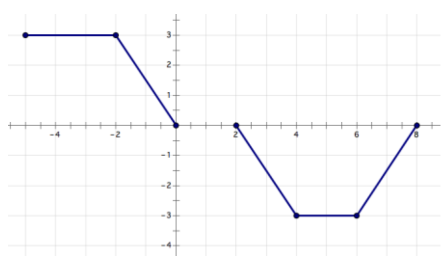
4.



Domain:

Range:

5.



Domain:

Range:

6. $f(x) = -2x + 7$

Domain:

Range:

7. $g(x) = 3\,(5)^x$

Domain:

Range:

8. The elements in the table define the entirety of the function.

x	h(x)
1	9
2	98
3	987
4	9876

Domain:

Range:

Set

Topic: Determine whether or not the relationship is a function.

Determine if the relationship presented is a function or not and provide a justification.

9. The distance a person is from the ground related to time as they ride a Ferris Wheel.

10. The amount of daylight during a day throughout the calendar year.

11. The value of a Volkswagen Bug convertible from time of first purchase in 1978 to now.

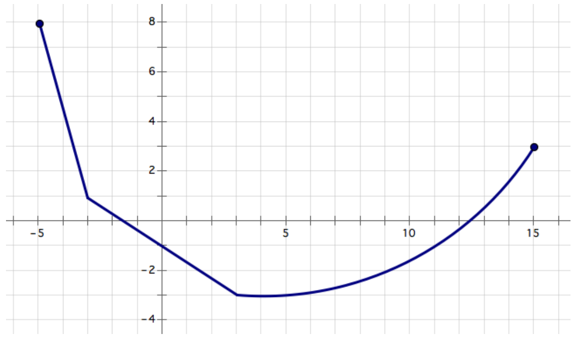
12. A person's name and their phone number.

13. The stadium in which a football player is playing related to the outcome of the game.

Go

Topic: Determining features of functions and finding solutions using functions.

14. For the graph given below provide a description of the function. Be sure to consider the following: decreasing/increasing, min/max, domain/range, etc.



15. For the given situation use the given function to find and interpret solutions.

Hope has been tracking the progress of her family as they travel across the country during their vacation and she has created a function, d(t)= 78t to model the progress they are making.

a. What would Hope be attempting to find if she writes "d(4)= 78(4)" ?

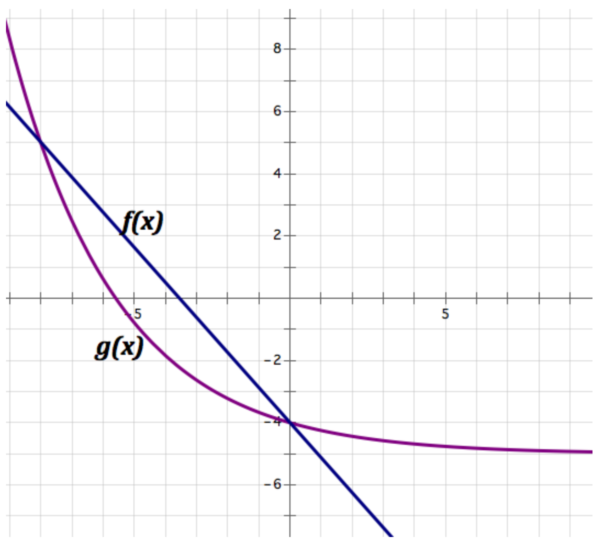
b. What would d(t)= 450 mean in this situation?

c. What would d(3.5) mean in this situation?

d. How could Hope use the function to find the time it would take to travel 800 miles?

Use the given representation of the functions to answer the questions.

16.



a. Where does $f(x) = g(x)$?

b. What is $g(0) + f(0)$?

c. On what interval(s) is $g(x) > f(x)$?

d. What is $g(-8) + f(-8)$?

5.9 Match That Function

A Practice Understanding Task

Welcome to Match That Function! To play, sort the deck of cards into sets by grouping cards together that describe a specific relationship. Each set is supposed to have four cards; however, one card is missing from every set. After you have sorted the cards into ten sets, create a fourth card for each set that would complete the set. Be sure to use a different representation than what is provided.

Ready, Set, Go!

Ready

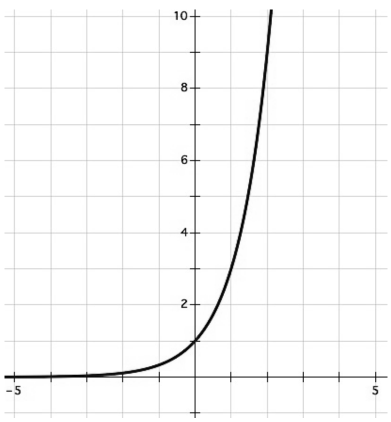
Topic: Find the output or input based on what is given.

For each function find the desired solutions.

1. $h(t) = 2t - 5$

a. $h(-4) =$ ____   b. $h(t) = 23, t =$ ____   c. $h(13) =$ ____   d. $h(t) = -33, t =$ _____

2. $g(x)$



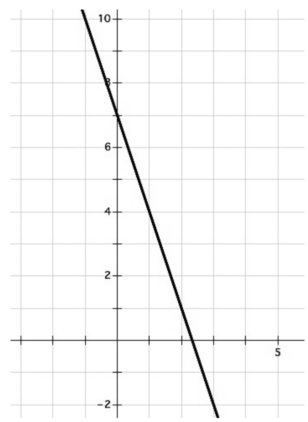
a. $g(2) =$ _____

b. $g(x) = 3$, $x =$ _____

c. $g(0) =$ ______

d. What is the explicit rule for $g(x)$

3. $r(x)$



a. $r(-1) =$ _____

b. $r(x) = 4$, $x =$ _____

c. $r(2) =$ _____

d. What is the explicit rule for $r(x)$

Set

Topic: Describing the key features of functions and creating a representation of a function given the key features.

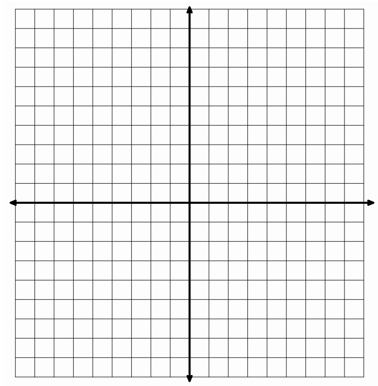
Use the given description of several of the key features of the function to sketch a possible graph of the function.

4. Domain contains all Real numbers between -2 and 3.

   Range contains all Real numbers between 3 and 7.

   The function is increasing from -2 to 0 and decreasing after 0.

   The function is not continuous at every point.

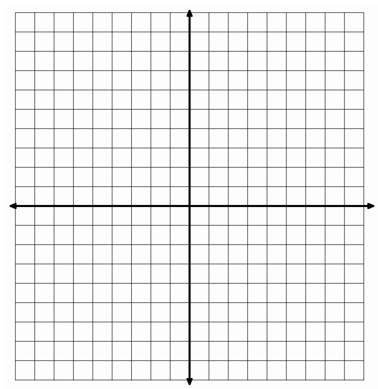


5. The function has a minimum at -5.

   The function has a maximum at 8.

   The function has two intervals on which it is decreasing and one interval on which it is increasing.

   The Domain of the functions contains all Real numbers from 1 to 9.

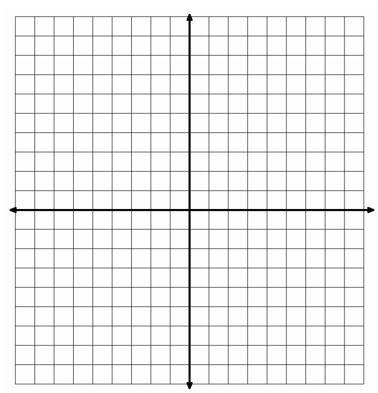


6. This function is not continuous anywhere.

   The function contains only seven elements in its domain.

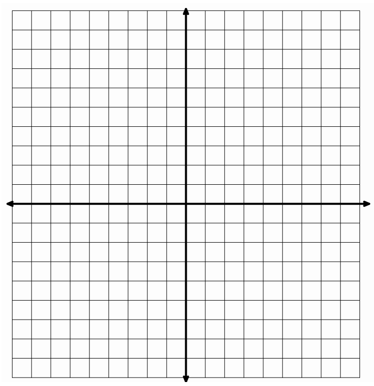
   The values of the domain are between -10 and 2.

   The values of the range are between -1 and 1.



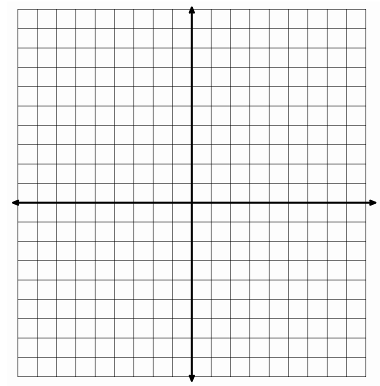
7. The function has three intervals on which its slope is zero.

The function has a maximum and a minimum.



8. The domain of the function is [-5, ¥)

The range of the function is [0, ¥)

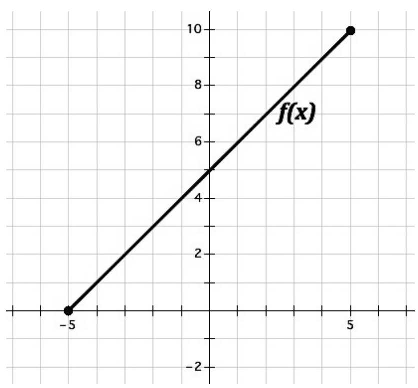


Go

Topic: Determine the following for each function: domain, range, discrete, continuous, increasing, decreasing, etc.

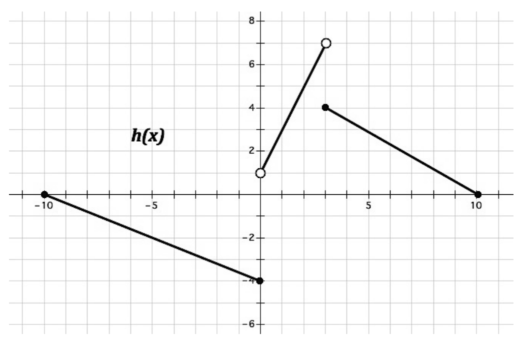
Given the representation of the function(s) provided determine the domain, range, and whether the function is discrete, continuous, increasing, decreasing, etc.

9.

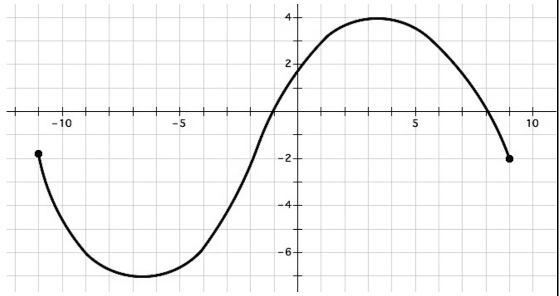


Description of Function:

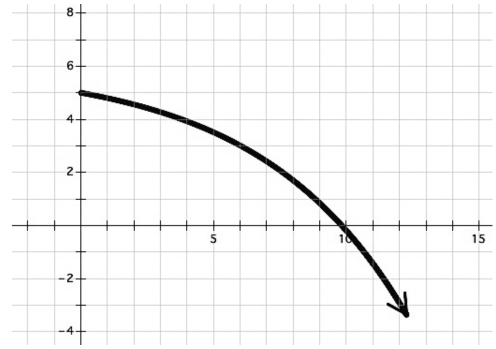
10.



Description of Function:

11. 

Description of Function:

12. 

Description of Function:

13. $f(0) = 2, f(n+1) = 3(f(n))$ Description of Function:

14. $g(x) = -9 + 4x$ Description of Function:

15. $f(x) = |x|$ Description of Function:

Secondary One Mathematics: An Integrated Approach
Module 6
Congruence, Construction and Proof

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 6 – Congruence, Construction and Proof

Classroom Task: Leaping Lizards! - A Develop Understanding Task
Developing the definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.1

Classroom Task: Is It Right? - A Solidify Understanding Task
Examining the slope of perpendicular lines (**G.CO.1, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.2

Classroom Task: Leap Frog– A Solidify Understanding Task
Determining which rigid-motion transformations carry one image onto another congruent image (**G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.3

Classroom Task: Leap Year – A Practice Understanding Task
Writing and applying formal definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.2, G.CO.4, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.4

Classroom Task: Symmetries of Quadrilaterals – A Develop Understanding Task
Finding rotational symmetry and lines of symmetry in special types of quadrilaterals (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.5

Classroom Task: Symmetries of Regular Polygons – A Solidify Understanding Task
Examining characteristics of regular polygons that emerge from rotational symmetry and lines of symmetry (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.6

Classroom Task: Quadrilaterals-Beyond Definition – A Practice Understanding Task
Making and justifying properties of quadrilaterals using symmetry transformations (**G.CO.3, G.CO.4, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.7

Classroom Task: Can You Get There From Here? – A Develop Understanding Task
Describing a sequence of transformations that will carry congruent images onto each other (**G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.8

Classroom Task: Congruent Triangles – A Solidify Understanding Task
Establishing the ASA, SAS and SSS criteria for congruent triangles (**G.CO.6, G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.9

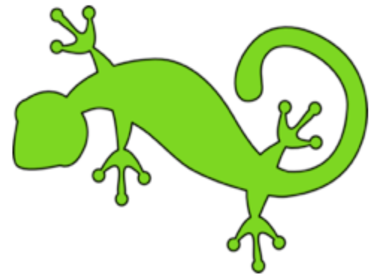
Classroom Task: Congruent Triangles to the Rescue – A Practice Understanding Task
Working with systems of linear equations, including inconsistent and dependent systems
(**G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.10

Classroom Task: Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct rhombuses and squares
(**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.11

Classroom Task: More Things Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct parallelograms, equilateral triangles and inscribed hexagons (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.12

Classroom Task: Justifying Constructions – A Solidify Understanding Task
Examining why compass and straightedge constructions produce the desired objects (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.13

Classroom Task: Construction Blueprints – A Practice Understanding Task
Writing procedures for compass and straightedge constructions (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.14



6.1 Leaping Lizards!

A Develop Understanding Task

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns) or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:

{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the **pre-image** and **image** anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them

Lazy Lizard

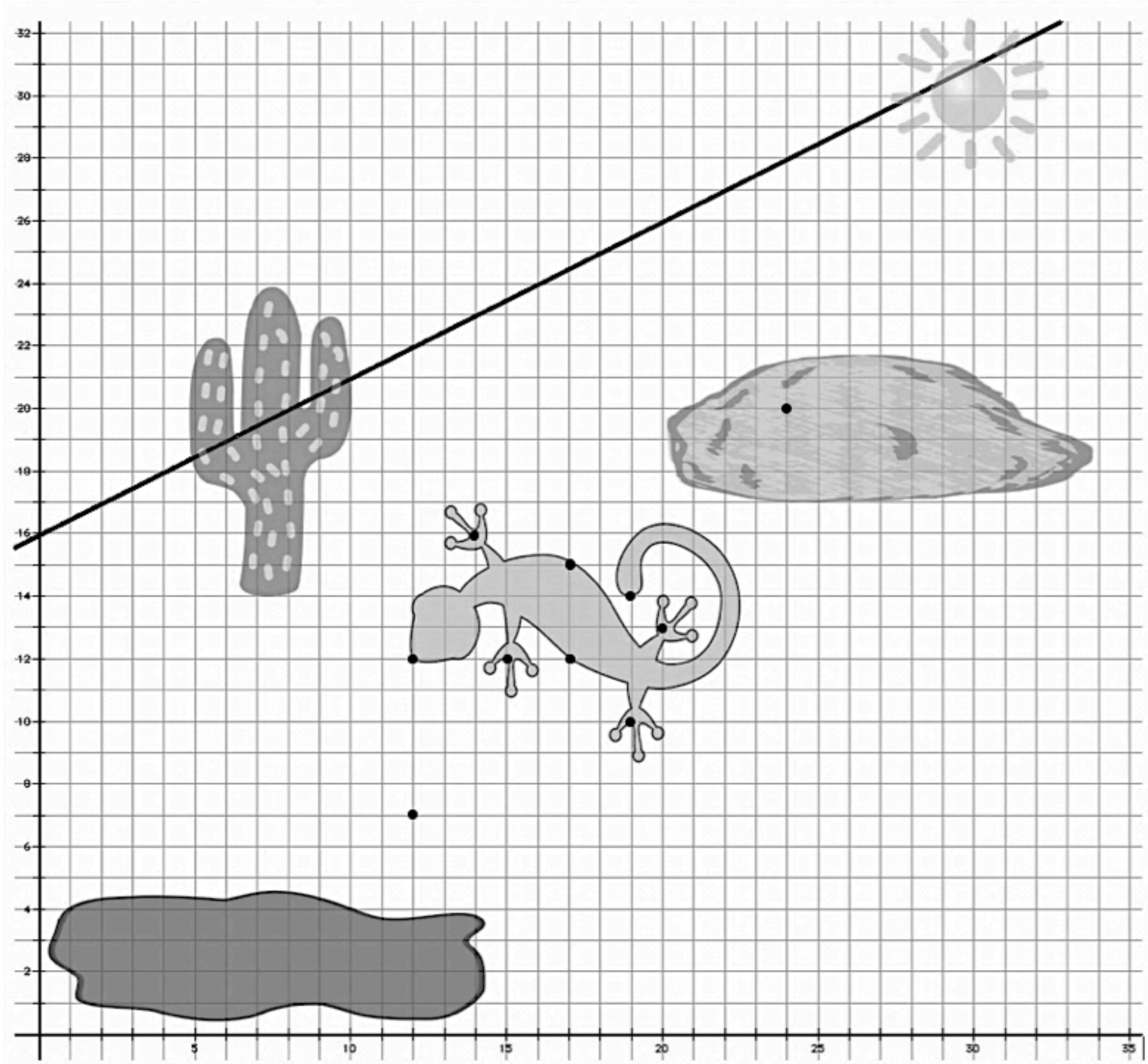
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

Lunging Lizard

Rotate the lizard 90° about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard

Reflect the lizard about given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.



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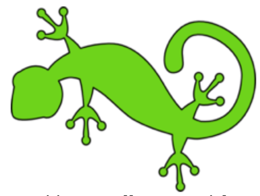
Ready, Set, Go!



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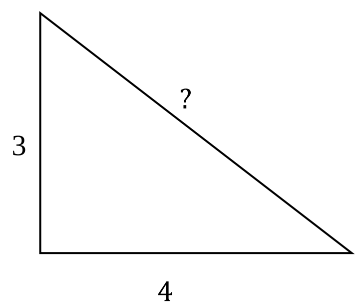
Ready

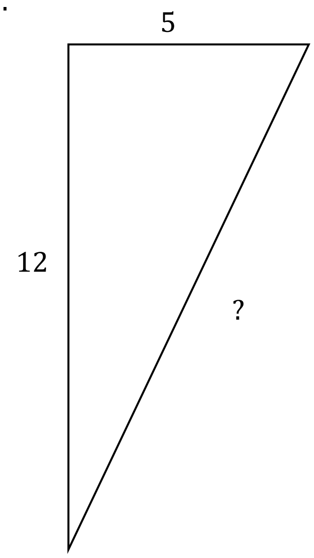
Topic: Pythagorean Theorem

For each of the following right triangles determine the number units measure for the missing side.

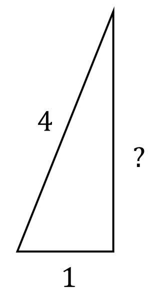
1.



2.



3.



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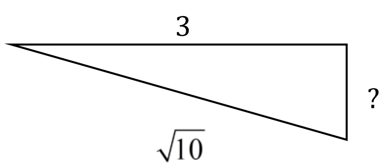
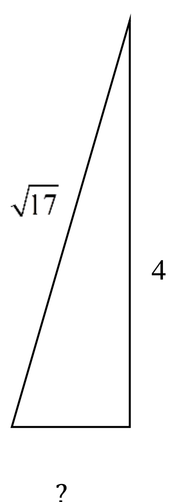
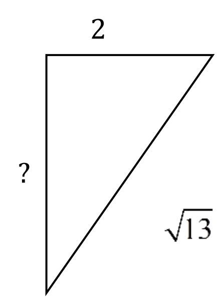


5.



6.



Set

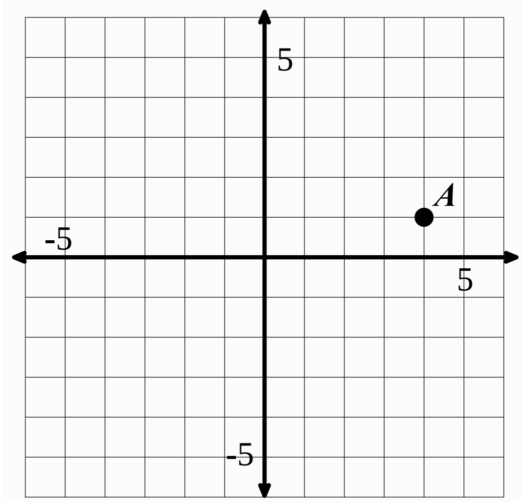
Topic: Transformations

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A'

b. Reflect point A over x-axis, label as A''

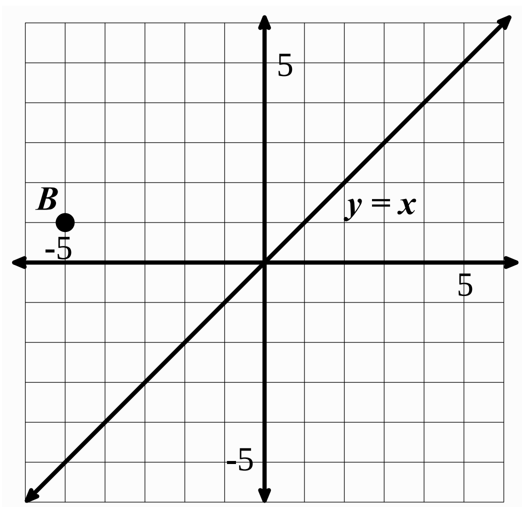
c. Apply the rule $(x-2\,, y-5)$, to point A and label A'''



8a. Reflect point B over the line $y = x$, label as B'

b. Rotate point B 180° about the origin, label as B''

c. Translate point B the point up 3 and right 7 units, label as B'''

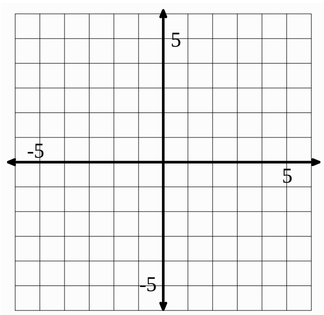


Go

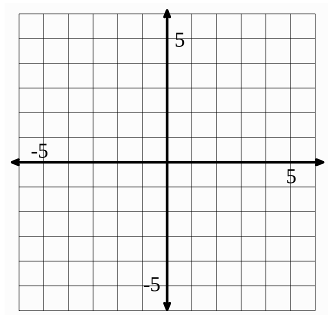
Topic: Graphing linear equations

Graph each equation on the coordinate grid provided. Extend the line as far as the grid will allow.

9. $y = 2x - 3$

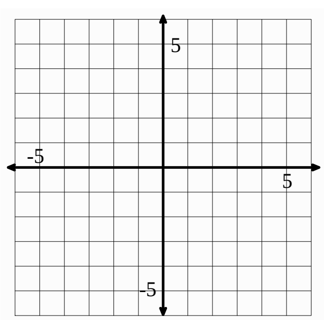


10. $y = -2x - 3$

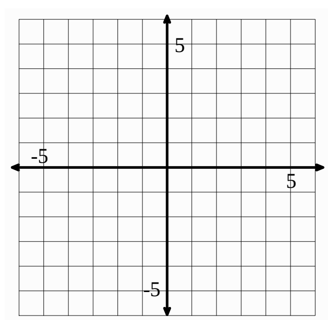


11. What similarities and differences are there between the equations in number 13 and 14?

12. $y = \frac{2}{3}x + 1$

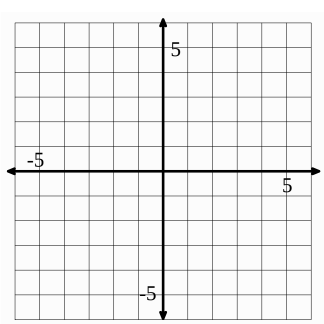


13. $y = -\frac{3}{2}x + 1$

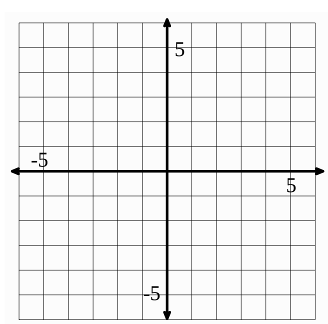


14. What similarities and differences are there between the equations in number 15 and 16?

15. $y = x + 1$



16. $y = x - 3$



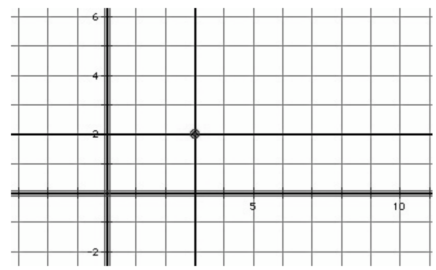
17. What similarities and differences are there between the equations in number 15 and 16?

6.2 Is It Right?
A Solidify Understanding Task

In *Leaping Lizards* you probably thought a lot about perpendicular lines, particularly when rotating the lizard about a 90° angle or reflecting the lizard across a line.

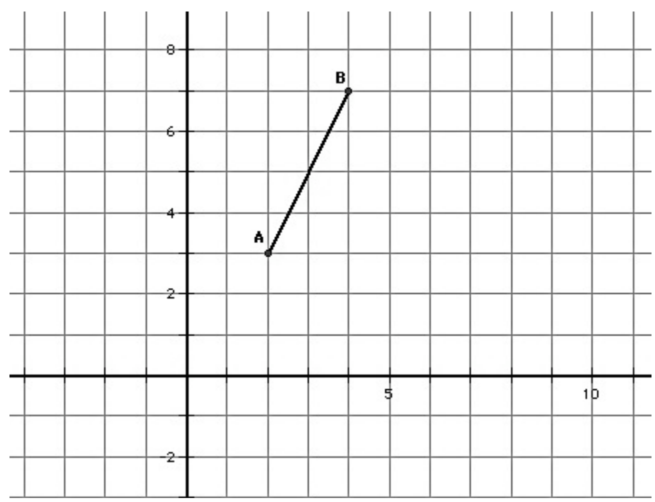
In previous tasks, we have made the observation that *parallel lines have the same slope*. In this task we will make observations about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: *Horizontal and vertical lines are perpendicular.* For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.



But what if a line or line segment is not horizontal or vertical?
How do we determine the slope of a line or line segment that will be perpendicular to it?

Experiment 1



1. Consider the points A (2, 3) and B (4, 7) and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

2. Locate a third point C (x, y) on the coordinate grid, so the points A (2, 3), B (4, 7) and C (x, y) form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

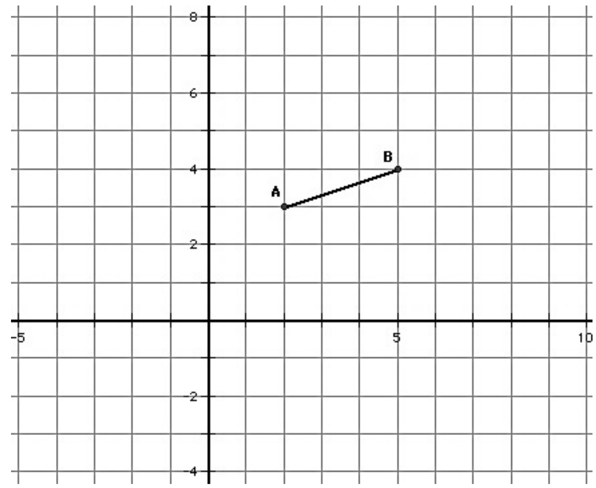
3. Explain how you know that the triangle you formed contains a right angle?

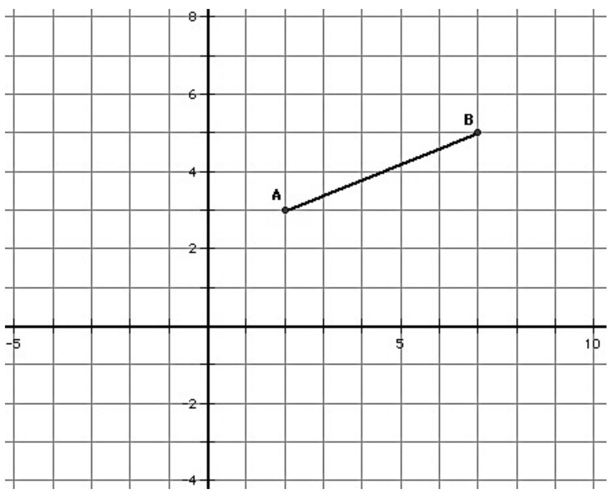
4. Now rotate this right triangle 90° about the vertex point (2, 3). Explain how you know that you have rotated the triangle 90°.

5. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

Experiment 2

Repeat steps 1-5 above for the points A (2, 3) and B (5, 4).



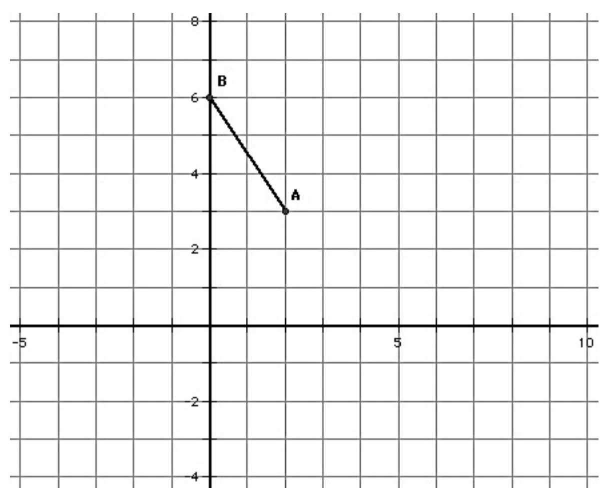


Experiment 3

Repeat steps 1-5 above for the points A (2, 3) and B (7, 5).

Experiment 4

Repeat steps 1-5 above for the points A (2, 3) and B (0, 6).



Based on experiments 1-4, state an observation about the slopes of perpendicular lines.

While this observation is based on a few specific examples, can you create an argument or justification for why this is always true? (Note: You will examine a formal proof of this observation in the next module.)

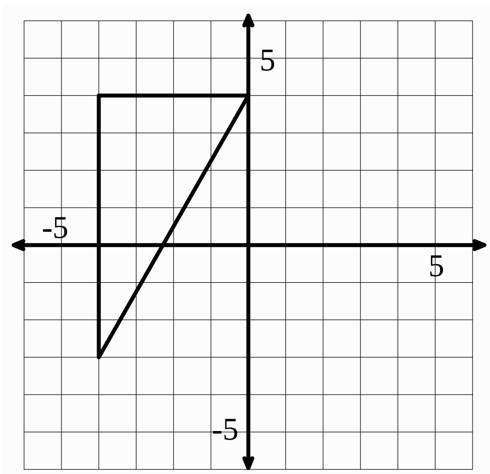
Ready, Set, Go!

Ready

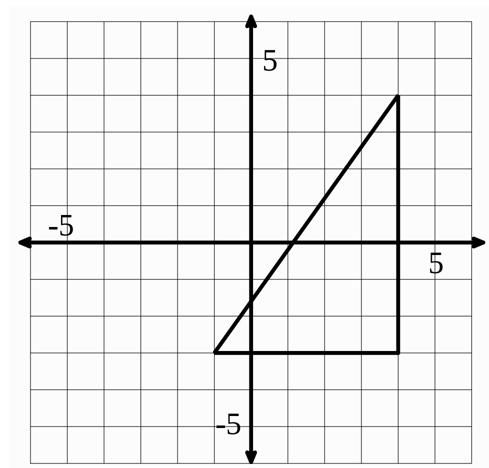
Topic: Finding Distance using Pythagorean Theorem

Use the coordinate grid to find the length of each side of the triangles provided.

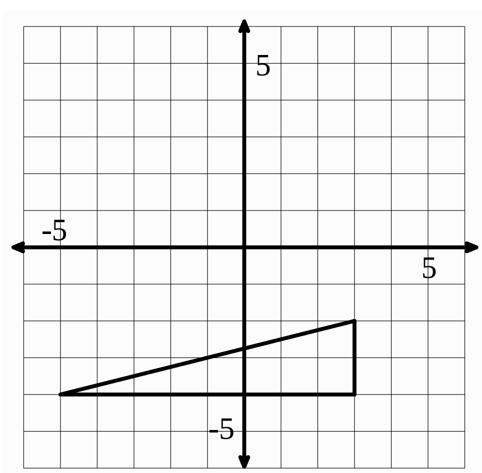
1.



2.



3.



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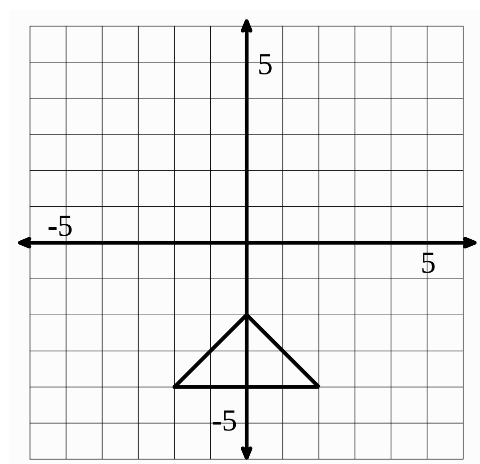
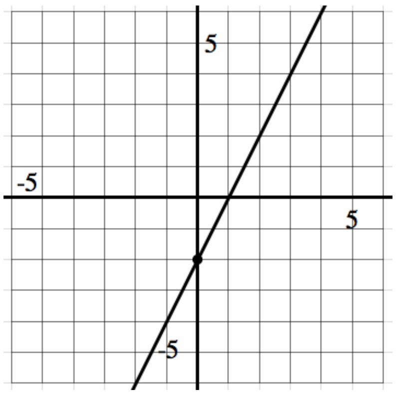


Set

Topic: Slopes of parallel and perpendicular lines.

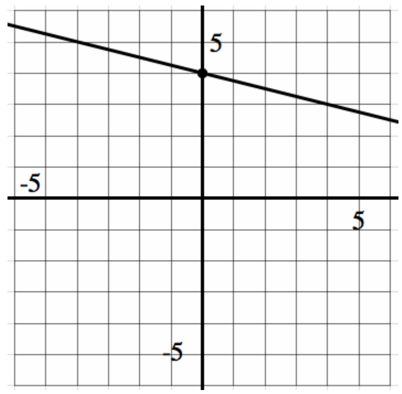
5. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

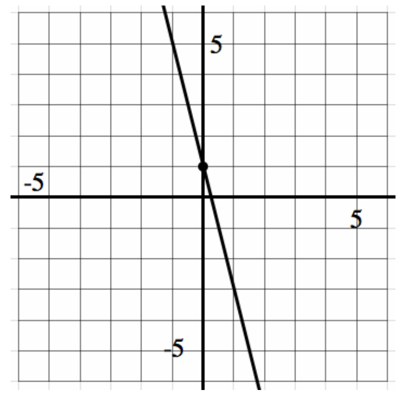
6. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

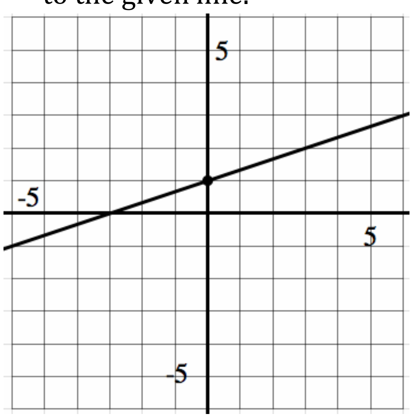
7. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

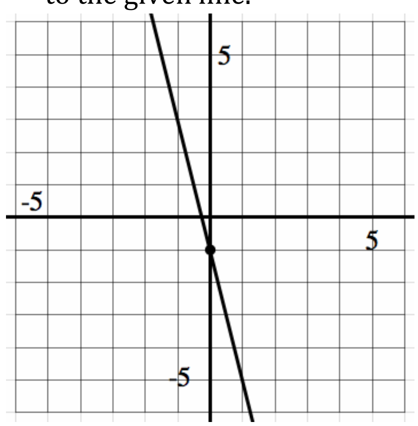
8. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

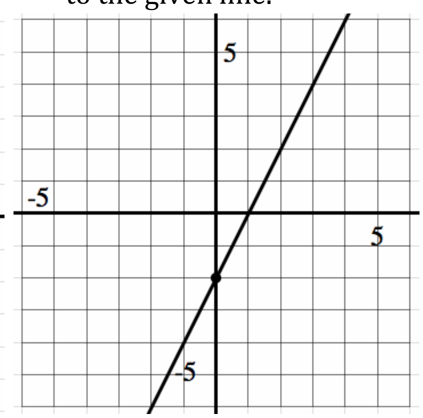
9. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

10. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

Go

Topic: Solve the following equations.

Solve each equation for the indicated variable.

11. $3(x-2) = 5x + 8$; Solve for x.

12. $-3 + n = 6n + 22$; Solve for n.

13. $y - 5 = m(x-2)$; Solve for x.

14. $Ax + By = C$; Solve for y.

6.3 Leap Frog

A Solidify Understanding Task

Josh is animating a scene where a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent". In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.

- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.

- If you decide the transformation is a translation you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.

- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	

images this page:
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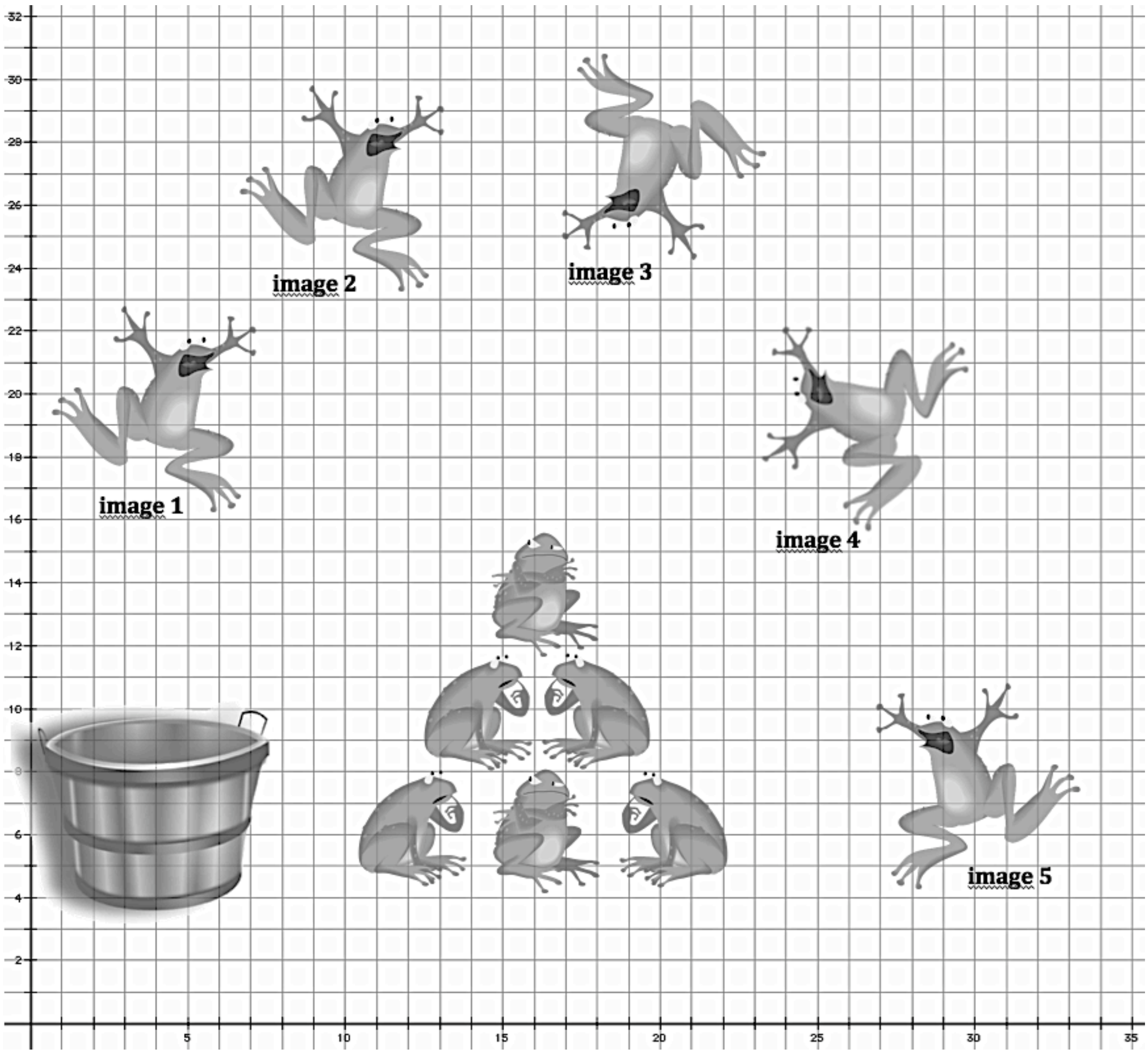


Ready, Set, Go!

Ready

Topic: Basic Rotations and Reflections of objects

In each problem there will be a preimage and several images based on the give preimage. Determine which of the images are rotations of the given preimage and which of them are reflections of the preimage. If an image appears to be created as the result of a rotation and a reflection then state both.

1.

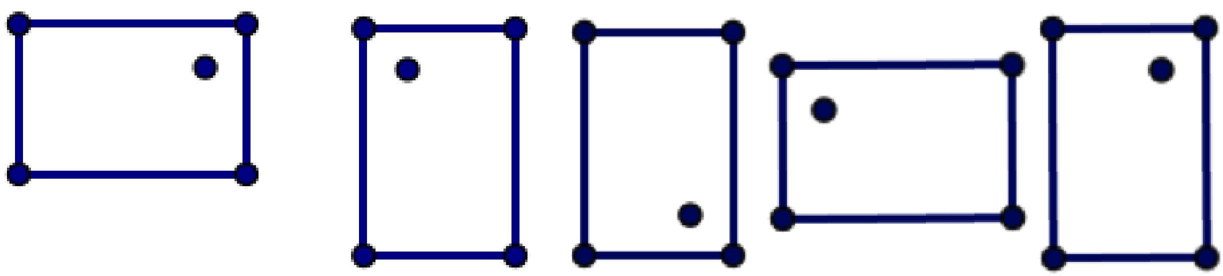


Image A Image B Image C Image D

Pre-Image

2.

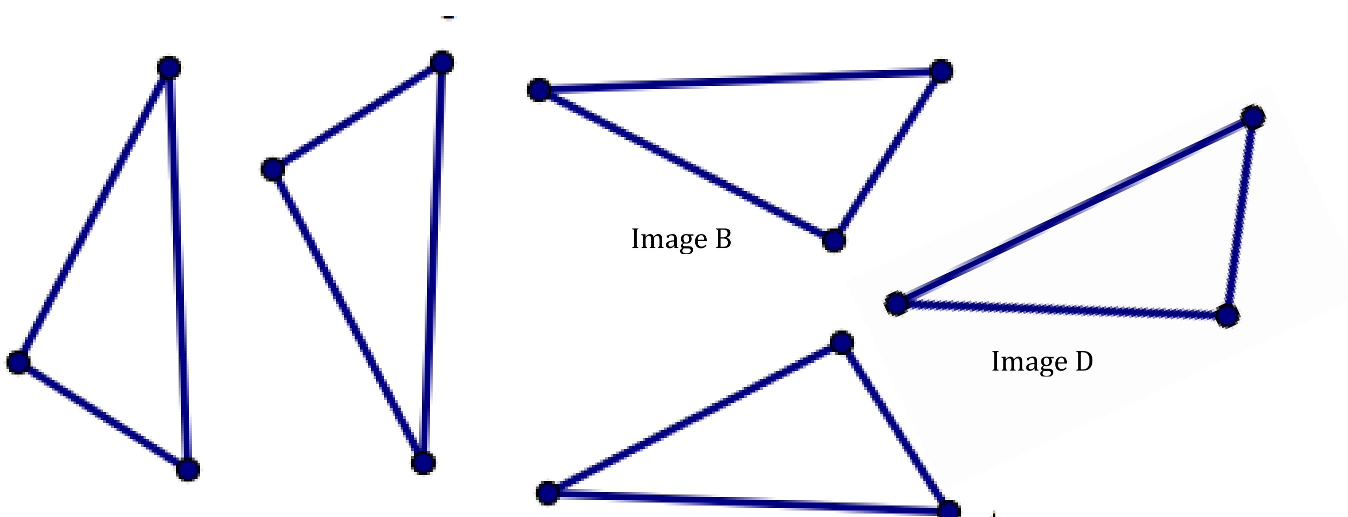


Image B

Image D

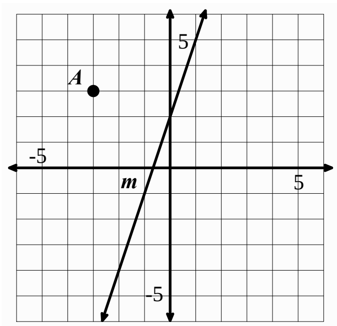
Pre-Image Image A Image C

Set

Topic: Reflecting and Rotating points

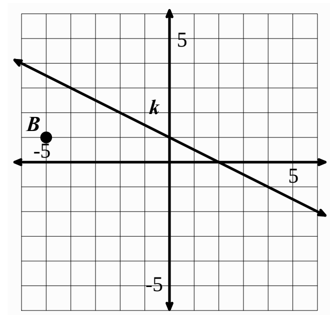
On each of the coordinate grids there is a labeled point and line. Use the line as a line of reflection to reflect the given point and create its reflected image over the line of reflection.

3.



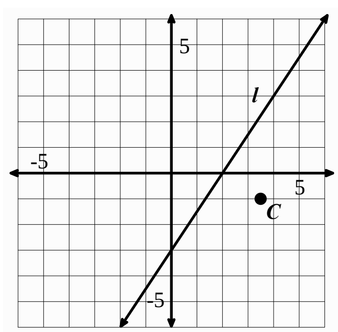
Reflect point ***A*** over line ***m*** and label the image ***A'***

4.



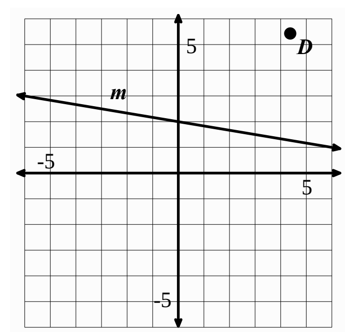
Reflect point ***B*** over line ***k*** and label the image ***B'***

5.



Reflect point ***C*** over line ***l*** and label the image ***C'***

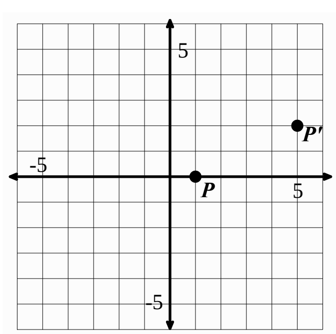
6.



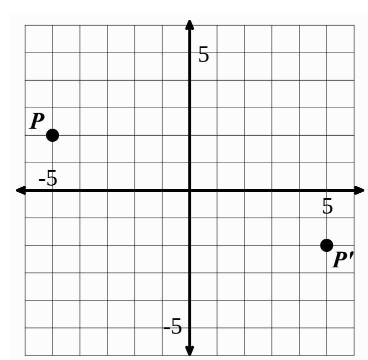
Reflect point ***D*** over line ***m*** and label the image ***D'***

For each pair of point, *P* and *P'* draw in the line of reflection that would need to be used to reflect *P* onto *P'*. Then find the equation of the line of reflection.

7.

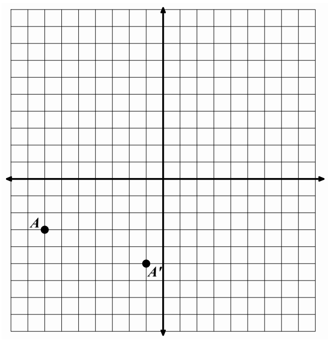


8.

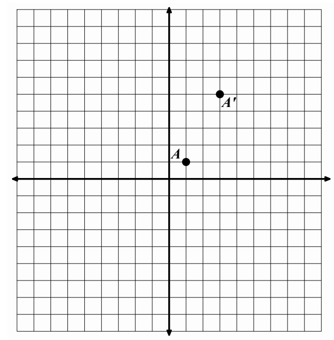


For each pair of point, *A* and *A'* draw in the line of reflection that would need to be used to reflect *A* onto *A'*. Then find the equation of the line of reflection. Also, draw a line connecting *A* to *A'* and find the equation of this line. Compare the slopes of the lines of reflection containing *A* and *A'*.

9.



10.



Go

Topic: Slopes of parallel and perpendicular lines and finding both distance and slope between two points.

For each linear equation write the slope of a line parallel to the given line.

11. $y = -3x + 5$

12. $y = 7x - 3$

13. $3x - 2y = 8$

For each linear equation write the slope of a line perpendicular to the given line.

14. $y = -\frac{2}{7}x + 5$

15. $y = \frac{1}{5}x - 4$

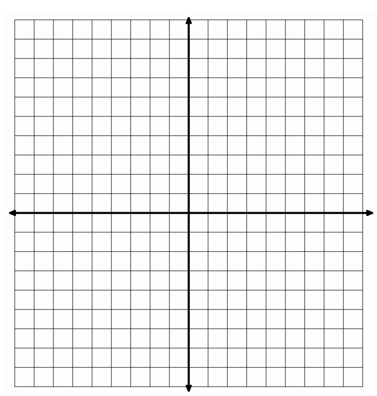
16. $3x + 5y = -15$

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. You may use the graph to help you as needed.

17. (-2, -3) (1, 1)
a. Slope: b. Distance:

18. (-7, 5) (-2, -7)
a. Slope: b. Distance:

19. (2, -4) (3, 0)
a. Slope: b. Distance:



6.4 Leap Year

A Practice Understanding Task

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Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining since images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational since each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," she explains, "you need 366 pages."

So now Carlos and Clarita have the dilemma of having to create an extra page to insert between February 28 and March 1. Here are the planner pages they have already designed.

February 28

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

An **angle** is the union of two rays that share a common endpoint.

An **angle of rotation** is formed when a ray is rotated about its endpoint. The ray that marks the preimage of the rotation is referred to as the "initial ray" and the ray that marks the image of the rotation is referred to as the "terminal ray."

Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.

March 1

Why are there 360° in a circle?

One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance in its path relative to the earth approximately 1/360 of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.)

Another theory is that the Babylonians first divided a circle into parts by inscribing a hexagon consisting of 6 equilateral triangles inside a circle. The angles of the equilateral triangles located at the center of the circle were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10 like our number system).

Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts.

Part 1

Since the theme for the facts for February is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

1. A translation of a set of points in a plane . . .

2. A rotation of a set of points in a plane . . .

3. A reflection of a set of points in a plane . . .

4. Translations, rotations and reflections are rigid motion transformations because . . .

Carlos and Clarita used these words and phrases in their definitions: perpendicular bisector, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, preserves distance and angle measures.

Revise your definitions so they also use these words or phrases.

Part 2

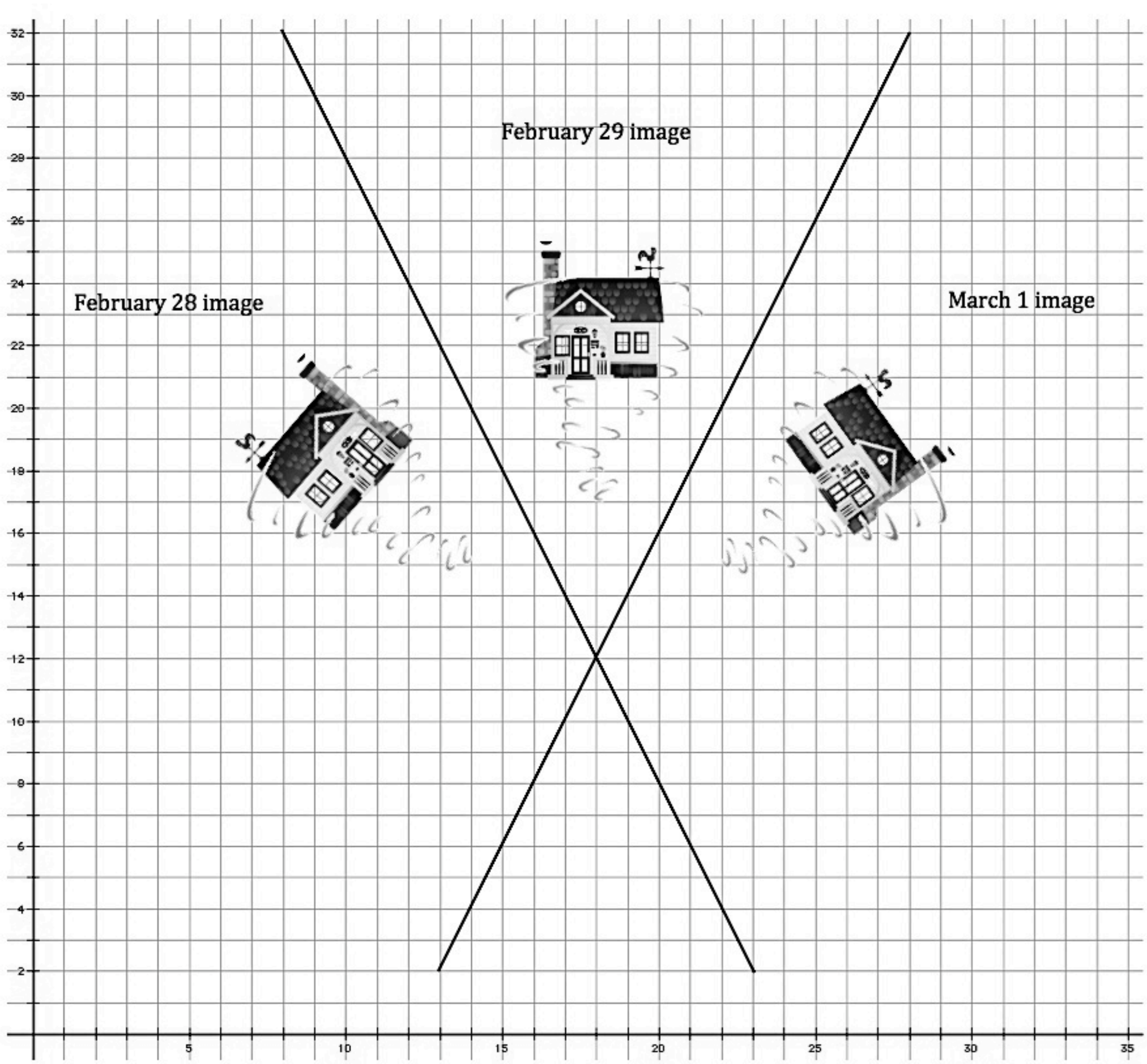
In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from the Wizard of Oz as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that he can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.

Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as he intended. For example,

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?



Images this page:
http://openclipart.org/detail/168722/simple-farm-pack-by-viscious-speed
www.clker.com/clipart.tornado-gray

Ready, Set, Go!

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Ready

Topic: Defining geometric shapes and components

For each of the geometric words below write a definition of the object that addresses the essential elements. Also, list necessary attributes and characteristics.

1. Quadrilateral:

2. Parallelogram:

3. Rectangle:

4. Square:

5. Rhombus:

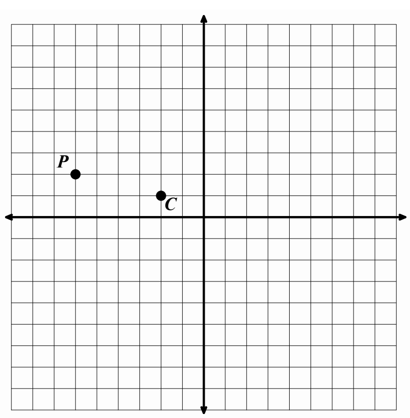
6. Trapezoid:

Set

Topic: Reflections and Rotations, composing reflections to create a rotation

Perform the indicated rotations.

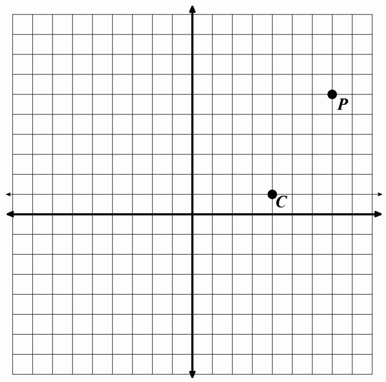
7.



Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P***'.

With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P***''.

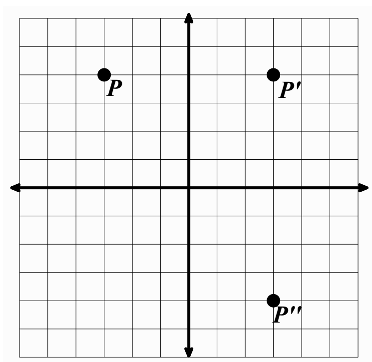
8.



Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P'***.

With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P''***.

9.

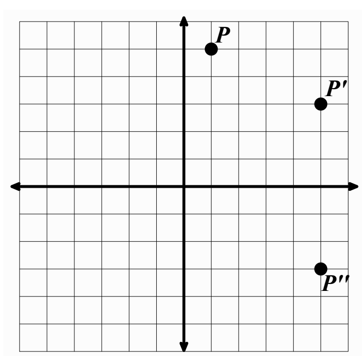


a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

10.

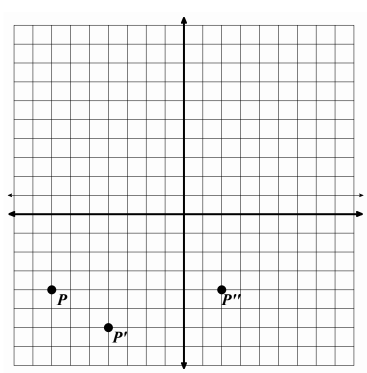


a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

11.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

Go

Topic: Rotations about the origin

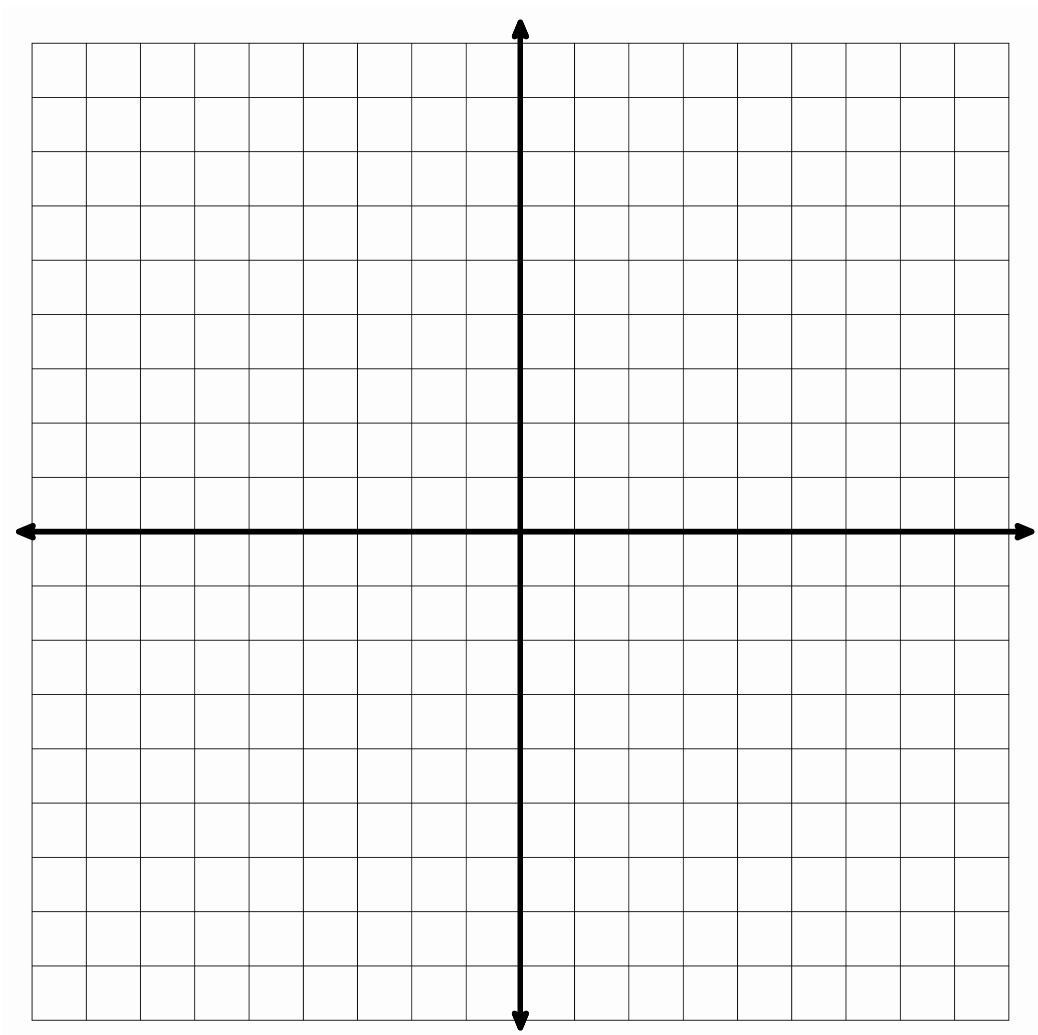
Plot the given coordinate and then perform the indicated rotation in a clockwise direction around the origin, the point (0 , 0), and plot the image created. State the coordinates of the image.

12. Point ***A*** (4, 2) rotate 180^0
    Coordinates for Point ***A***' (___ , ___)

13. Point ***B*** (-5, -3) rotate 90^0 clockwise
    Coordinates for Point ***B***' (___ , ___)

14. Point ***C*** (-7, 3) rotate 180^0
    Coordinates for Point ***C***' (___ , ___)

15. Point ***D*** (1, -6) rotate 90^0 clockwise
    Coordinates for Point ***D***' (___ , ___)



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6.5 Symmetries of Quadrilaterals

A Develop Understanding Task

A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

1. A **rectangle** is a quadrilateral that contains four right angles. Is it possible to reflect or rotate a rectangle onto itself?

For the rectangle shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rectangle onto itself.

Describe the rotations and/or reflections that carry a rectangle onto itself. (Be as specific as possible in your descriptions.)

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel. Is it possible to reflect or rotate a parallelogram onto itself?

For the parallelogram shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the parallelogram onto itself.

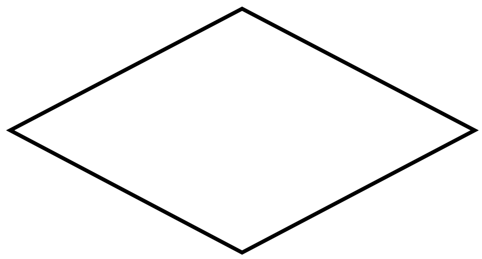
Describe the rotations and/or reflections that carry a parallelogram onto itself. (Be as specific as possible in your descriptions.)

3. A **rhombus** is a quadrilateral in which all sides are congruent. Is it possible to reflect or rotate a rhombus onto itself?

For the rhombus shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rhombus onto itself.



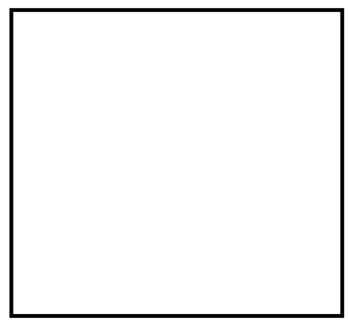
Describe the rotations and/or reflections that carry a rhombus onto itself. (Be as specific as possible in your descriptions.)

4. A **square** is both a rectangle and a rhombus. Is it possible to reflect or rotate a square onto itself?

For the square shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the square onto itself.



Describe the rotations and/or reflections that carry a square onto itself. (Be as specific as possible in your descriptions.)

5. A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find

- any lines of symmetry, or
- any centers of rotational symmetry

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

Ready, Set, Go!

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Ready

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

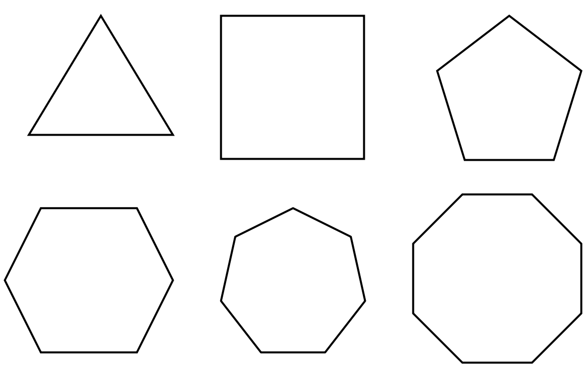
2. Fill in the names of each polygon based on the number of sides the polygon has.

Number of Sides	Name of Polygon
3	
4	
5	
6	
7	
8	
9	
10	

Set

Topic: Lines of symmetry and diagonals

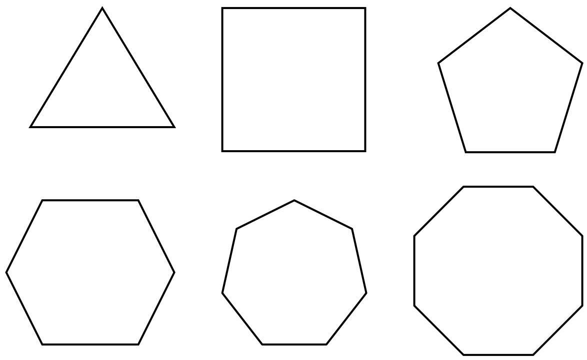
3. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a ***n***-sided polygon.



Number of Sides	Number of lines of symmetry
3	
4	
5	
6	
7	
8	
n	

4. Find

all of the diagonals in each regular polygon. Fill in the table including an expression for the number of diagonals in a ***n***-sided polygon.



Number of Sides	Number of diagonals
3	
4	
5	
6	
7	
8	
n	

5. Are all lines of symmetry also diagonals? Explain.

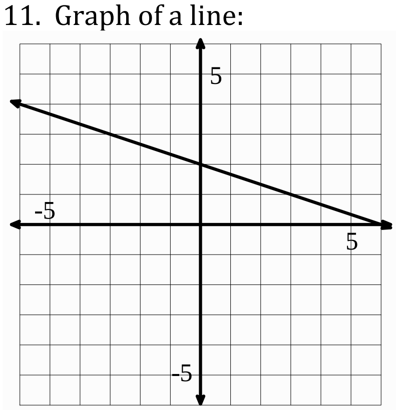
6. Are all diagonals also lines of symmetry? Explain.

7. What shapes will have diagonals that are not lines of symmetry? Name some and draw them.

8. Will all parallelograms have diagonals that are lines of symmetry? If so, draw and explain. If not draw and explain.

Go

Topic: Equations for parallel and perpendicular lines.

<table>
<tr><th></th><th>Find the equation of a line PARALLEL to the given info and through the indicated point.</th><th>Find the equation of a line PERPENDICULAR to the given line and through the indicated point.</th></tr>
<tr><td>9. Equation of a line:
$y = 4x + 1$.</td><td>a. Parallel line through point (-1 , -7):</td><td>b. Perpendicular to the line line through point (-1 , -7):</td></tr>
<tr><td>10. Table of a line:
<table><tr><th>x</th><th>y</th></tr><tr><td>3</td><td>-8</td></tr><tr><td>4</td><td>-10</td></tr><tr><td>5</td><td>-12</td></tr><tr><td>6</td><td>-14</td></tr></table></td><td>a. Parallel line through point (3 , 8):</td><td>b. Perpendicular to the line through point (3 , 8):</td></tr>
<tr><td>11. Graph of a line:

</td><td>a. Parallel line through point (2 , -9):</td><td>b. Perpendicular to the line through point (2 , -9):</td></tr>
</table>

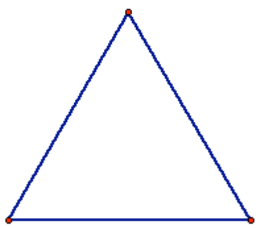
6.6 Symmetries of Regular Polygons

A Solidify Understanding Task

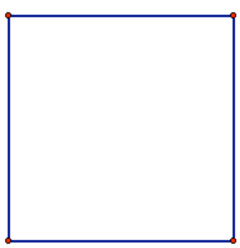
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

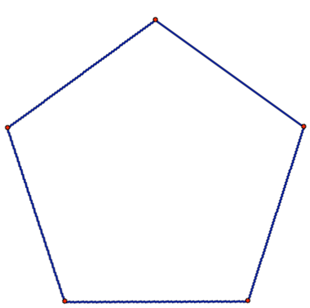
1. An equilateral triangle



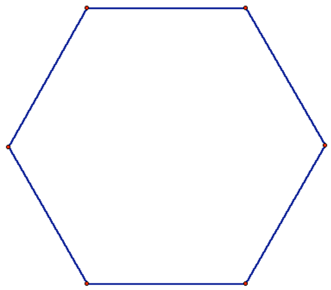
2. A square



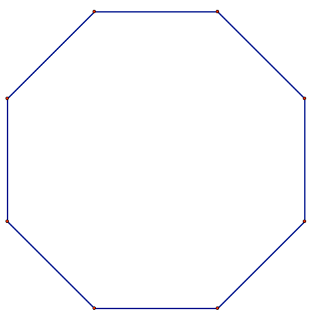
3. A regular pentagon



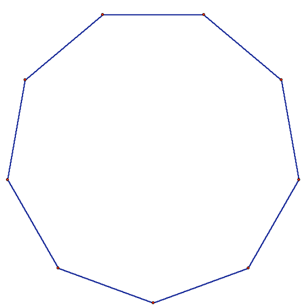
4. A regular hexagon



5. A regular octagon



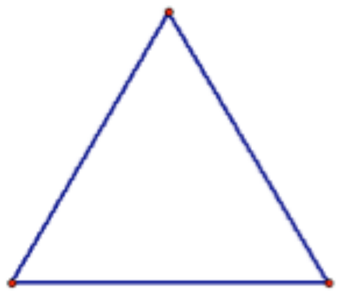
6. A regular nonagon



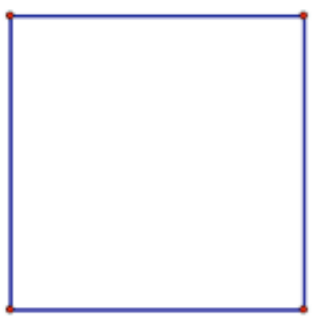
What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

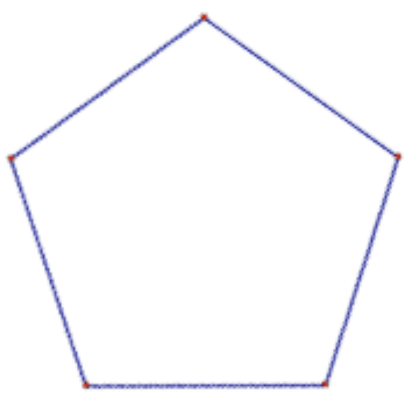
1. An equilateral triangle



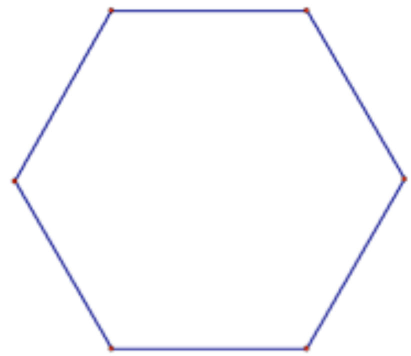
2. A square



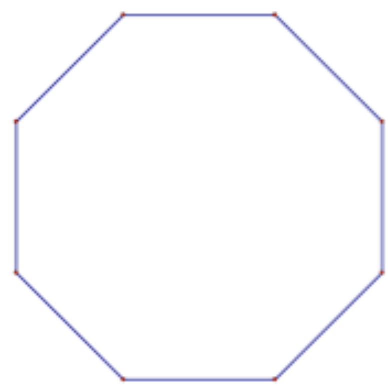
3. A regular pentagon



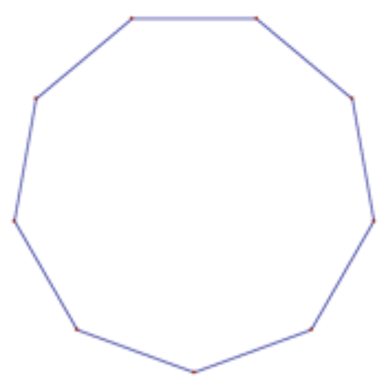
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



Ready, Set, Go!

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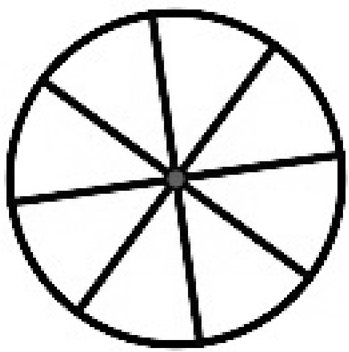
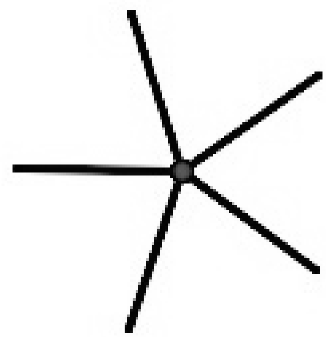
Ready

Topic: Rotation as a transformation, what does it mean?

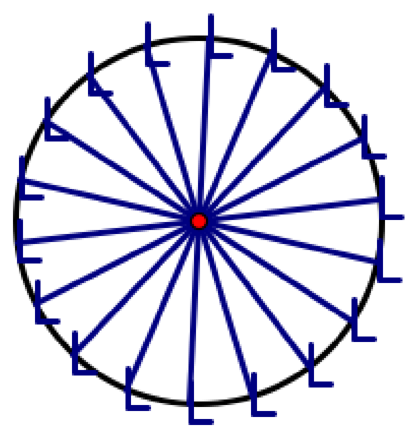
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



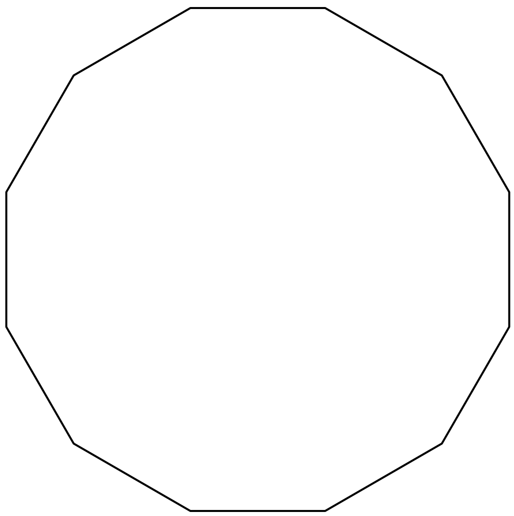
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



Set

Topic: Finding angels of rotation for regular polygons.

4. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



5. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

6. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

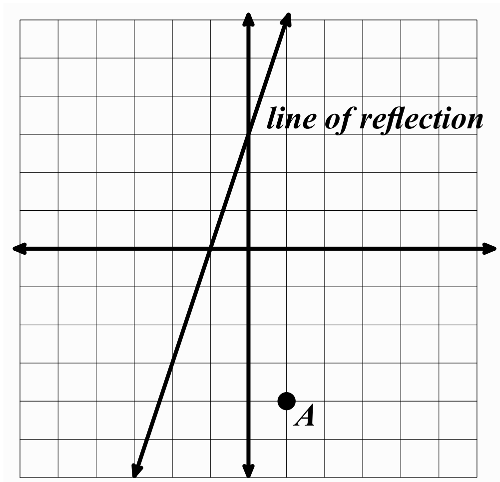
7. How many sides does a regular polygon have that has an angle of rotation equal to 18^0? Explain.

8. How many sides does a regular polygon have that has an angle of rotation equal to 20^0? How many lines of symmetry will it have?

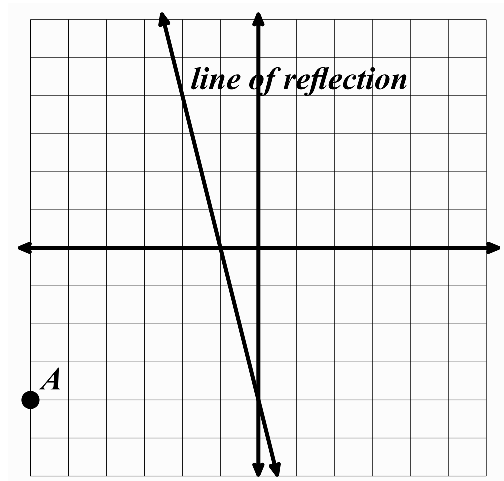
Go

Topic: Reflecting and Rotating points on the coordinate plane.

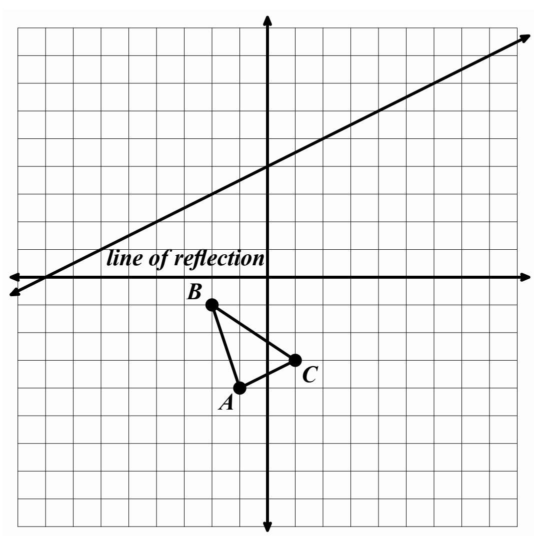
9. Reflect point ***A*** over the line of reflection and label the image ***A'***.



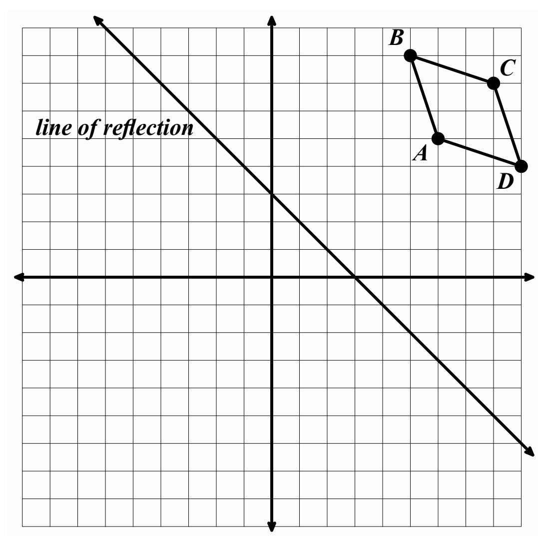
10. Reflect point ***A*** over the line of reflection and label the image ***A'***.



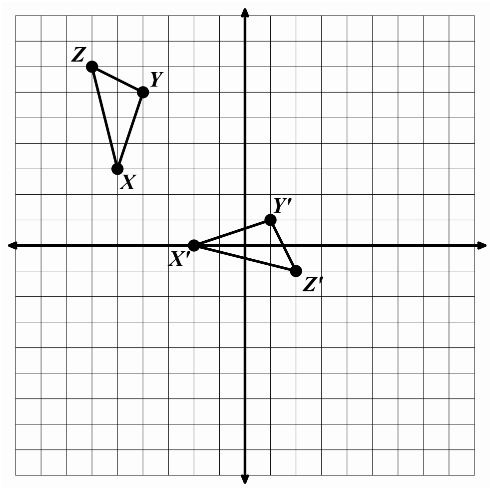
11. Reflect triangle ***ABC*** over the line of reflection and label the image ***A'B'C'***.



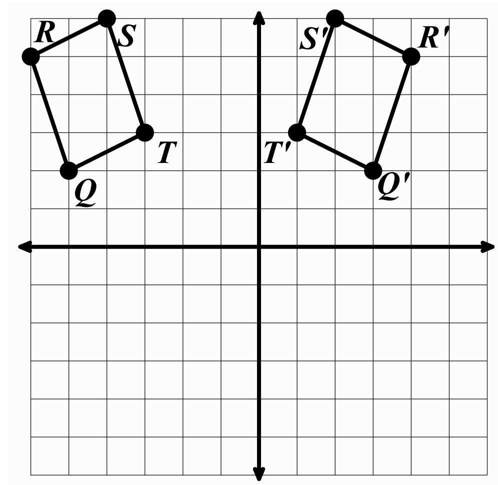
12. Reflect parallelogram ***ABCD*** over the line of reflection and label the image ***A'B'C'D'.***



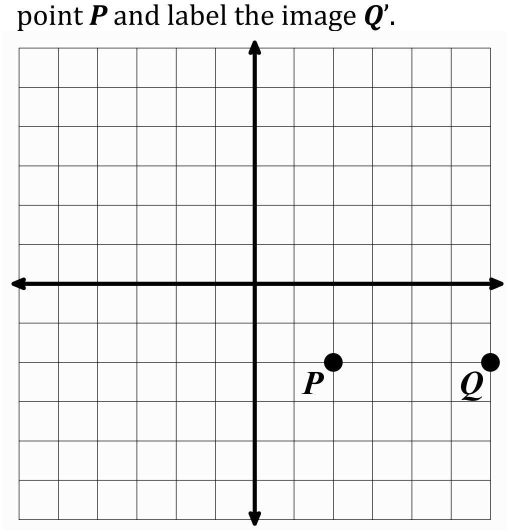
13. Given triangle ***XYZ*** and its image ***X'Y'Z'*** draw the line of reflection that was used.



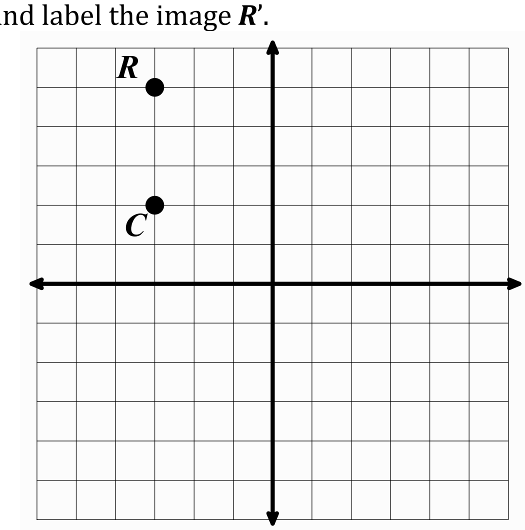
14 Given parallelogram ***QRST*** and its image ***Q'R'S'T'*** draw the line of reflection that was used.



15. Using point ***P*** as a center of rotation. Rotate point ***Q*** 120^0 clockwise about point ***P*** and label the image ***Q'***.



16. Using point ***C*** as the center or rotation. Rotate point ***R*** 270^0 counter-clockwise about point ***C*** and label the image ***R'***.



6.7 Quadrilaterals—Beyond Definition

A Practice Understanding Task

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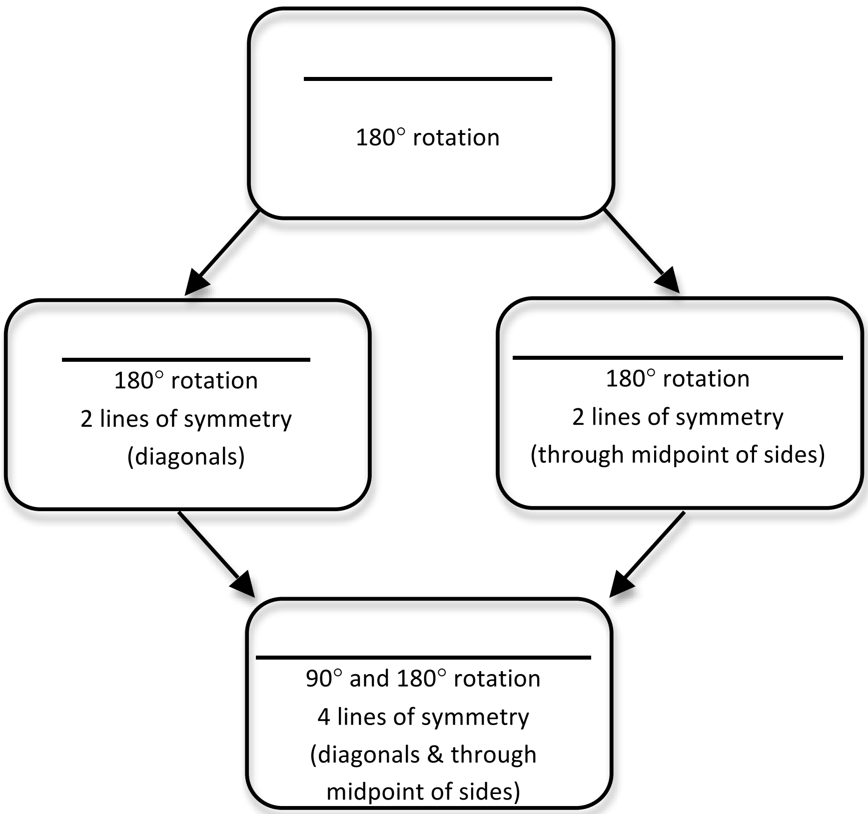
We have found that many different quadrilaterals possess line and/or rotational symmetry.

In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.

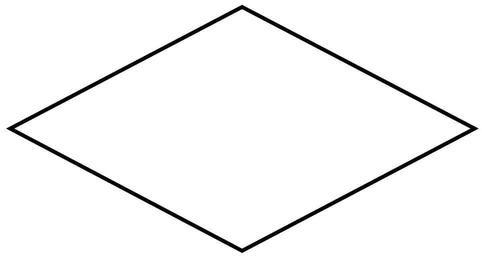
1. A **rectangle** is a quadrilateral that contains four right angles.

Based on what you know about transformations, what else can we say about rectangles besides the defining property that all four angles are right angles? Make a list of additional properties of rectangles that seem to be true based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals.

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

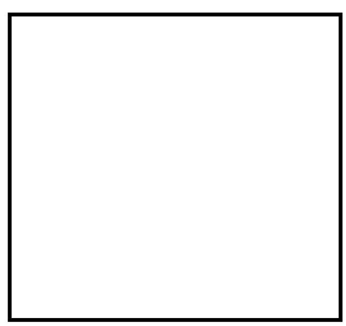
Based on what you know about transformations, what else can we say about parallelograms besides the defining property that opposite sides of a parallelogram are parallel? Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles and the diagonals.

3. A **rhombus** is a quadrilateral in which all four sides are congruent.



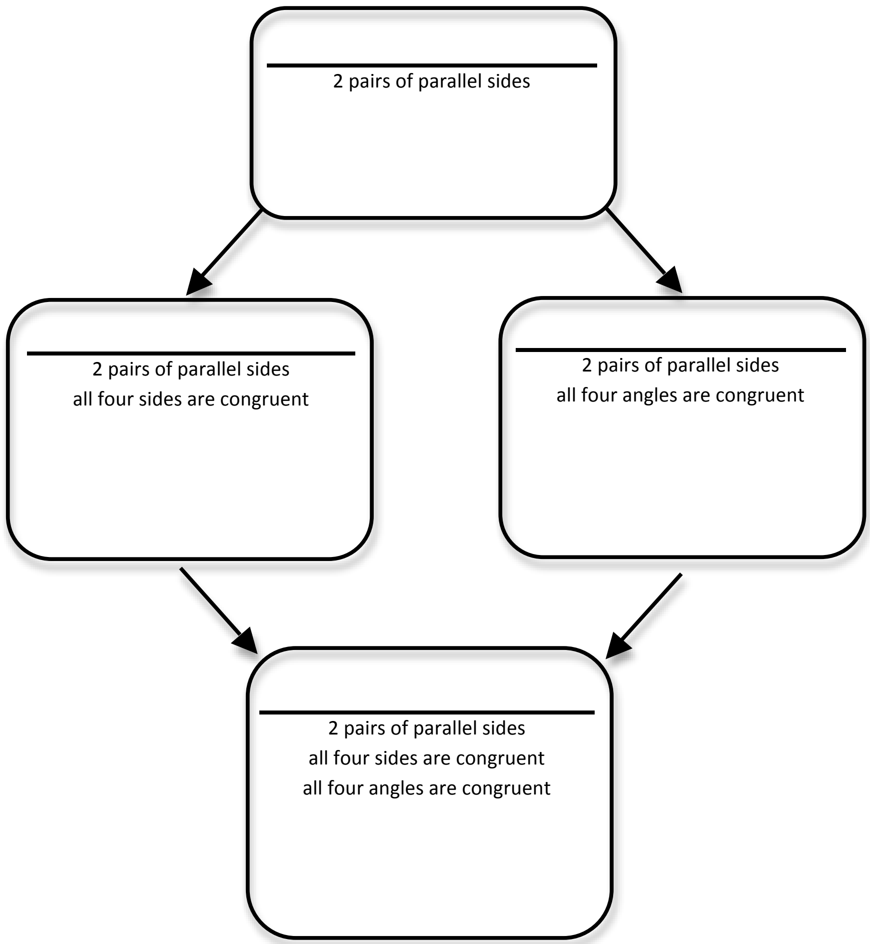
Based on what you know about transformations, what else can we say about a rhombus besides the defining property that all sides are congruent? Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles and the diagonals.

4. A **square** is both a rectangle and a rhombus.



Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles and the diagonals.

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and highlighted in the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

Ready, Set, Go!

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Ready

Topic: Defining Congruence and Similarity.

1. What do you know about two figures if they are congruent?

2. What do you need to know about two figures to be convinced that the two figures are congruent?

3. What do you know about two figures if they are similar?

4. What do you need to know about two figures to be convinced that the two figures are similar?

Set

Topic: Classifying quadrilaterals based on their properties.

Using the information given determine the most accurate classification of the quadrilateral.

5. Has 180^0 rotational symmetry.

6. Has 90^0 rotational symmetry.

7. Has two lines of symmetry that are diagonals.

8. Has two lines of symmetry that are not diagonals.

9. Has congruent diagonals.

10. Has diagonals that bisect each other.

11. Has diagonals that are perpendicular.

12. Has congruent angles.

Go

Topic: Slope and distance

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points.

13. (-3 , -2) , (0 , 0)

a. Slope: b. Distance:

14. (7 , -1) , (11 , 7)

a. Slope: b. Distance:

15. (-10 , 13) , (-5 , 1)

a. Slope: b. Distance:

16. (-6 , -3) , (3 , 1)

a. Slope: b. Distance:

17. (5 , 22) , (17 , 28)

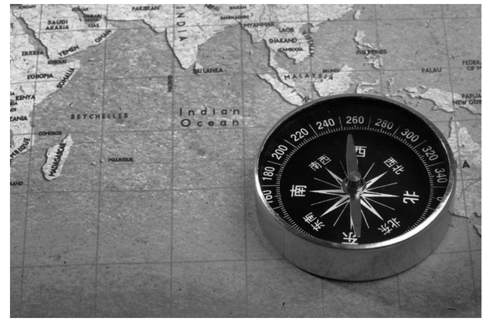
a. Slope: b. Distance:

18. (1 , -7) , (6 , 5)

a. Slope: b. Distance:

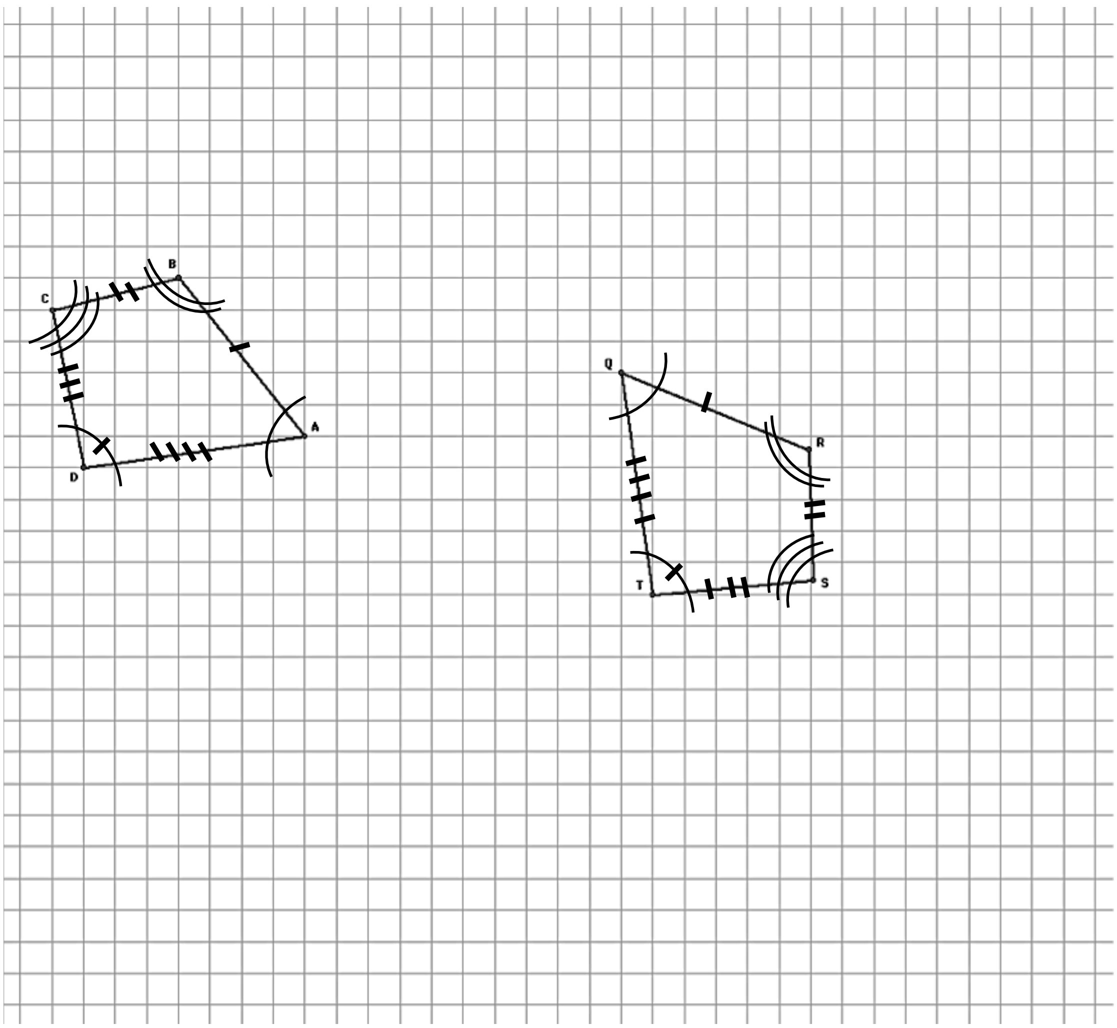
6. 8 Can You Get There From Here?

A Develop Understanding Task



The two quadrilaterals shown below, quadrilateral *ABCD* and quadrilateral *QRST* are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



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Ready, Set, Go!

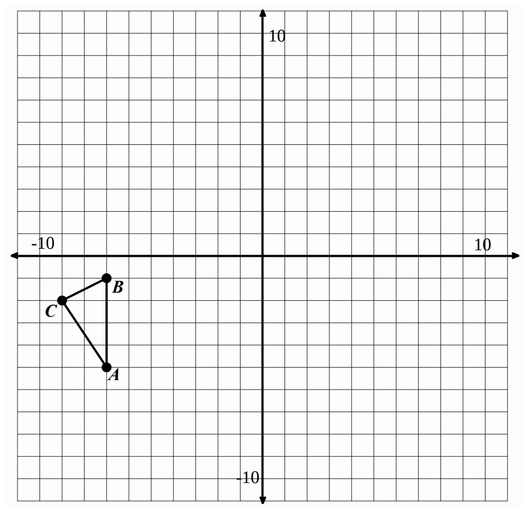
Ready

Topic: Performing a sequence of transformations.

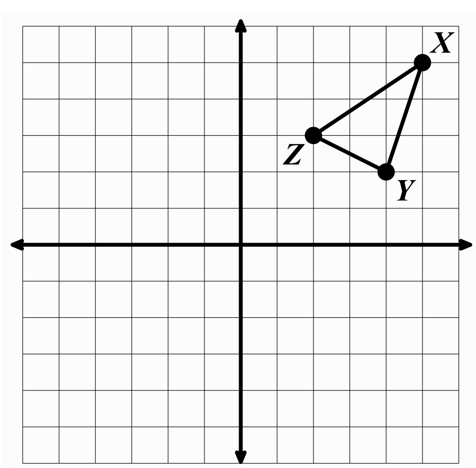
The given figures are to be used as pre-images. Perform the stated transformations to obtain an image. Label the corresponding parts of the image in accordance with the pre-image.

1. Reflect triangle ***ABC*** over the line $y = x$ and label the image ***A'B'C'***.

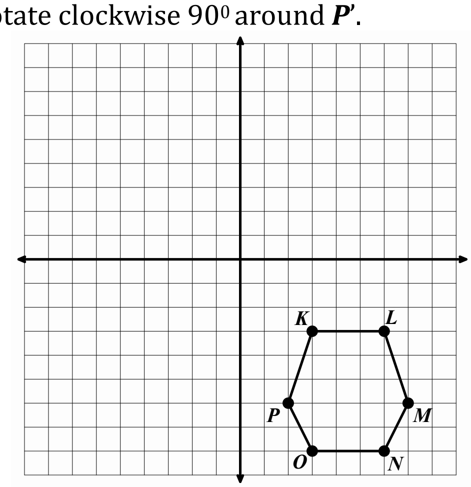
   Rotate triangle ***A'B'C'*** 180^0 counter clockwise around the origin and label the image ***A''B''C''***.



2. Reflect over the line $y = -x$.

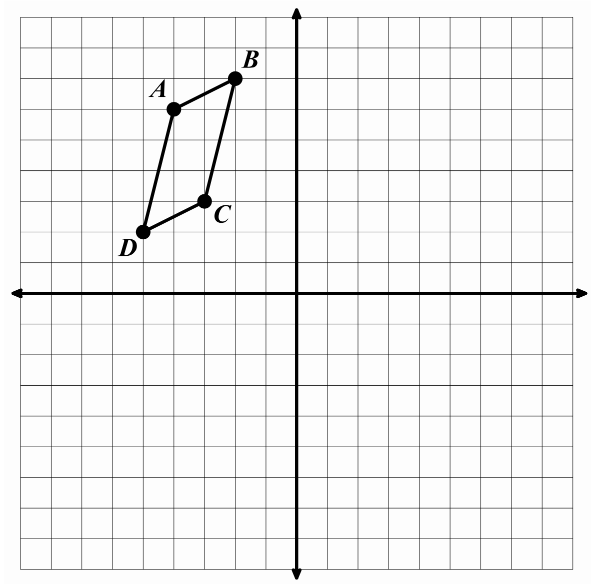


3. Reflect over y-axis and then Rotate clockwise 90^0 around ***P'***.



4. Reflect quadrilateral ABCD over the line $y = 2 + x$ and label the image A'B'C'D'.

Rotate quadrilateral A'B'C'D' counter-clockwise 90^0 around (-2, -3) as the center of rotation label the image A''B''C''C''.

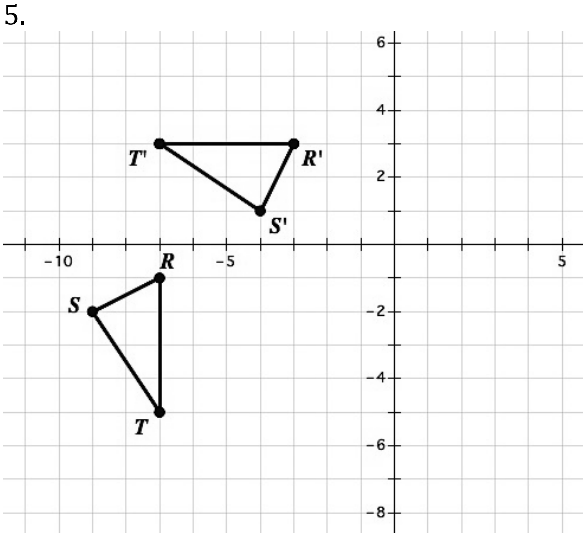


Set

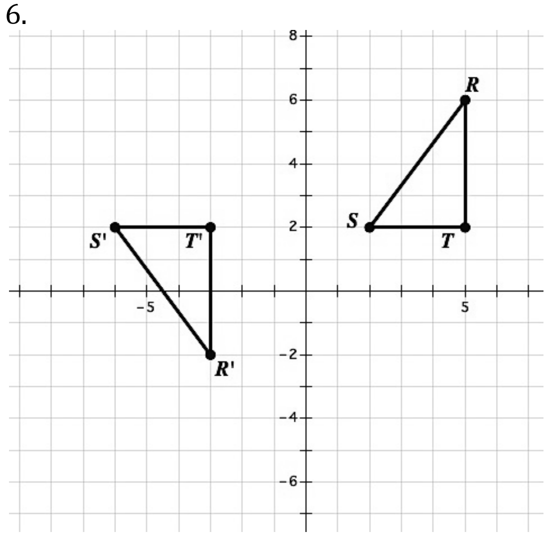
Topic: Find the sequence of transformations.

Find the sequence of transformations that will carry triangle ***RST*** onto triangle ***R'S'T'***. Clearly describe the sequence of transformations below each grid.

5.



6.

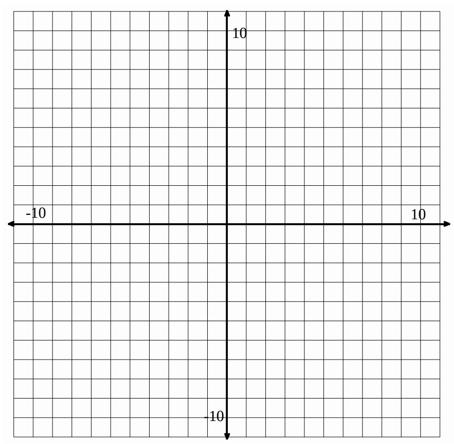


Go

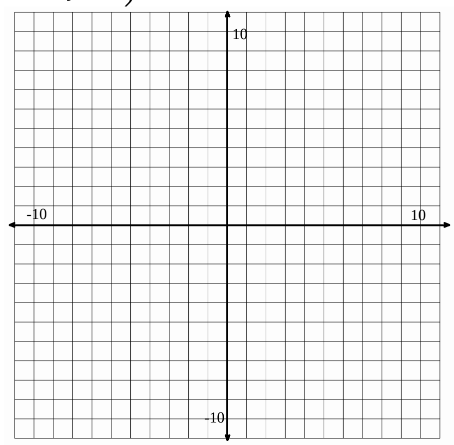
Topic: Graphing functions and making comparisons.

Graph each pair of functions and make an observation about how the functions compare to one another.

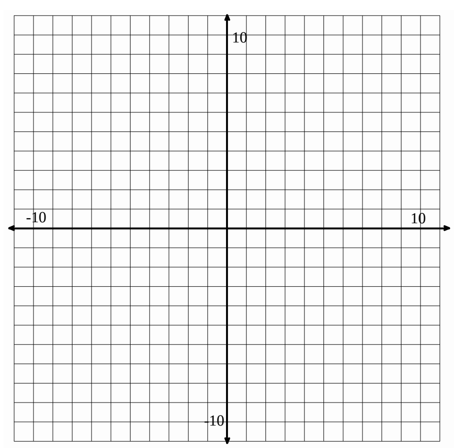
7. $y = \frac{1}{3}x - 1$
   $y = -3x - 1$



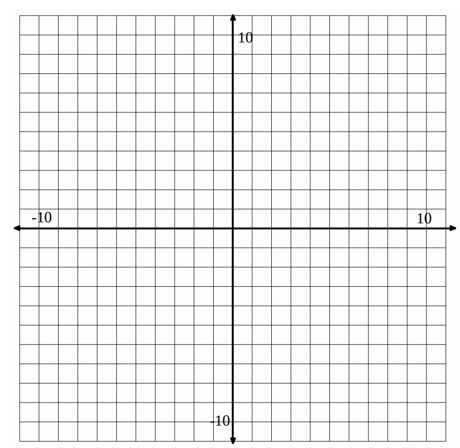
8. $y = -\frac{2}{3}x + 5$
   $y = \frac{3}{2}x + 5$

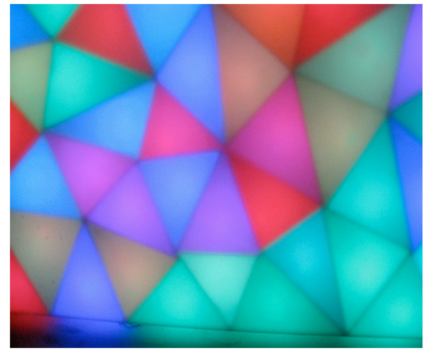


9. $y = \frac{1}{4}x + 2$
   $y = -\frac{1}{4} + 2$



10. $y = 2^x$
    $y = -2^x$





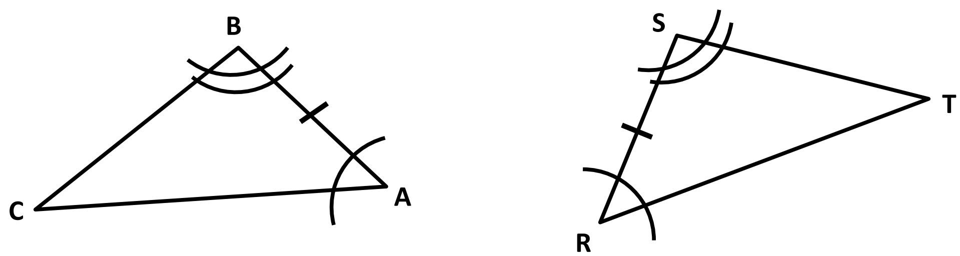
6.9 Congruent Triangles

A Solidify Understanding Task

Zac and Sione are trying to decide how much information they need to know about two triangles before they can convince themselves that the two triangles are congruent.

They are wondering if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria their teacher refers to as ASA—is enough to know that the two triangles are congruent. They are trying to justify that this would be so.

To start reasoning about the congruence of the two triangles, Zac and Sione have created the following diagram in which they have marked an ASA relationship between the triangles.



1. Based on the diagram, which angles have Zac and Sione indicated are congruent? Which sides?

2. To convince themselves that the two triangles are congruent, what else would Zac and Sione need to know?

Zac's Argument

"I know what to do," said Zac. "We can translate point A until it coincides with point R, then rotate $\overline{AB}$ about point R until it coincides with $\overline{RS}$. Finally, we can reflect ΔABC across $\overleftrightarrow{RS}$ and then everything coincides so the triangles are congruent." [Zac and Sione's teacher has suggested they use the word "coincides" when they want to say that two points or line segments occupy the same position on the plane. They like the word, so they plan to use it a lot.]

What do you think about Zac's argument? Does it convince you that the two triangles are congruent? Does it leave out any essential ideas that you think need to be included?

3. Write a paragraph explaining your reaction to Zac's argument:

Sione isn't sure that Zac's argument is really convincing. He asks Zac, "How do you know point C coincides with point T after you reflect the triangle?"

4. How do you think Zac might answer Sione's question?

While Zac is trying to think of an answer to Sione's question he adds this comment, "And you really didn't use all of the information about the corresponding congruent parts of the two triangles."

"What do you mean?" asked Zac.

Sione replied, "You started using the fact that $\angle A \cong \angle R$ when you translated ΔABC so that vertex A coincides with vertex R. And you used the fact that $\overline{AB} \cong \overline{RS}$ when you rotated $\overline{AB}$ to coincide with $\overline{RS}$, but where did you use the fact that $\angle B \cong \angle S$?"

"Yeah, and what does it really mean to say that two angles are congruent?" Zac added. "Angles are more than just their vertex points."

5. How might thinking about Zac and Sione's questions help improve Zac's argument?

<u>Sione's Argument</u>

"I would start the same way you did, by translating point A until it coincides with point R, rotating $\overline{AB}$ about point R until it coincides with $\overline{RS}$, and then reflecting ΔABC across $\overleftrightarrow{RS}$," Sione said. "But then I would want to convince myself that points C and T coincide. I know that an angle is made up of two rays that share a common endpoint. Since I know that $\overrightarrow{AB}$ coincides with $\overrightarrow{RS}$ and $\angle A \cong \angle R$, that means that $\overrightarrow{AC}$ coincides with $\overrightarrow{RT}$. Likewise, I know that $\overrightarrow{BA}$ coincides with $\overrightarrow{SR}$ and $\angle B \cong \angle S$, so $\overrightarrow{BC}$ must coincide with $\overrightarrow{ST}$. Since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ intersect at point C, and $\overrightarrow{RT}$ and $\overrightarrow{ST}$ intersect at point T, points C and T must also coincide because the corresponding rays coincide. Therefore, $\overline{BC} \cong \overline{ST}$, $\overline{CA} \cong \overline{TR}$, and $\angle C \cong \angle T$ because both angles are made up of rays that coincide!"

At first Zac was confused by Sione's argument, but he drew diagrams and carefully marked and sketched out each of his statements until it started to slowly make sense.

6. Do the same kind of work that Zac did to make sense of Sione's argument. What parts of his argument are unclear to you? What ideas did sketching out the words of his proof help you to clarify?

Sione's argument suggests that ASA is sufficient criteria for determining if two triangles are congruent. Now Zac and Sione are wondering about other criteria, such as SAS or SSS, or perhaps even AAA (which Zac immediately rejects because he thinks two triangles can have the same angle measures but be different sizes).

7. Draw two triangles that have SAS congruence. Be sure to mark you triangles to show which sides and which angles are congruent.

8. Write out a sequence of transformations to show that the two triangles potentially coincide.

9. If Sione were to examine your work in #8, what questions would he wonder about?

10. How can you use the given congruence criteria (SAS) to resolve Simone's wonderings?

Repeat 7-10 for SSS congruence.

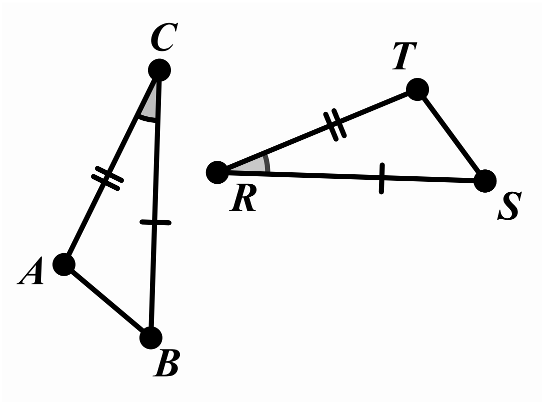
Ready, Set, Go!

Ready

Topic: Corresponding parts of figures and transformations

Given the figures in each sketch with congruent angles and sides marked, first list the parts of the figures that correspond (For example, in #1, $\angle C \cong \angle R$) Then determine a reflection occurred as part of the sequence of transformations that was used to create the image.

1.

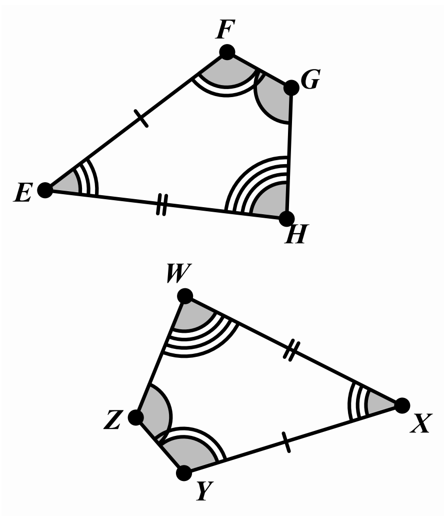


Congruencies

$\angle C \cong \angle R$

Reflected? Yes or No

2.



Congruencies

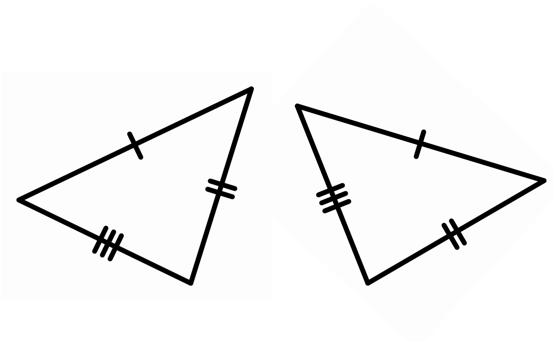
Reflected? Yes or No

Set

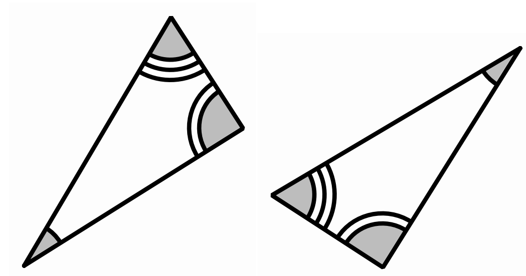
Topic: Triangle Congruencies

Explain whether or not the triangles are congruent, similar, or neither based on the markings that indicate congruence.

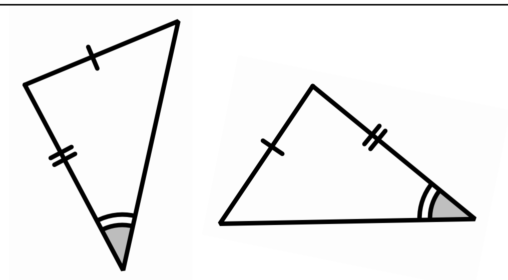
3.



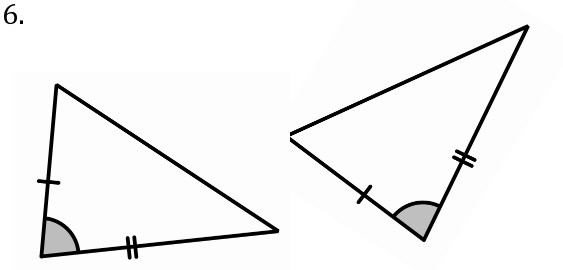
4.



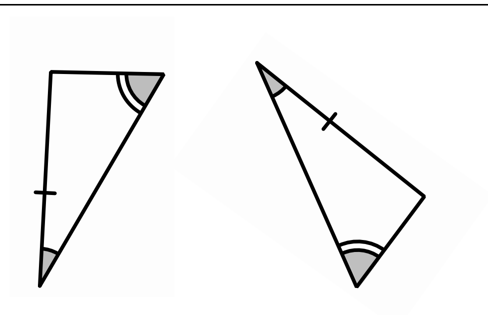
5.



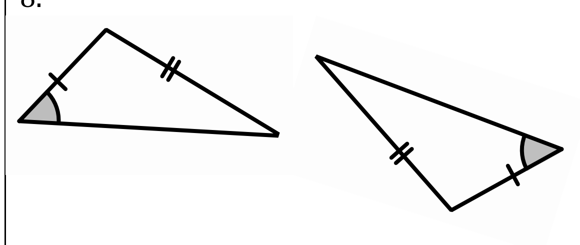
6.



7.



8.



Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.

8. $\Delta\, ABC \;\cong\; \Delta\, PQR$

9. $\Delta\, XYZ \;\cong\; \Delta\, KLM$

Go

Topic: Review of solving equations and finding recursive rules for sequences.

Solve each equation for *t*.

10. $\frac{3t-4}{5} = 5$

11. $10 - t = 4t + 12 - 3t$

12. $P = 5t - d$

13. $xy - t = 13t + w$

Use the given sequence of number to write a recursive rule for the *n*th value of the sequence.

14. 5, 15, 45, …

15. $\frac{1}{2}$, 0, -$\frac{1}{2}$, -1, …

16. 3, -6, 12, -24, …

17. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, …

6.10 Congruent Triangles to the Rescue

A Practice Understanding Task

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Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent.

Zac: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side.

Sione: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

Zac: But I've folded lots of isosceles triangles in half, and it always seems to work.

Sione: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the line through the vertex point of the angle made up of the two congruent sides and the midpoint of the third side in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could she use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the “crease line”? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral *A*, *B*, *C*, and *D*, and label the point of intersection of the two diagonals as point *N.*

2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent.

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$

3. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

- the diagonals bisect each other
- the diagonals are congruent
- the diagonals are perpendicular to each other
- the diagonals bisect the angles of the quadrilateral

Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

Ready, Set, Go!

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Ready

Topic: Defining bisectors of angles and perpendicular bisectors.

1. Based on the meaning of "bisect", which means to split into two equal parts, what would it mean to *bisect* an angle? Describe in words and also provide visuals to communicate the meaning of angle bisector.

2. What does it mean if you have a *perpendicular bisector* of a line segment? Provide both written explanation and visual sketches to communicate the meaning of perpendicular bisector.

Set

Topic: Use congruent triangle criteria and transformations to justify conjectures.

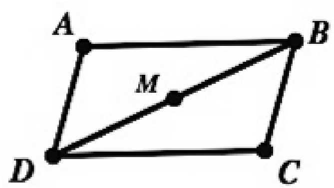
In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

3. True statements:

Point ***M*** is the midpoint of $\overline{DB}$

$\angle ABD \cong \angle BDC$

$\overline{AB} \cong \overline{DC}$



Conjecture: Ð*A* @Ð*C*

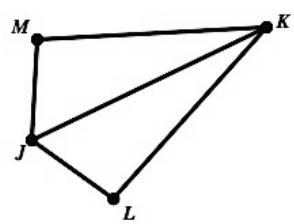
a. Is the conjecture correct?

b. Argument to prove you are right:

4. True statements

$\angle KJL \cong \angle KJM$

$\overline{JL} \cong \overline{JM}$



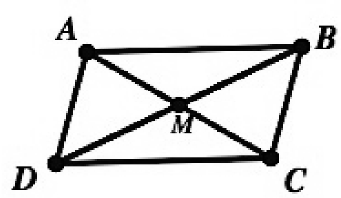
Conjecture: $\overline{JK}$ bisects $\angle MKL$

a. Is the conjecture correct?

b. Argument to prove you are right:

5. True statements

ΔADM is a 180° rotation of ΔCMB



Conjecture: $\Delta ABM \cong \Delta CDM$

a. Is the conjecture correct?

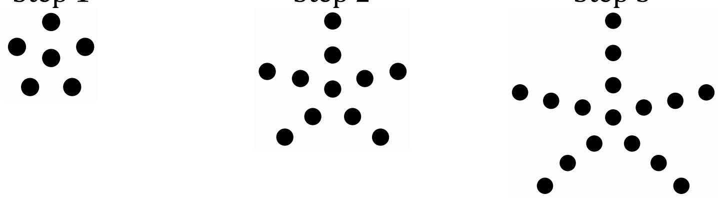
b. Argument to prove you are right:

Go

Topic: Create both explicit and recursive rules for the visual patterns.

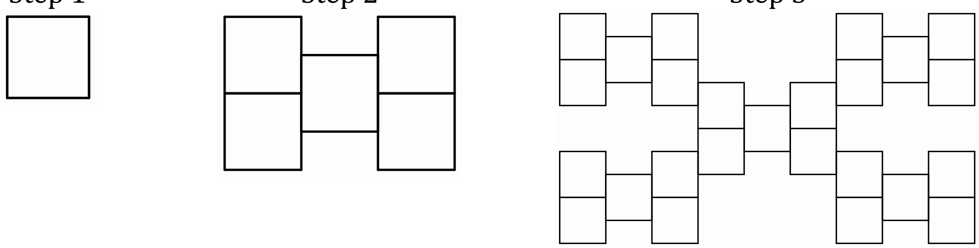
6. Find an explicit function rule and a recursive rule for dots in step *n*.

Step 1 Step 2 Step 3



7. Find an explicit function rule and a recursive rule for squares in step *n.*

Step 1 Step 2 Step 3



Find an explicit function rule and a recursive rule for the values in each table.

8.

Step	Value
1	1
2	11
3	21
4	31

9.

n	$f(n)$
2	16
3	8
4	4
5	2

10.

n	$f(n)$
1	-5
2	25
3	-125
4	625

6.11 Under Construction
A Develop Understanding Task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Anciently, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools you can construct all kinds of geometric shapes.

Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the following sketch.

Now the hard work begins. We can't just keep drawing line segments, because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

Constructing a rhombus

Knowing what you know about circles and line segments, how might you locate point C on the ray in the diagram above so the distance from B to C is the same as the distance from B to A?

1. Describe how you will locate point C and how you know $\overline{BC} \cong \overline{BA}$, then construct point C on the diagram above.

Now that we have three of the four vertices of the rhombus, we need to locate point D, the fourth vertex.

2. Describe how you will locate point D and how you know $\overline{CD} \cong \overline{DA} \cong \overline{AB}$, then construct point D on the diagram above.

Constructing a Square (A rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given $\overline{RS}$ below, fold and crease the paper so that point R is reflected onto point S. Based on the definition of reflection, what do you know about this "crease line"?

You have “constructed” a perpendicular bisector of $\overline{RS}$ by using a paper-folding strategy. Is there a way to construct this line using a compass and straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the “crease line”. When you have located at least two points on the “crease line” use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of $\overline{RS}$.

Now that you have created a line perpendicular to $\overline{RS}$ we will use the right angle formed to construct a square.

5. Label the midpoint of $\overline{RS}$ on the diagram above as point *M.* Using segment $\overline{RM}$ as one side of the square, and the right angle formed by segment $\overline{RM}$ and the perpendicular line drawn through point *M* as the beginning of a square. Finish constructing this square on the diagram above. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

Ready, Set, Go!

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Ready

Topic: Tools for construction and geometric work.

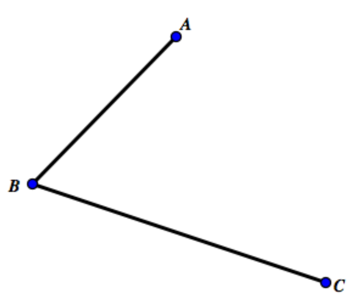
1. Using your compass draw several concentric circles that have point A as a center and then draw those same sized concentric circles that have B as a center. What do you notice about where all the circles with center A intersect all the corresponding circles with center B?

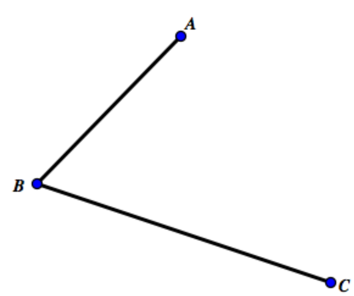
2. In the problem above you have demonstrated one way to find the midpoint of a line segment. Explain another way that a line segment can be bisected without the use of circles.

Set

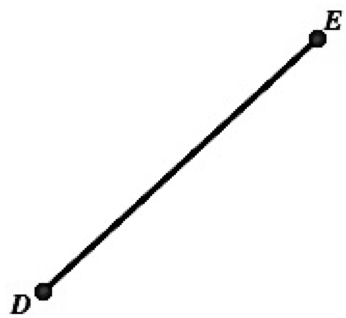
Topic: Constructions with compass and straight edge.

3. Bisect the angle below do it with compass and straight edge as well as with paper folding.

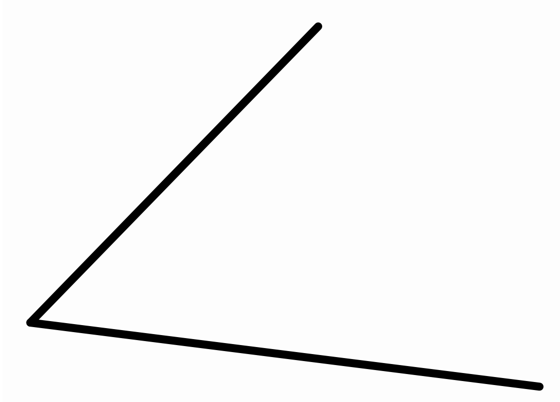




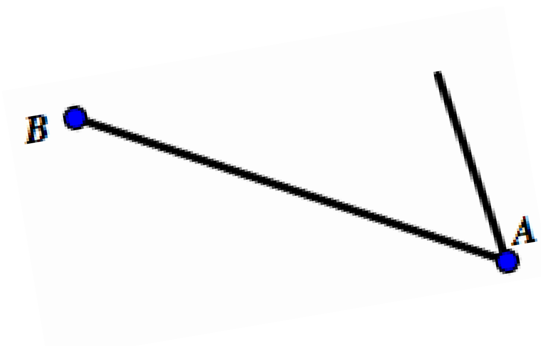
4. Copy the segment below using construction tools of compass and straight edge, label the image ***D'E'***.



5. Copy the angle below using construction tool of compass and straight edge.

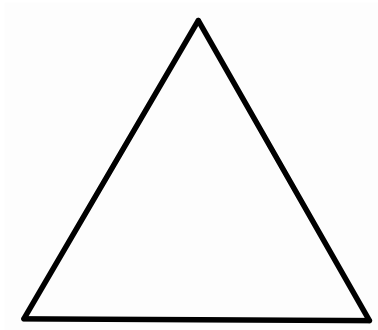


6. Construct a rhombus on the segment AB that is given below and that has point A as a vertex. Be sure to check that your final figure is a rhombus.



7. Construct a square on the segment CD that is given below. Be sure to check that your final figure is a square.

8. Given the equilateral triangle below, find the center of rotation of the triangle using compass and straight edge.



Go

Topic: Solving systems of equations review.

Solve each system of equations. Utilize substitution, elimination, graphing or matrices.

9. $\begin{cases} x = 11 + y \\ 2x + y = 19 \end{cases}$

10. $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

11. $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

12. $\begin{cases} y = -x + 1 \\ y = 2x + 1 \end{cases}$

13. $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$

14. $\begin{cases} 4x - y = 7 \\ -6x + 2y = 8 \end{cases}$

6.12 More Things Under Construction
A Develop Understanding Task

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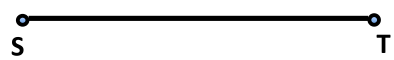
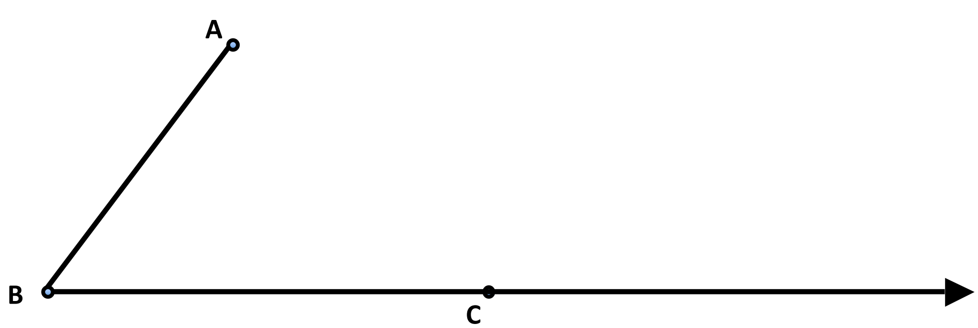
Constructing an Equilateral Triangle

Like a rhombus, an equilateral triangle has three congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given $\overline{ST}$ below as one side of the equilateral triangle.



Constructing a Parallelogram

To construct a parallelogram we will need to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to segment $\overline{AB}$ through point C on the diagram below. Since we have observed that parallel lines have the same slope, the line through point C will be parallel to $\overline{AB}$ only if the angle formed by the line and $\overrightarrow{CD}$ is congruent to $\angle ABC$. Can you describe and illustrate a strategy that will construct an angle with vertex at point C and a side parallel to $\overline{AB}$? (Hint: We know that corresponding parts of congruent triangles are congruent, so perhaps we can begin by constructing some congruent triangles.)



Constructing a Hexagon Inscribed in a Circle

Because regular polygons have rotational symmetry, they can be *inscribed* in a circle. The *circumscribed* circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

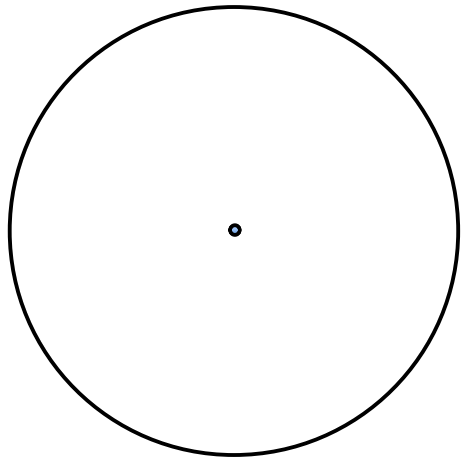
We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

1. Sketch a diagram of such a decomposition.

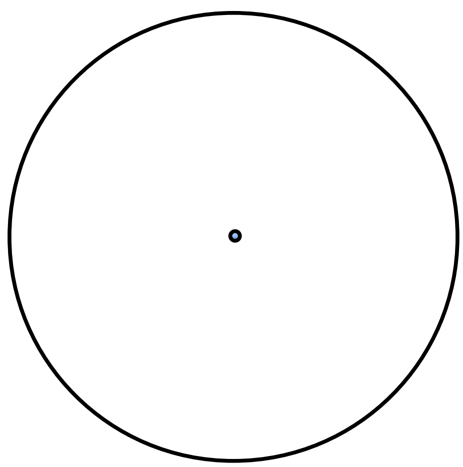
2. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?

3. The six vertices of the hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are *chords* of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? Be able to justify how you know this is so.

4. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the circle given below.



Modify your work with the hexagon to construct an equilateral triangle inscribed in the circle given below.



Describe how you might construct a square inscribed in a circle.

Ready, Set, Go!

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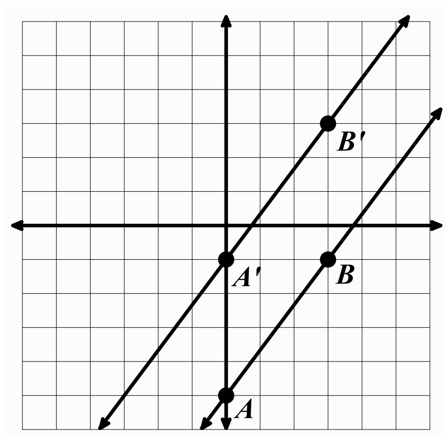
Ready

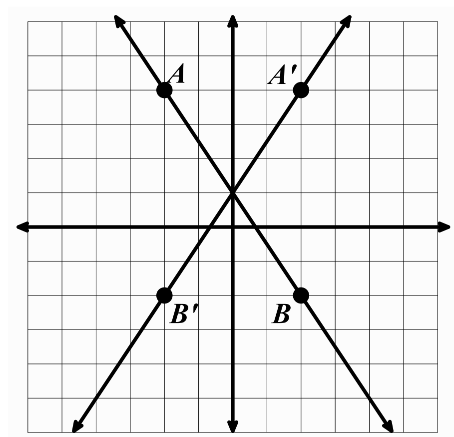
Topic: Transformations of lines, algebraic and geometric thoughts.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, label them, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.



2.



1. a. Description of Transformation:

   b. Equation for pre-image:

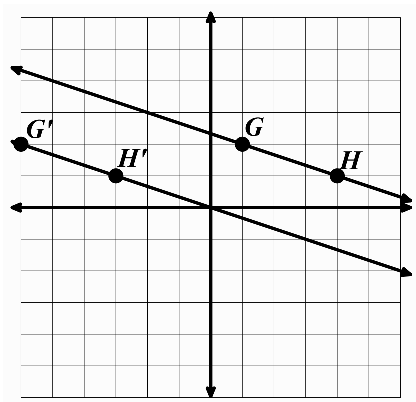
   c. Equation for image:

2. a. Description of Transformation:

   b. Equation for pre-image:

   c. Equation for image:

3.

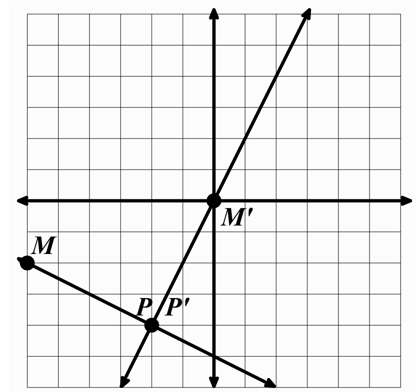


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4.



a. Description of Transformation:

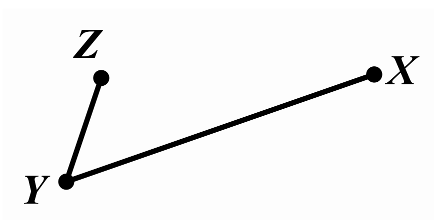
b. Equation for pre-image:

c. Equation for image:

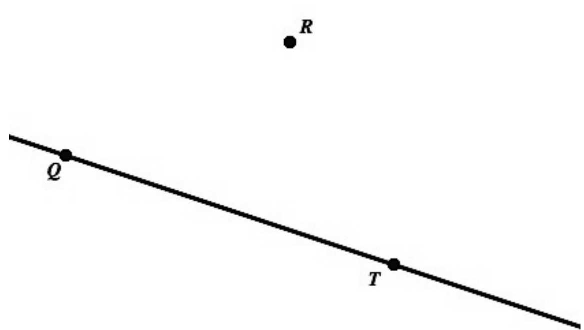
Set

Topic: Geometric Constructions using compass and straight edge.

5. Construct a parallelogram given sides $\overline{XY}$ and $\overline{YZ}$ and $\angle XYZ$.



6. Construct a line parallel to $\overline{QT}$ and through point ***R.***



7. Given segment $\overline{AB}$ show all points ***C*** such that $\triangle ABC$ is an isosceles triangle.

8. Given segment $\overline{AB}$ show all points C such that $\triangle ABC$ is a right triangle.

Go

Topic: Triangle congruence and properties of polygons.

9. What is the minimum amount of information needed to determine that two triangles are congruent? List all possible combinations of needed criteria.

10. What is a line of symmetry and what is a diagonal? Are they the same thing? Could they be the same in a polygon? If so give an example, if not explain why not.

11. How is the number of lines of symmetry for a *regular* polygon connected to the number of sides of the polygon? How is the number of diagonals for a polygon connected to the number of sides?

12. What do right triangles have to do with finding distance between points on a coordinate grid?

6.13 Justifying Constructions

A Solidify Understanding Task

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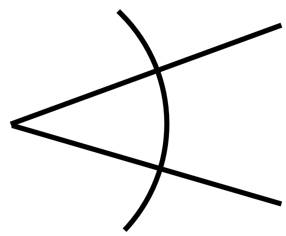
Compass and straightedge constructions can be justified using such tools as:

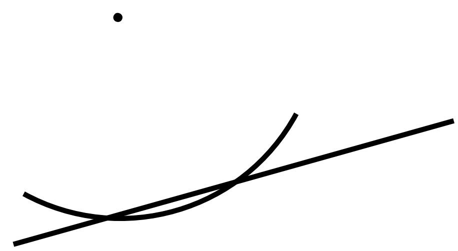
- the definitions and properties of the rigid-motion transformations
- identifying corresponding parts of congruent triangles
- using observations about sides, angles and diagonals of special types of quadrilaterals

Study the steps of the following procedure for *constructing an angle bisector*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

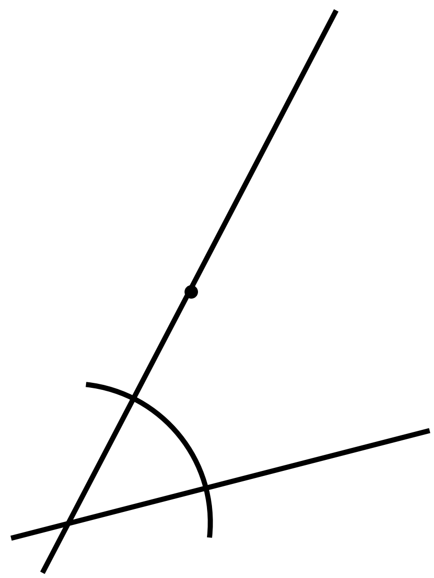
Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for *constructing a line perpendicular to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.	
Without changing the span of the compass, locate a second point not on the given line, by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for constructing a line parallel to a given line through a given point.

Steps	Illustration
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc, and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Ready, Set, Go!

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Ready

Topic: Rotation symmetry for regular polygons and transformations

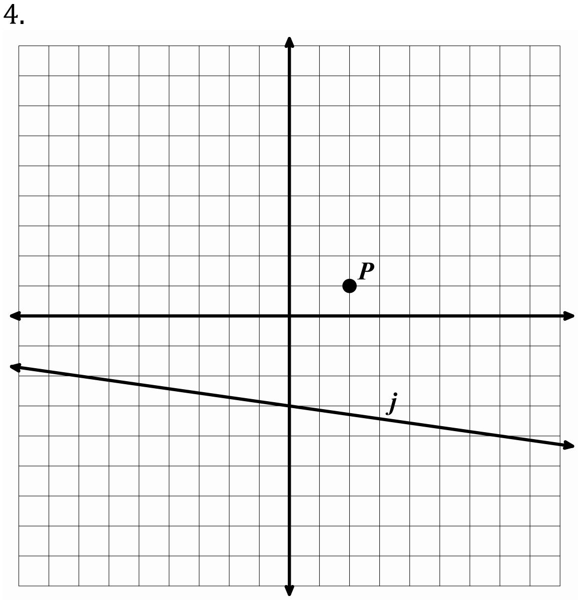
1. What angles of rotational symmetry are there for a pentagon?

2. What angles of rotational symmetry are there for a hexagon?

3. If a regular polygon has an angle of rotational symmetry that is 40^0, how many sides does the polygon have?

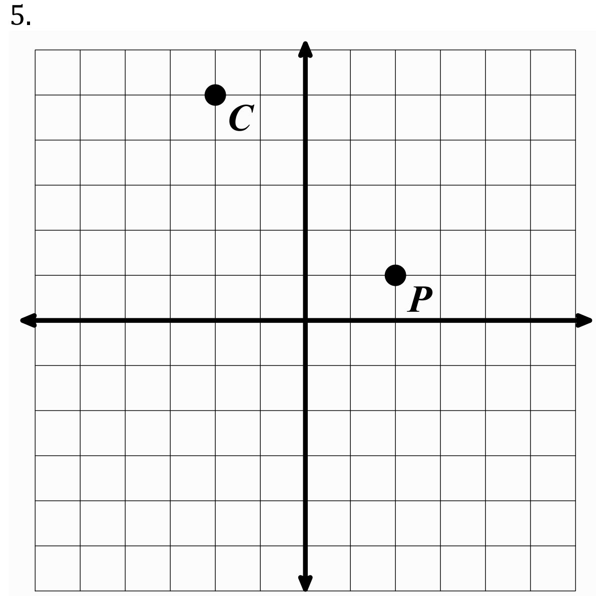
On each given coordinate grid below perform the indicated transformation.

4.



Reflect point ***P*** over line ***j***.

5.

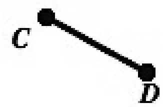


Rotate point ***P*** 90^0 clockwise around point ***C***.

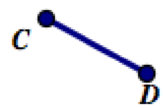
Set

Topic: Constructing regular polygons inscribed in a circle.

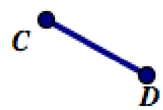
6. Construct an isosceles triangle that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the triangle.



7. Construct a hexagon that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the hexagon.



8. Construct a square that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the square.



Go

Topic: Finding distance and slope.

For each pair of given coordinate points find distance between them and find the slope of the line that passes through them. Show all your work.

9. (-2, 8) , (3, -4)
   a. Slope: b. Distance:

10. (-7, -3) , (1, 5))
   a. Slope: b. Distance:

11. (3, 7) , (-5, 9)
   a. Slope: b. Distance:

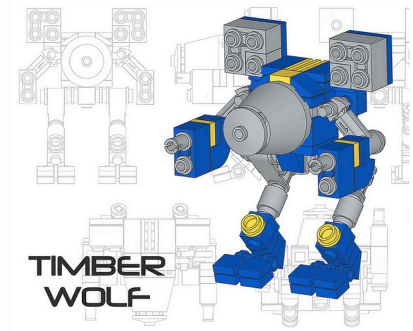
12. (1, -5) (-7, 1)
   a. Slope: b. Distance:

13. (-10, 31) (20, 11)
   a. Slope: b. Distance:

14. (16, -45) (-34, 75)
   a. Slope: b. Distance:

6.14 Construction Blueprints

A Practice Understanding Task



For each of the following straightedge and compass constructions, illustrate or list the steps for completing the construction and give an explanation for why the construction works. You explanations may be based on rigid-motion transformations, congruent triangles, or properties of quadrilaterals.

Purpose of the construction	Illustration and/or steps for completing the construction	Justification of why this construction works
Copying a segment	1. Set the span of the compass to match the distance between the two endpoints of the segment. 2. Without changing the span of the compass, draw an arc on a ray centered at the endpoint of the ray. The second endpoint of the segment is where the arc intersects the ray.	The given segment and the constructed segment are radii of congruent circles.
Copying an angle		
Bisecting a segment		
Bisecting an angle		
Constructing a perpendicular bisector of a line segment		
Constructing a perpendicular to a line through a given point		

Constructing a line parallel to a given line through a given point		
Constructing an equilateral triangle		
Constructing a regular hexagon inscribed in a circle		

Ready, Set, Go!

Ready

Topic: Connecting tables with transformations.

For each function find the outputs that fill in the table. Then describe the relationship between the outputs in each table.

1. $f(x) = 3x$ $\qquad$ $g(x) = 3x - 5$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

2. $t(x) = 2x$ $\qquad$ $h(x) = 2x - 5$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

Relationship between $t(x)$ and $h(x)$:

3. $f(x) = 2x$ $\qquad$ $g(x) = 2(x - 3)$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

4. $t(x) = 4x$ $\qquad$ $h(x) = 4^{(x-3)}$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

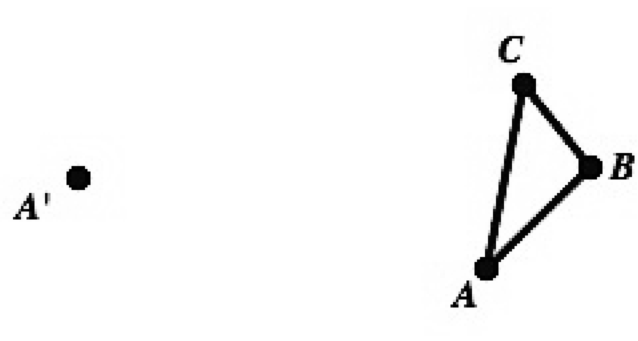
Relationship between $t(x)$ and $h(x)$:

Set

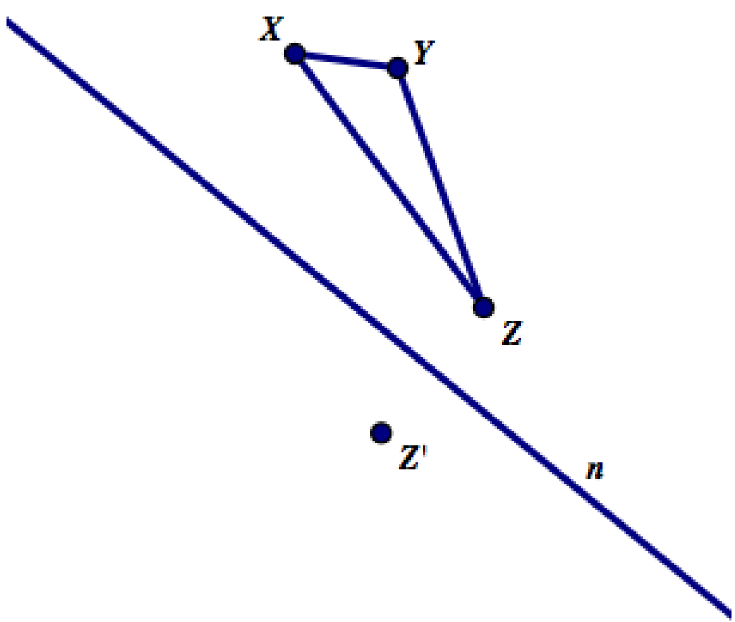
Topic: Constructing transformations

In each problem below use compass and straight edge to construct the transformation that is described.

5. Construct D A'B'C' so that it is a translation of D ABC. (Hint: parallel lines may be useful.)



6. Construct D X'Y'Z' so that it is a reflection of D XYZ over line ***m***.
(Hint: perpendicular lines may be useful.)



Go

Topic: Transformations and triangle congruence.

Determine whether or not the statement is true or false. If true, explain why. If false, explain why not or provide a counterexample.

7. If one triangle can be transformed so that one of its angles and one of its sides coincide with another triangles angle and side then the two triangles are congruent.

8. If one triangle can be transformed so that two of its sides and any one of its angles will coincide with two sides and an angle from another triangle then the two triangles will be congruent.

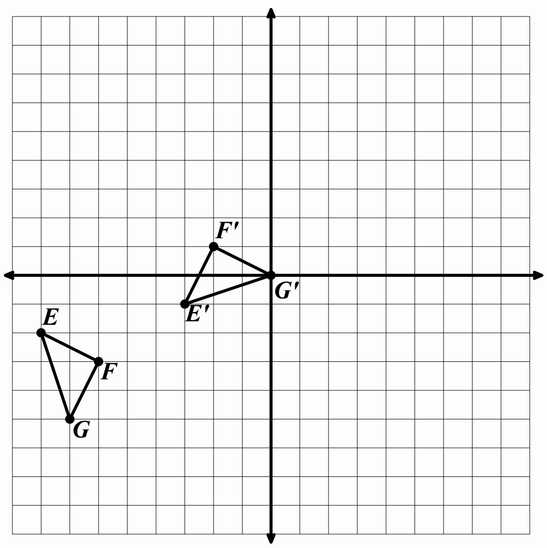
9. If all three angles of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

10. If all three sides of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

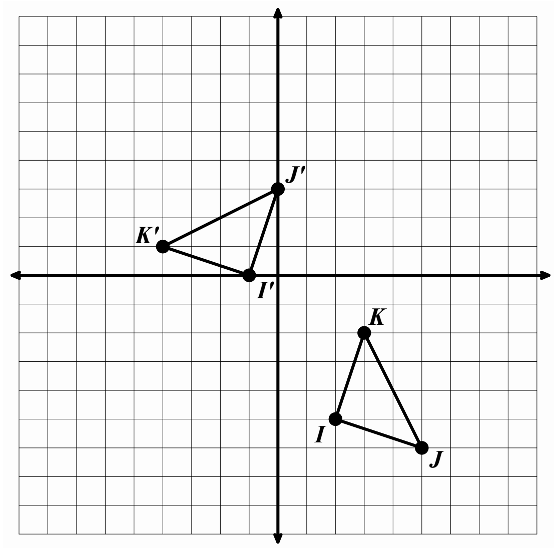
11. For any two congruent polygons there is a sequence of transformations that will transform one of the polygons onto the other.

Find the point of rotation for each of the figures below.

12.

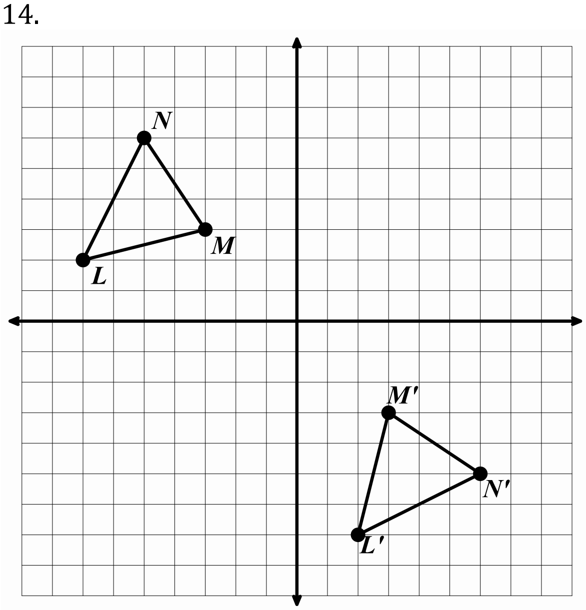


13.

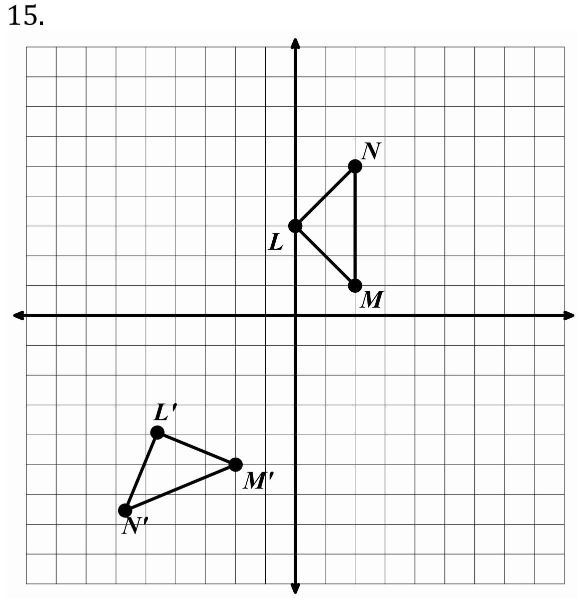


Find the line of reflection for each of the figures drawn below.

14.



15.



Secondary One Mathematics: An Integrated Approach
Module 7
Connecting Algebra and Geometry

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 7 – Connecting Algebra and Geometry

Classroom Task: 7.1 Go the Distance- A Develop Understanding Task
Use coordinates to find distances and determine the perimeter of geometric shapes **(G.GPE.7)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.1

Classroom Task: 7.2 Slippery Slopes – A Solidify Understanding Task
Prove slope criteria for parallel and perpendicular lines **(G.GPE.5)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.2

Classroom Task: 7.3 Prove It! – A Solidify Understanding Task
Use coordinates to algebraically prove geometric theorems **(G.GPE.4)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.3

Classroom Task: 7.4 Training Day– A Solidify Understanding Task
Write the equation f(t)= m(t) + k by comparing parallel lines and finding k **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.4

Classroom Task: 7.5 Training Day Part II – A Practice Understanding Task
Determine the transformation from one function to another **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.5

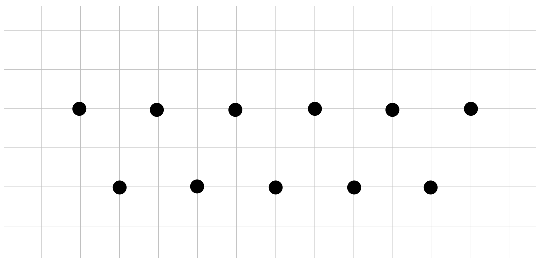
Classroom Task: 7.6 Shifting Functions – A Practice Understanding Task
Translating linear and exponential functions using multiple representations **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.6

7.1 Go the Distance

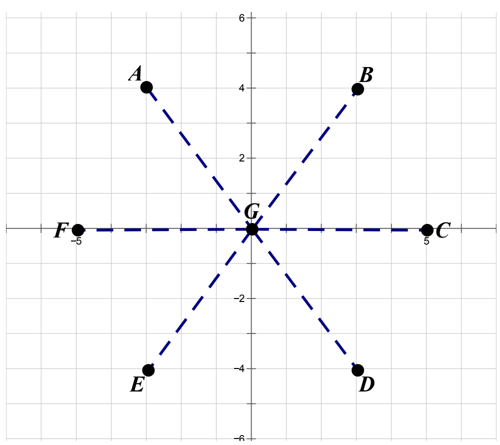
A Develop Understanding Task

http://www.flickr.com/photos/briemckinneyxo/

The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one girl in the middle to six other girls. On the grid, their pattern looks like this:

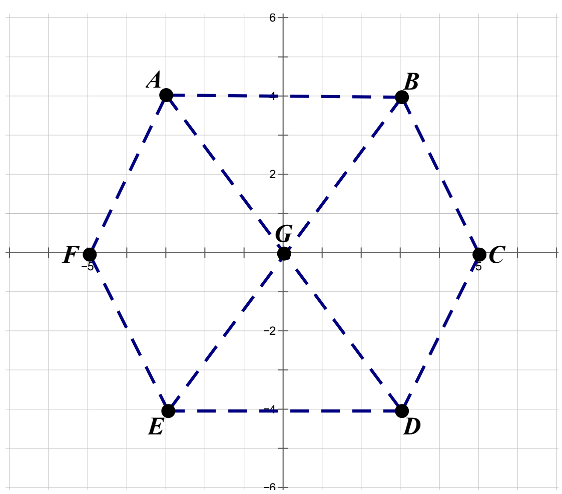


The question the girls have is how long to make the ribbons. Some girls think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

2. Explain how you found the length of each ribbon.

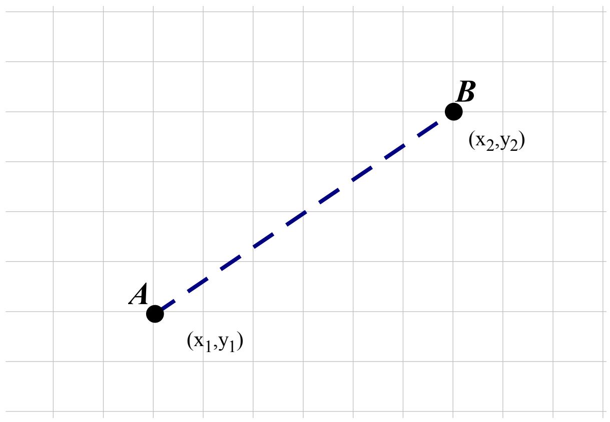
When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1, y_1) and B (x_2, y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, using points A and B.

5. Use the process you came up with in #4 to find the distance between two points located at (-1, 5) and (2, -6)

6. Use you process to find the perimeter of the hexagon pattern shown in #3.

Name: Connecting Algebra and Geometry | 7.1

Ready, Set, Go!

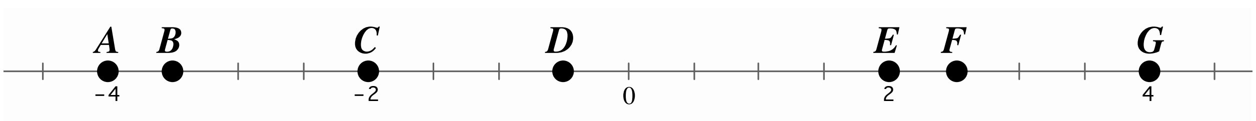
http://www.flickr.com/photos/briemckinneyxo/

Ready

Topic: Finding the distance between two points

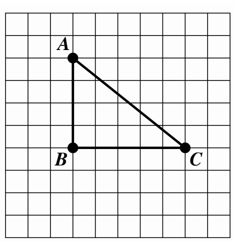
Use the number line to find the distance between the given points. (The notation AB means the distance between points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8.



a. Find AB

b. Find BC

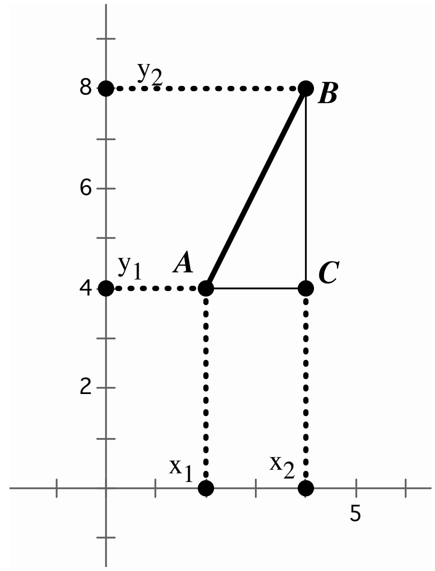
c. Find AC

9. Why is it easier to find the distance between points A and B and points B and C than it is to find the distance between A and C?

10. Explain how to find the distance between points A and C.

Set

Topic: Slope triangles and the distance formula.



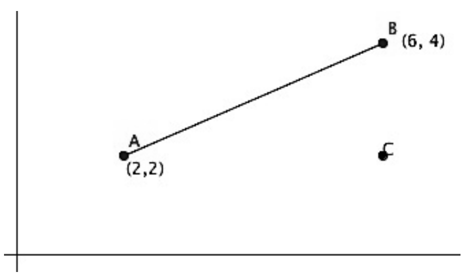
Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2-y_1}{x_2-x_2}$ *where m is the slope.*

11a. What does the value ($y_2 - y_1$) tell you?

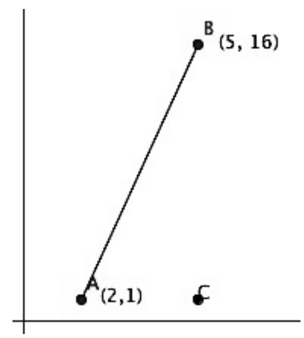
b. What does the value ($x_2 - x_1$) tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and performing the Pythagorean Theorem. In this exercise try to develop a more efficient method of finding the length of a line segment by using the meaning of ($y_2 - y_1$) and ($x_2 - x_1$) combined with the Pythagorean Theorem.

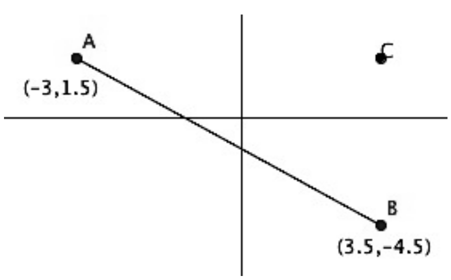
12. Find AB.



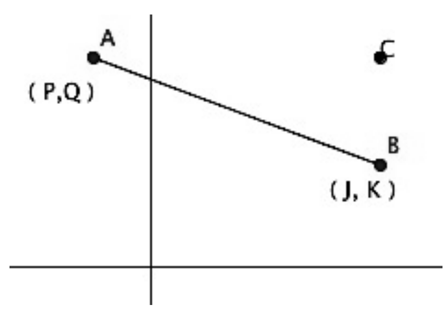
13. Find AB.



14. Find AB.



15. Find AB.



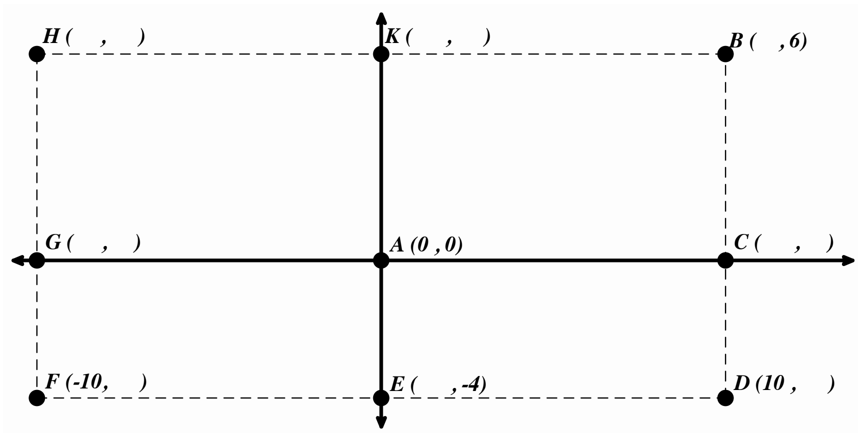
Name: Connecting Algebra and Geometry 7.1

Go Topic: Rectangular coordinates

Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

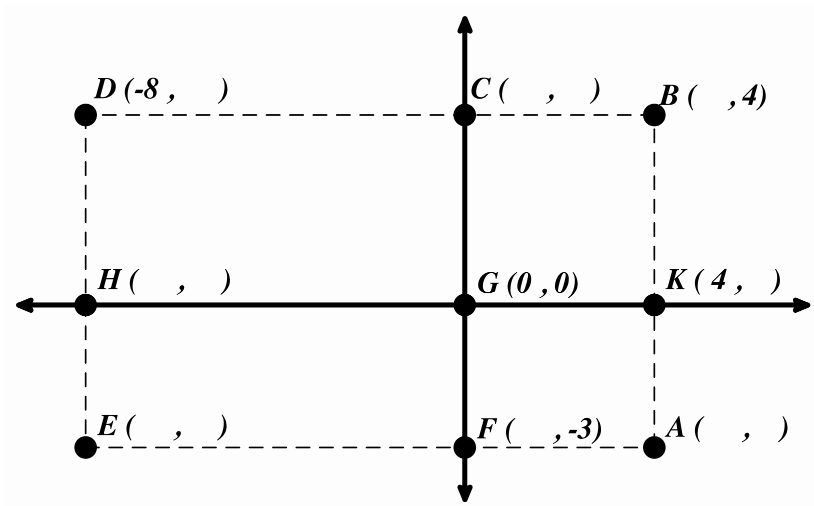
16a. Find HB

b. Find BD



17a. Find DB

b. Find CF



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/the-coordinate-plane

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/distance-formula

7.2 Slippery Slopes

A Solidify Understanding Task

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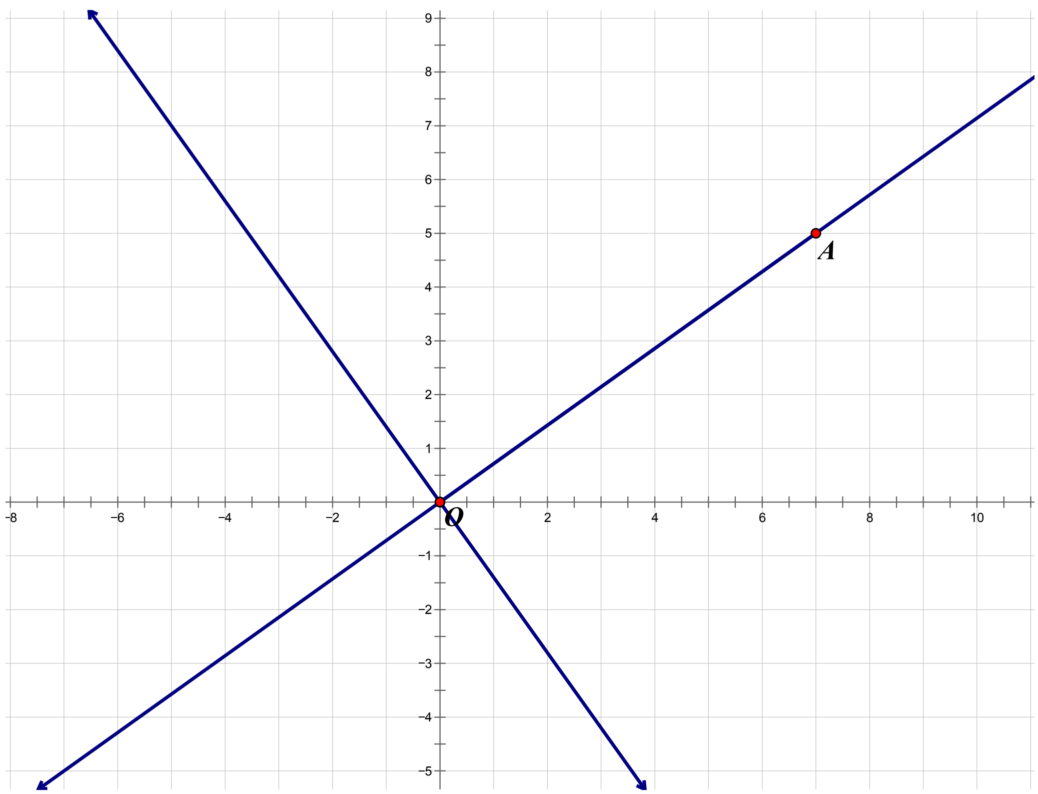
While working on “Is It Right?” in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let’s start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

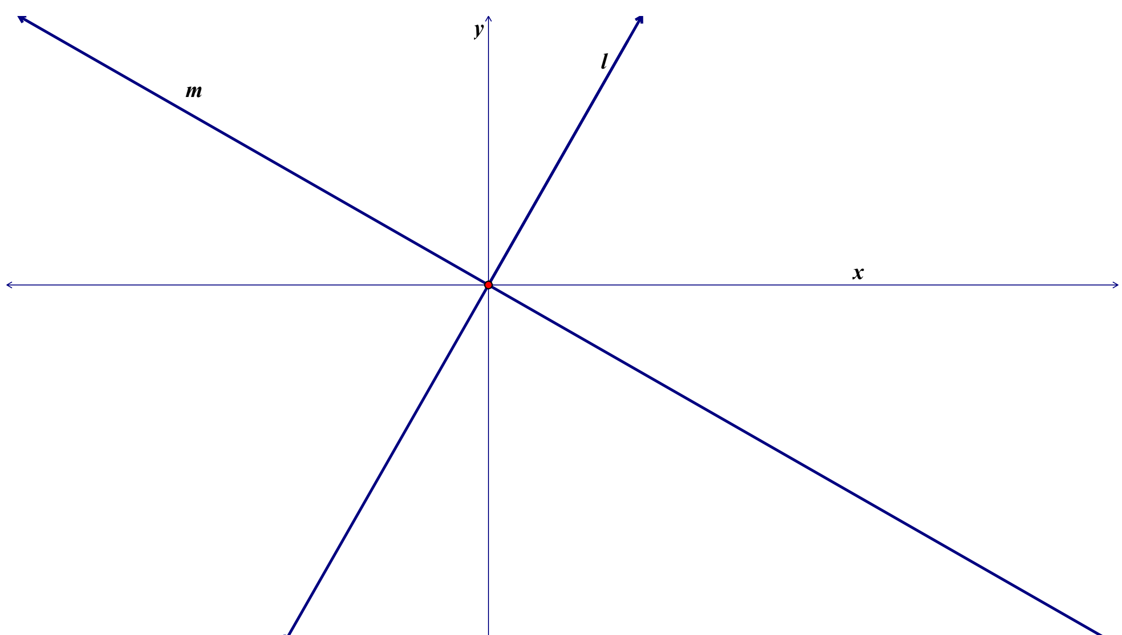
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with this theorem.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle.

6. What is the slope of line *l*?

   Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

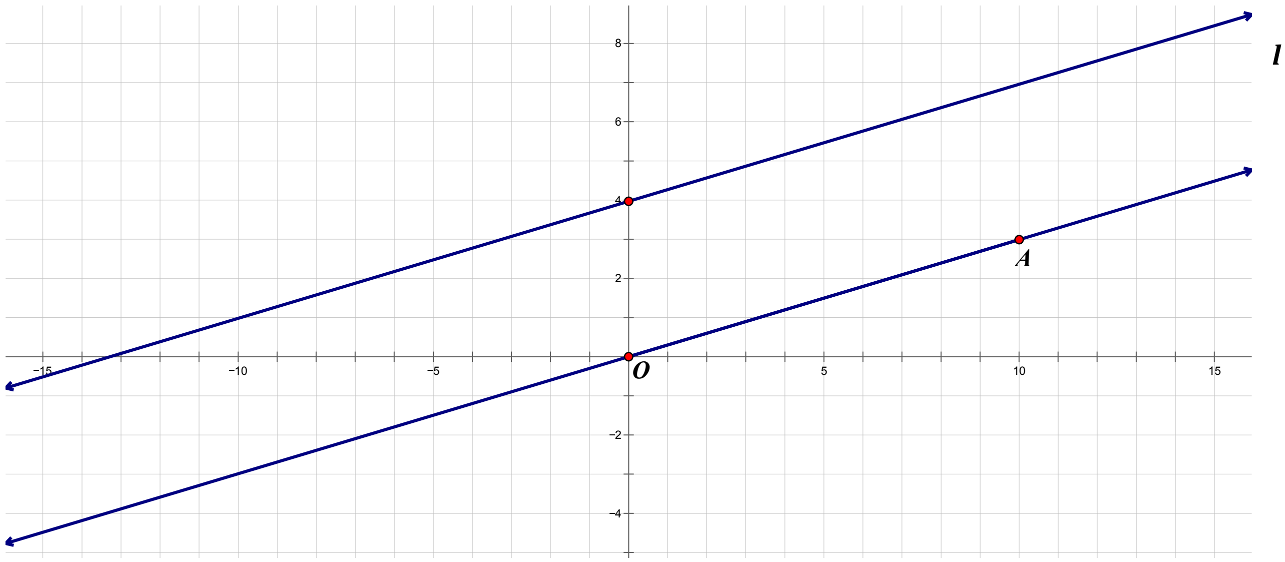
10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

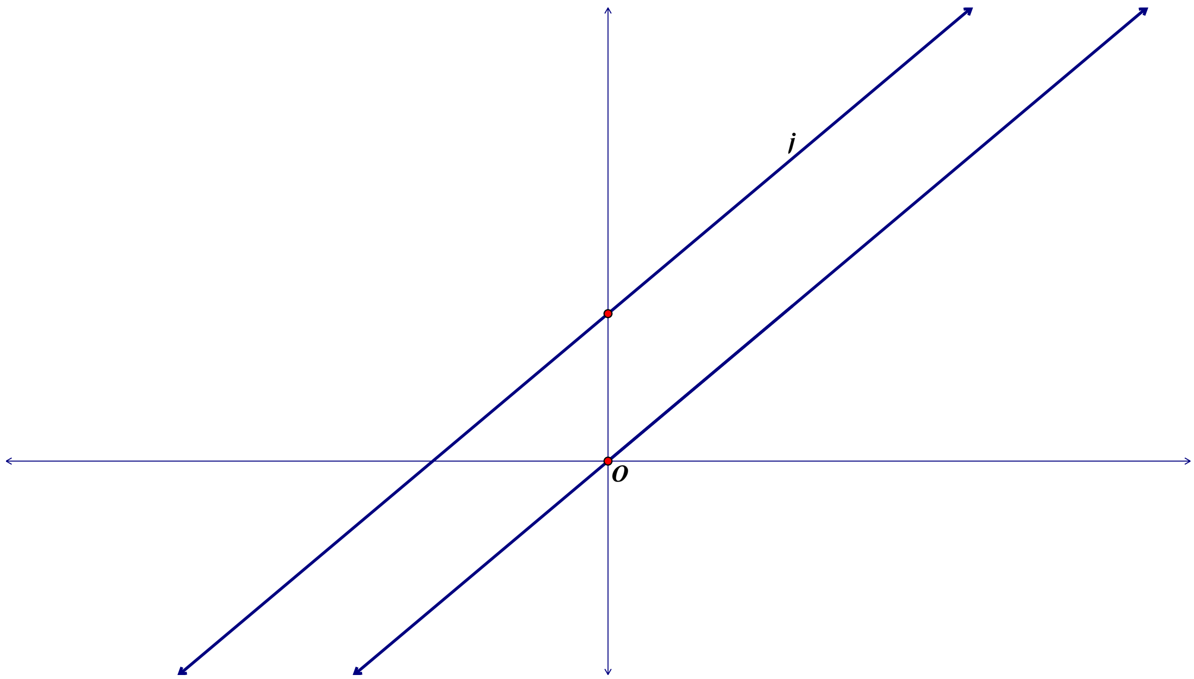
Draw the slope triangle from point A. What is the slope of $\overleftrightarrow{OA}$?



What translation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto line l?

What must be true about the slope of line l? Why?

Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



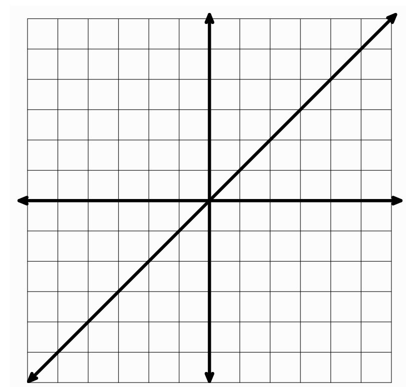
Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

Ready, Set, Go!

Ready

Topic: Graphing lines.

The graph at the right is of the line $f(x) = x$.



1a. On the same grid, graph a parallel line that is 3 units above it.

b. Write the equation of the new line. ___________________________

c. Write the y-intercept of the new line as an ordered pair.

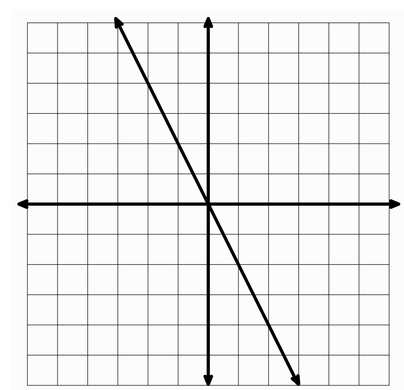
d. Write the x-intercept of the new line as an ordered pair.

e. Write the equation of the new line in point-slope form using the y-intercept.

f. Write the equation of the new line in point-slope form using the x-intercept.

g. Explain in what way the equations are the same and in what way they are different.

The graph at the right is of the line $f(x) = -2x$.



2a. On the same grid, graph a parallel line that is 4 units below it.

b. Write the equation of the new line. _________________________

c. Write the y-intercept of the new line as an ordered pair.

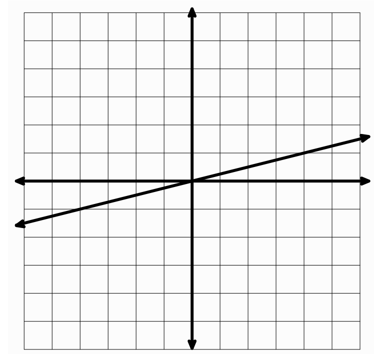
d. Write the x-intercept as an ordered pair.

e. Write the equation of the new line in point-slope form using the y-intercept

f. Write the equation of the new line in point-slope form using the x-intercept.

g. Explain in what way the equations are the same and in what way they are different.

Name: Connecting Algebra and Geometry | 7.2



The graph at the right is of *f(x)* = ¼ x

3a. Graph a parallel line 2 units below.

b. Write the equation of the new line.

c. Write the y-intercept as an ordered pair.

d. Write the x-intercept as an ordered pair.

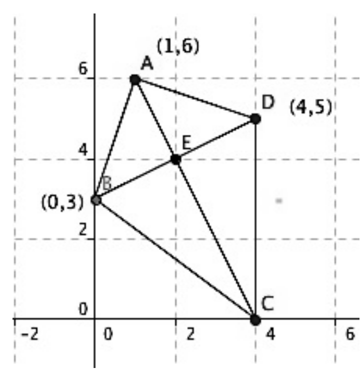
e. Write the equation of the new line in point-slope form using the y-intercept

f. Write the equation of the new line in point-slope form using the x-intercept

g. Explain in what way the equations are the same and in what way they are different.

Set

Topic: Verifying and Proving Geometric Relationships



The quadrilateral at the right is called a **kite.**

Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically.
(A symbol may be used more than once.)

$\cong \quad \perp \quad || \quad < \quad > \quad =$

Proof

4. $\overline{BC}$ ________ $\overline{DC}$ ____

____

5. $\overline{BD}$ ________ $\overline{AC}$ ____

____

6. $\overline{AB}$ ________ $\overline{BC}$ ____

____

7. ΔABC _______ ΔADC ____

____

8. $\overline{BE}$ ________ $\overline{ED}$ ____

____

9. $\overline{AE}$ ________ $\overline{ED}$ ____

____

10. $\overline{AC}$ _________ $\overline{BD}$ ____

____

Go

Topic: Writing equations of lines.

Write the equation of the line in standard form using the given information.

11. Slope: -¼ point (12, 5)

12. A (11, -3) , B (6, 2)

13. x-intercept: -2, y-intercept: -3

14. All x values are -7, y can be anything

15. Slope: ½ x-intercept: 5

16. E (-10, 17) , G (13, 17)

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-using-x-and-y-intercepts

7.3 Prove It!

A Solidify Understanding Task

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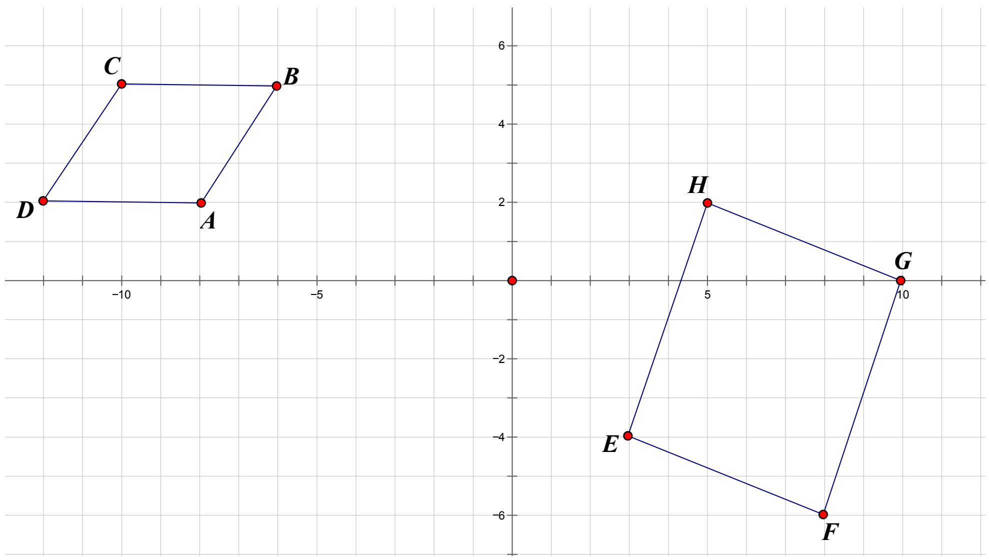
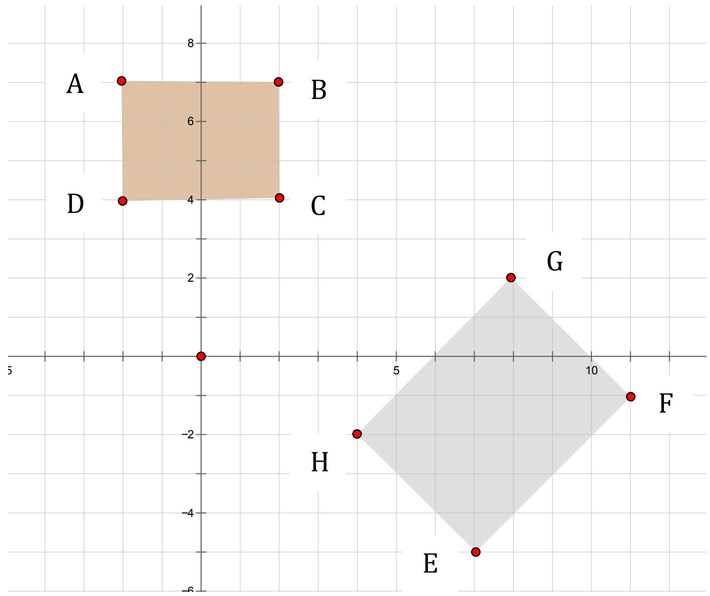
In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.



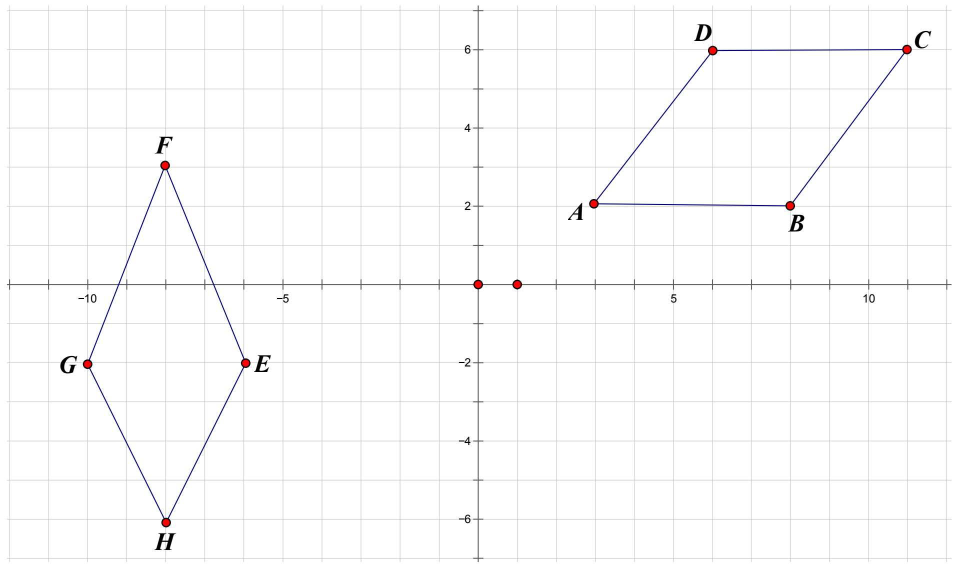
Is ABCD a parallelogram? Explain how you know.

Is EFGH a parallelogram? Explain how you know.



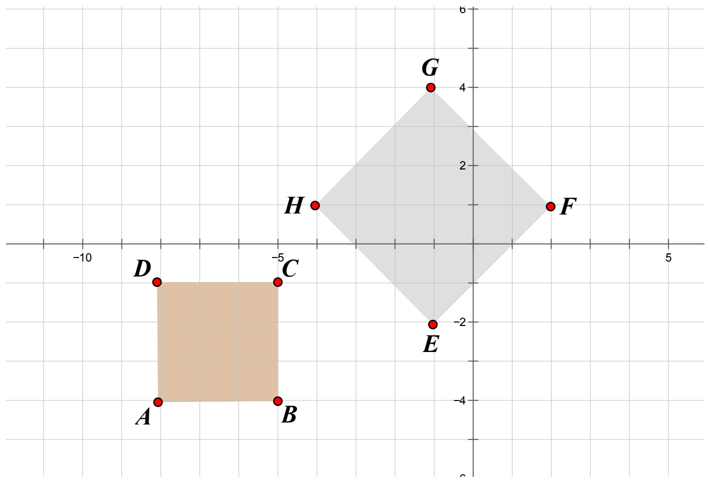
Is ABCD a rectangle? Explain how you know.

Is EFGH a rectangle? Explain how you know.



Is ABCD a rhombus? Explain how you know.

Is EFGH a rhombus? Explain how you know.



Is ABCD a square? Explain how you know.

Is EFGH a square? Explain how you know.

Name: Connecting Algebra and Geometry | 7.3

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Ready, Set, Go!

Ready

Topic: Tables of value

Find the value of *f (x)* for the given domain. Write *x* and *f (x)* as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

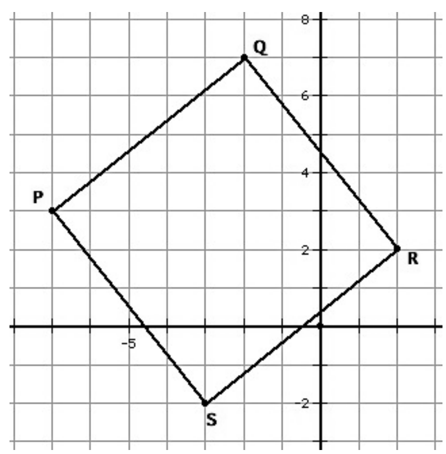
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

Set

Topic: Characteristics of rectangles and squares

4a. Is the figure below a rectangle? (Justify your answer)

b. Is the figure a square? (Justify your answer)

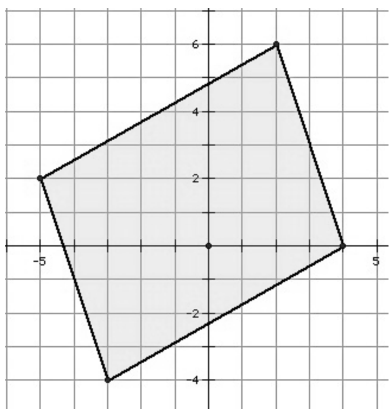


Name: Connecting Algebra and Geometry | 7.3

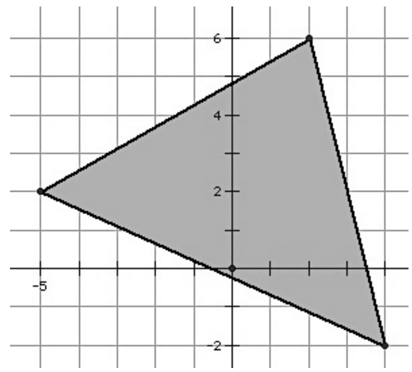
Go

Find the perimeter of each figure below. Round to the nearest hundredth.

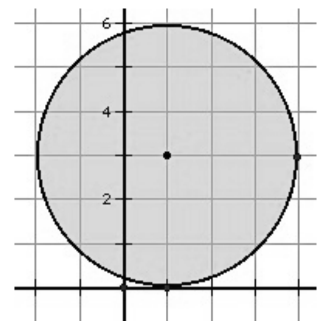
5.



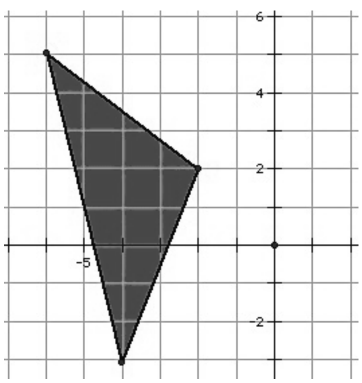
6.



7.



8.



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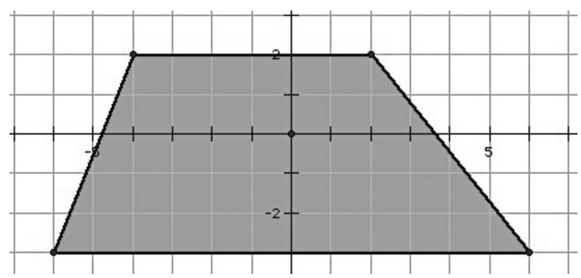

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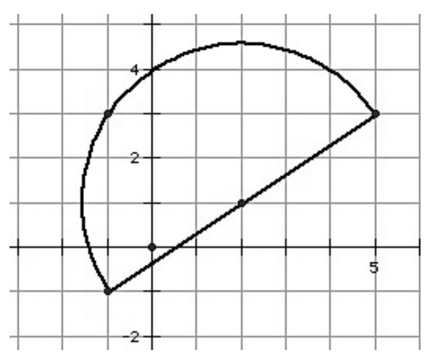
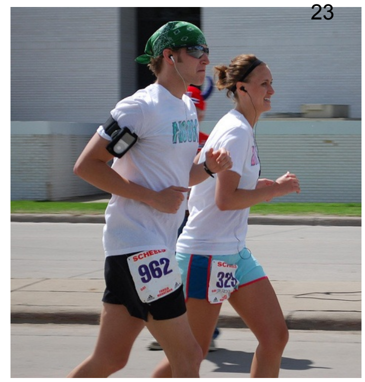


Need Help? Check out these related videos:

http://www.khanacademy.org/math/geometry/basic-geometry/v/perimeter-and-area-of-a-non-standard-polygon

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/distance-formula



http://www.flickr.com/photos/fargomoorheadcvb/

7.4 Training Day

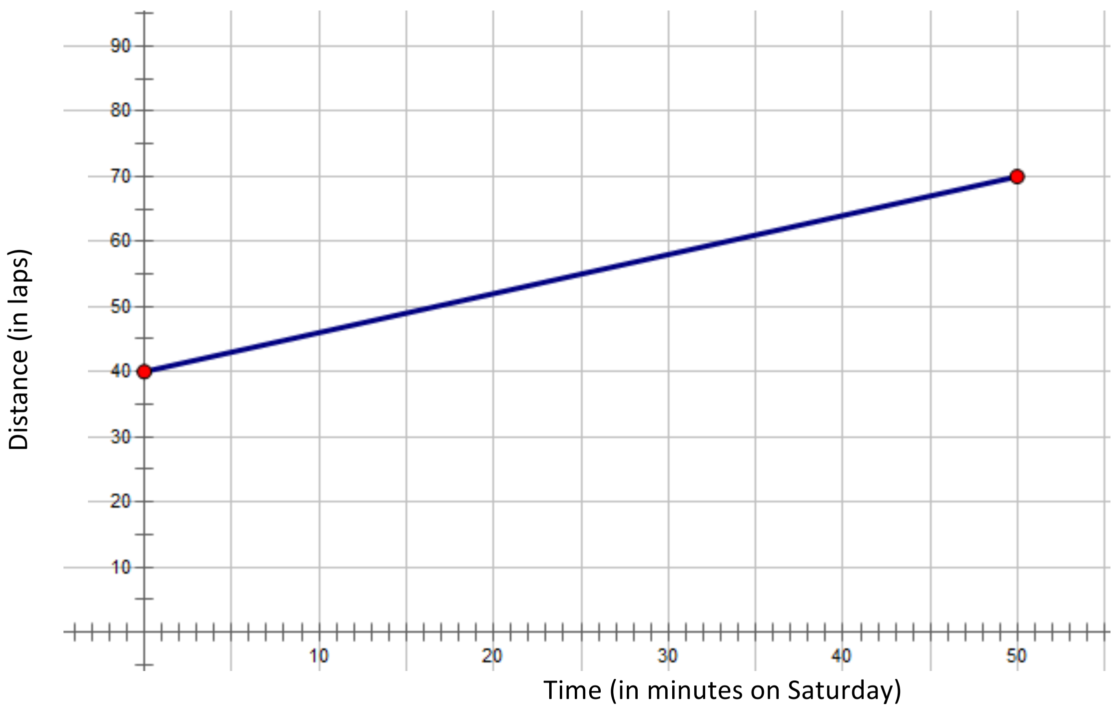
A Develop Understanding Task

Fernando and Mariah are training for six weeks to run in the Salt Lake half- marathon. To train, they run laps around the track at Eastland High School. Since their schedules do not allow them to run together during the week, they each keep a record of the total number of laps they run throughout the week and then always train together on Saturday morning. The following are representations of how each person kept track of the total number of laps that they ran throughout the week plus the number of laps they ran on Saturday.

Fernando's data:

Time (in minutes on Saturday)	0	10	20	30	40	50
Distance (in laps)	60	66	72	78	84	90

Mariah's data:



What observations can be made about the similarities and differences between the two trainers?

1. Write the equation, *m(t),* that models Mariah's distance.

2. Fernando and Mariah both said they ran the same rate during the week when they were training separately. Explain in words how Fernando's equation is similar to Mariah's. Use the sentence frame: The rate of both runners is the same throughout the week, however, Fernando ____.

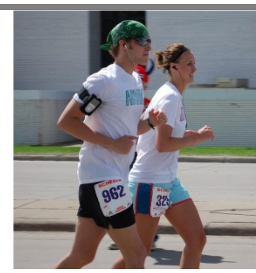
3. In mathematics, sometimes one function can be used to build another. Write Fernando's equation, *f(t),* by starting with Mariah's equation, *m(t).*

   f(t) =

4. Use the mathematical representations given in this task (table and graph) to model the equation you wrote for number 3. Write in words how you would explain this new function to your class.

Name: Connecting Algebra and Geometry 7.4

Ready, Set, Go!

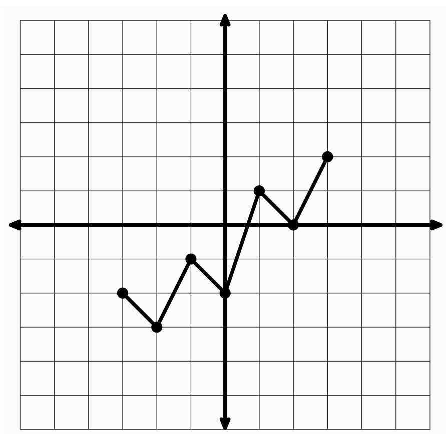


http://www.flickr.com/photos/fargomoorheadcvb

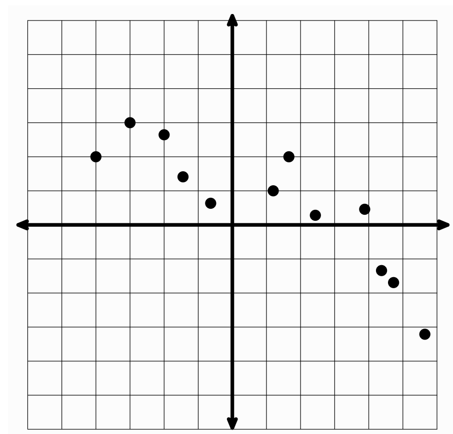
Ready

Topic: Vertical transformations of graphs

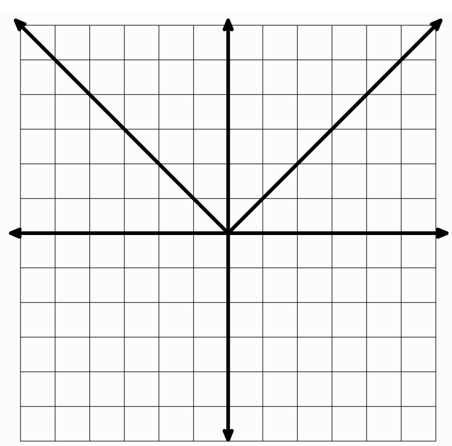
1. Use the graph below to draw a new graph that is translated up 3 units.



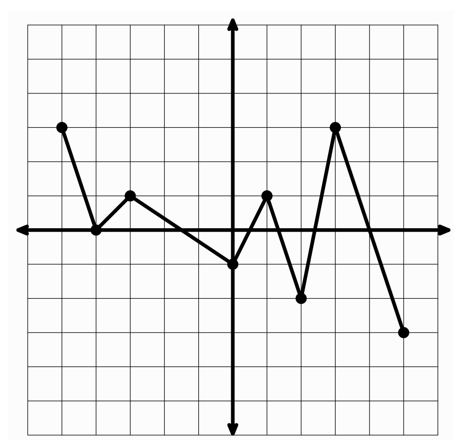
2. Use the graph below to draw a new graph that is translated down 1 unit.



3. Use the graph below to draw a new graph that is translated down 4 units.



4. Use the graph below to draw a new graph that is translated down 3 units.



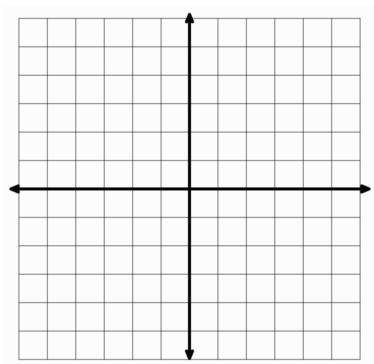
Name: Connecting Algebra and Geometry | 7.4

Set

You are given the equation of *f(x)* and the transformation *g(x) = f(x) + k*. Graph both *f (x)* and *g (x) and* the linear equation for *g(x)* below the graph.

5. $f(x) = 2x - 4$

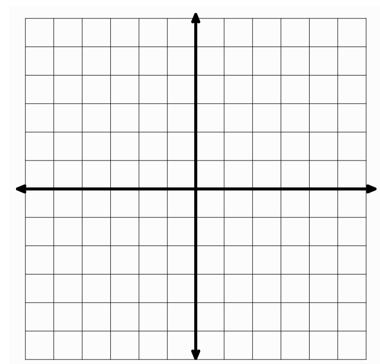
$g(x) = f(x) + 3$



$g(x)$ = ________________________

6. $f(x) = 0.5x$

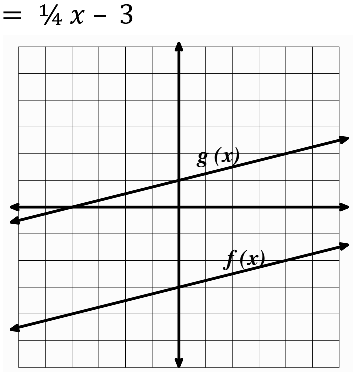
$g(x) = f(x) - 3$



$g(x)$ = ________________________

Based on the given graph, write the equation of *g(x)* in the form of *g(x) = f(x) + k.* Then simplify the equation of *g(x)* into slope-intercept form. The equation of *f(x)* is given.

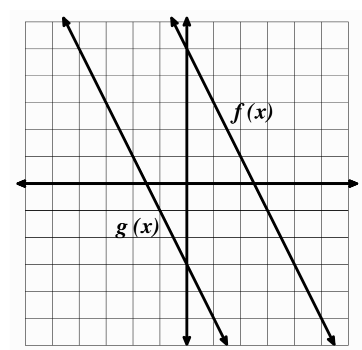
7. $f(x) = \frac{1}{4}x - 3$



a. $g(x)$ = ________________________
Translation form

b. $g(x)$ = ________________________
Slope-Intercept form

8. $f(x) = -2x + 5$



a. $g(x)$ = ________________________
Translation form

b. $g(x)$ = ________________________
Slope-Intercept form

Name: Connecting Algebra and Geometry 7.4

Go

9. Fernando and Mariah are training for a half marathon. The chart below describes their workout for the week just before the half marathon. If four laps are equal to one mile, and if there are 13.1 miles in a half marathon, do you think Mariah and Fernando are prepared for the event? Describe how you think each person will perform in the race. Include who you think will finish first and what each person's finish time will be. Use the data to inform your conclusions and to justify your answers.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Fernando: Distance (in laps)	34	45	52	28	49	36
Time per day (in minutes)	60	72	112	63	88	58
Mariah: Distance (in laps)	30	48	55	44	38	22
Time per day (in minutes)	59	75	119	82	70	45

7.5 Training Day Part II

A Practice Understanding Task

http://www.flickr.com/photos/pdgoodman

Fernando and Mariah continued training in preparation for the half marathon. For the remaining weeks of training, they each separately kept track of the distance they ran during the week. Since they ran together at the same rate on Saturdays, they took turns keeping track of the distance they ran and the time it took. So they would both keep track of their own information, the other person would use the data to determine their own total distance for the week.

Week 2: Mariah had completed 15 more laps than Fernando before they trained on Saturday.

a. Complete the table for Mariah.

Time (in minutes on Saturday)	0	10	20	30	40	50	60
Fernando: Distance (in laps)	50	56	62	68	74	80	86
Mariah: Distance (in laps)							

b. Write the equation for Mariah as a transformation of Fernando. Equation for Mariah: *m(t) = f(t)* ________

Week 3: On Saturday morning before they started running, Fernando saw Mariah's table and stated, "My equation this week will be *f(t) = m(t)* + 30."

a. What does Fernando's statement mean?

b. Based on Fernando's translated function, complete the table.

Time (in minutes on Saturday)	0	20	40	60	70
Fernando: Distance (in laps)					
Mariah: Distance (in laps)	45	57	69	81	87

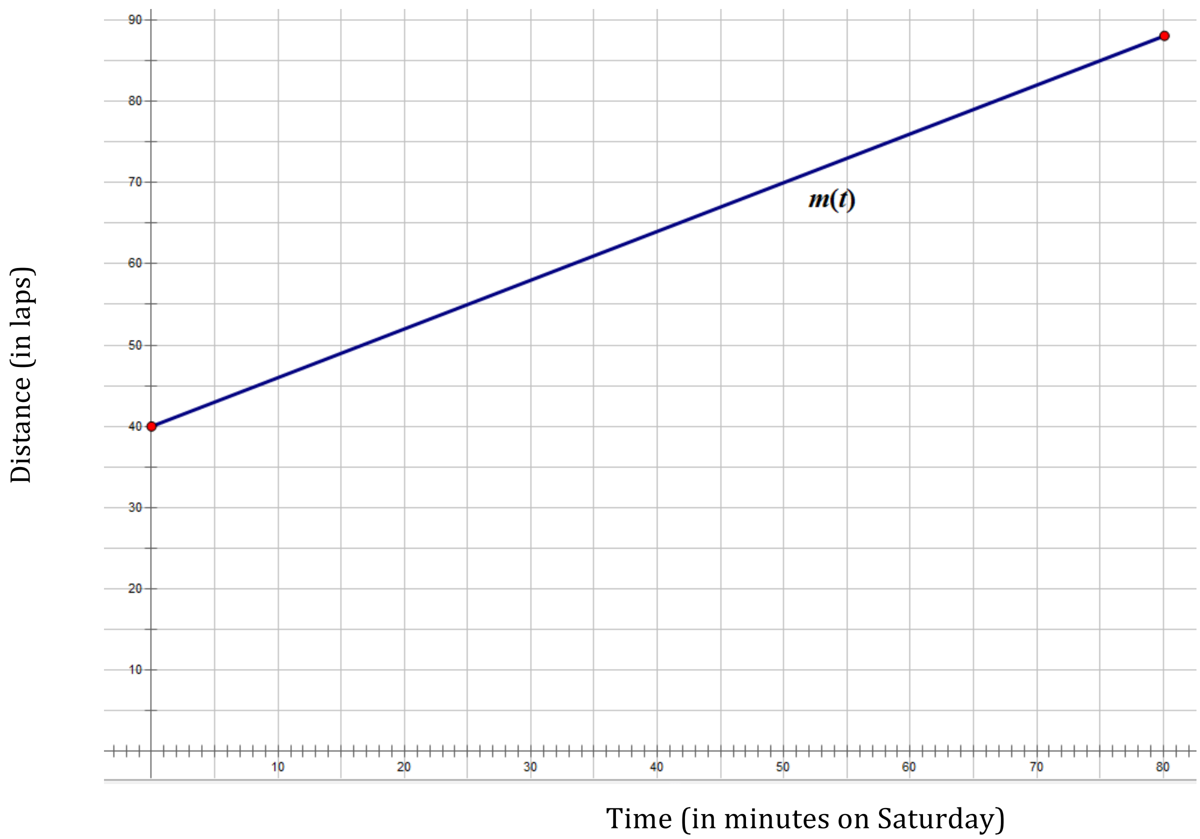
c. Write the equation for both runners:

d. Write the equation for Mariah, transformed from Fernando.

e. What relationship do you notice between your answers to parts c and d?

Week 4: The marathon is only a couple of weeks away!

a. Use Mariah's graph to sketch *f(t)*. *f(t)* = *m(t)* – 10



b. Write the equations for both runners.
c. What do you notice about the two graphs? Would this always be true if one person ran "*k*" laps more or less each week?

Week 5: This is the last week of training together. Next Saturday is the big day. When they arrived to train, they noticed they had both run 60 laps during the week.

a. Write the equation for Mariah given that they run at the same speed that they have every week.
b. Write Fernando's equation as a transformation of Mariah's equation.

What conjectures can you make about the general statement: "*g(x) = f(x)+k*" when it comes to linear functions?

Name: Connecting Algebra and Geometry | 7.5

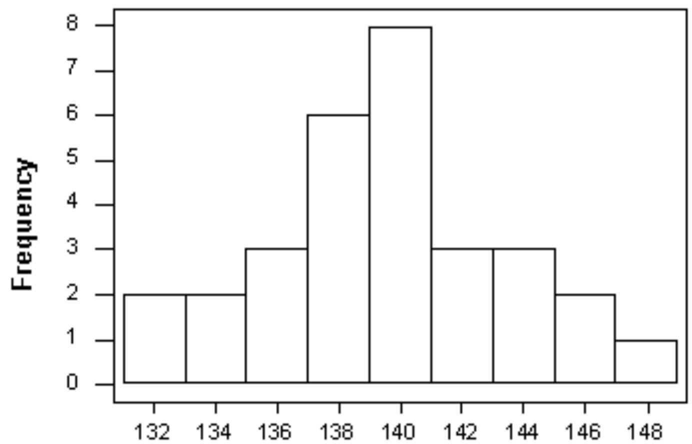
http://www.flickr.com/photos/pdgoodman

Ready, Set, Go!

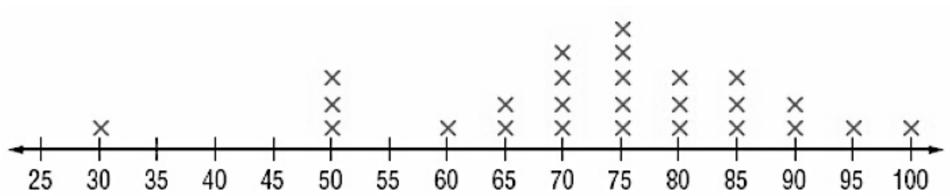
Ready

Topic: Identifying spread.

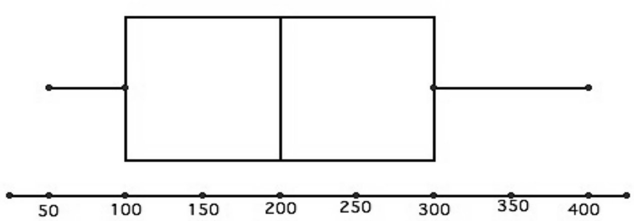
1. Describe the spread in the histogram below.



2. Describe the spread in the line plot below.



3. Describe the spread in the box and whisker plot.



Name: Connecting Algebra and Geometry | 7.5

Set

You are given information about $f(x)$ and $g(x)$. Rewrite $g(x)$ in translation form: $g(x) = f(x) + k$)

4. $f(x) = 7x + 13$
$g(x) = 7x - 5$

$g(x)$ = ____________________
Translation form

5. $f(x) = 22x - 12$
$g(x) = 22x + 213$

$g(x)$ = ____________________
Translation form

6. $f(x) = -15x + 305$
$g(x) = -15x - 11$

$g(x)$ = ____________________
Translation form

7.

x	f(x)	g(x)
3	11	26
10	46	61
25	121	136
40	196	211

$g(x)$ = ____________________
Translation form

8.

x	f(x)	g(x)
-4	5	-42
-1	-1	-48
5	-13	-60
20	-43	-90

$g(x)$ = ____________________
Translation form

9.

x	f(x)	g(x)
-10	4	-15.5
-3	7.5	-12
22	20	0.5
41	29.5	10

$g(x)$ = ____________________
Translation form

Go

Topic: Vertical and horizontal translations

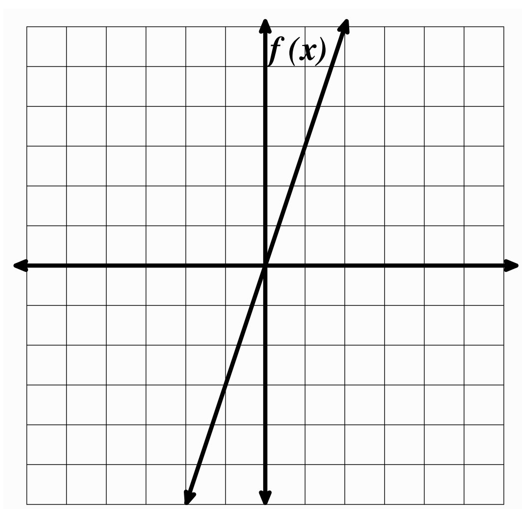
10. Use the graph of $f(x) = 3x$ to answer the following questions.

a. Sketch the graph of $g(x) = 3x - 2$ on the same grid.

b. Sketch the graph of $h(x) = 3(x - 2)$.

c. Describe how $f(x)$, $g(x)$, and $h(x)$ are different and how they are the same.

d. Explain in what way the parentheses affect the graph. Why do you think this is so?



7.6 Shifting Functions

A Practice Understanding Task

http://www.flickr.com/photodean/21202433@N08

Part I: Transformation of an exponential function.

The table below represents the property value of Rebekah's house over a period of four years.

Rebekah's Home

Time (years)	Property Value	Common Ratio
0	150,000	
1	159,000	
2	168,540	
3	178,652	
4	189,372	

Rebekah says the function $P(t) = 150{,}000(1.06)^t$ represents the value of her home.

1. Explain how this function is correct by using the table to show the initial value and the common ratio between terms.

Jeremy lives close to Rebekah and says that his house is always worth $20,000 more than Rebekah's house. Jeremy created the following table of values to represent the property value of his home.

Jeremy's Home

Time (years)	Property Value	Relationship to Rebekah's table
0	170,000	
1	179,000	
2	188,540	
3	198,652	
4	209,372	

When Rebekah and Jeremy tried to write an exponential function to represent Jeremy's property value, they discovered there was not a common ratio between all of the terms.

2. Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.

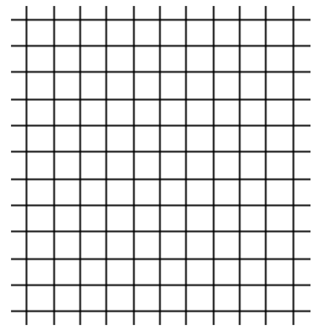
Part 2: Shifty functions.

Given the function *g(x)* and information about *f(x),*

- write the function for *f(x),*
- graph both functions on the set of axes, and
- show a table of values that compares *f(x)* and *g(x)*.

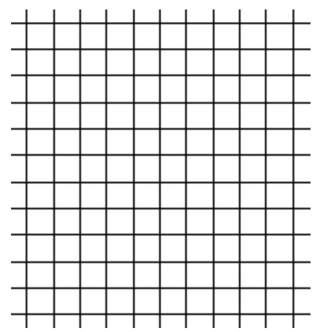
3. $If\ \ g(x) = 3(2)^x\ and\ \ f(x) = \ g(x) - 5, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



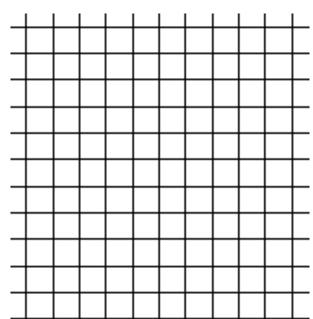
4. $If\ \ g(x) = 4(.5)^x\ and\ \ f(x) = \ g(x) + 3,\ then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



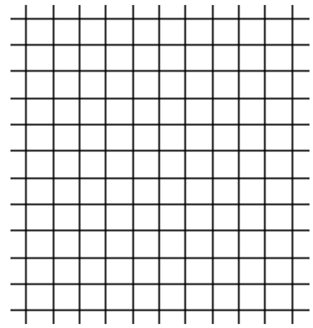
5. $If\ \ g(x) = 4x + 3\ and\ \ f(x) = \ g(x) + 7, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



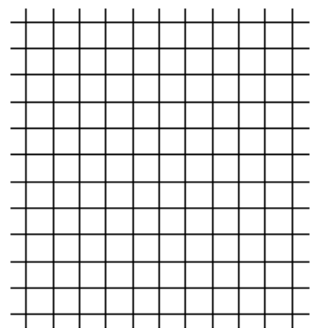
6. $If\ g(x) = 2x + 1\ and\ f(x) = g(x) - 4, then\ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



7. $If\ g(x) = -x\ and\ f(x) = g(x) + 3, then\ f(x) =$ ______________________________

x				
$f(x)$				
$g(x)$				



Part III: Communicate your understanding.

8. If *f(x) = g(x) + k,* describe the relationship between *f(x)* and *g(x).* Support your answers with tables and graphs.

Name: Connecting Algebra and Geometry | 7.6

Ready, Set, Go!

http://www.flickr.com/photodean/21202433@N08/

Ready

Topic: Finding percentages.

Mrs. Gonzalez noticed that her new chorus class had a lot more girls than boys in it. There were 32 girls and 17 boys. (Round answers to the nearest %.)

1. What percent of the class are girls?

2. What percent are boys?

3. 68% of the girls were sopranos.
 a.How many girls sang soprano?

 b. What percent of the entire chorus sang soprano?

4. Only 30% of the boys could sing bass.
 a. How many boys were in the bass section?

 b. What percent of the entire chorus sang bass?

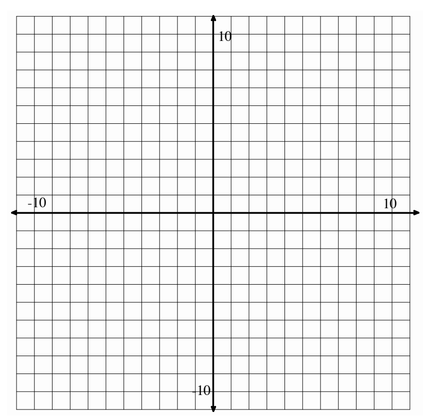
5. Compare the number of girls who sang alto to the number of boys who sang tenor. Which musical section is larger? Justify your answer.

Set

Topic: Graphing exponential equations

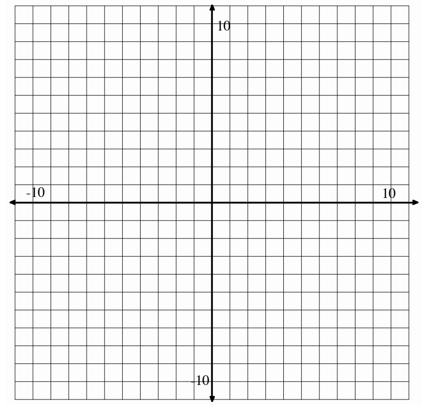
6. Think about the graphs of $y = 2^x$ and $y = 2^x - 4$.
 a. Predict what you think is the same and what is different.

 b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed when you subtracted 4. Identify in what way it changed.



7. Think about the graphs of $y = 2^x$ and $y = 2^{(x-4)}$

a. Predict what you think is the same and what is different.



b. Use your calculator to graph both equations on the same grid.
Explain what stayed the same and what changed.
Identify in what way it changed.

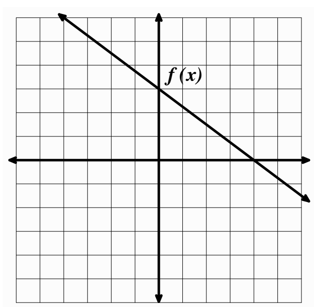
Go

Topic: Vertical translations of linear equations

The graph of $f(x)$ and the translation form equation of $g(x)$ are given. Graph $g(x)$ on the same grid and write the slope-intercept equation of $f(x)$ and $g(x)$.

8. $g(x) = f(x) - 5$

a.

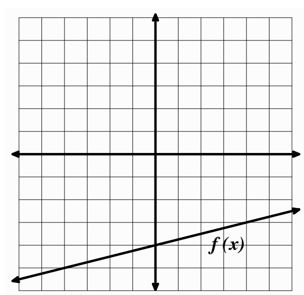


b. $f(x) =$ ________________

c. $g(x) =$ __________________
Slope-Intercept form

9. $g(x) = f(x) + 4$

a.

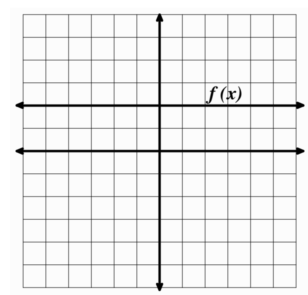


b. $f(x) =$ ________________

c. $g(x) =$ __________________
Slope-Intercept form

10. $g(x) = f(x) - 6$

a.



b. $f(x) =$ ________________

c. $g(x) =$ __________________
Slope-Intercept form

Need Help? Check out these related videos:

http://www.khanacademy.org/math/arithmetic/percents/v/identifying-percent-amount-and-base

Secondary One Mathematics: An Integrated Approach
Module 8
Modeling Data

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the
Utah State Office of Education

Module 8 – Modeling Data

Classroom Task: 8.1 Texting By the Numbers- A Solidify Understanding Task
Use context to describe data distribution and compare statistical representations **(S.ID.1, S.ID.3)**
Ready, Set, Go Homework: Modeling Data 8.1

Classroom Task: 8.2 Data Distributions – A Solidify/Practice Understanding Task
Describe data distributions and compare two or more data sets **(S.ID.1, S.ID.3)**
Ready, Set, Go Homework: Modeling Data 8.2

Classroom Task: 8.3 After School Activity – A Solidify Understanding Task
Interpret two way frequency tables **(S.ID.5)**
Ready, Set, Go Homework: Modeling Data 8.3

Classroom Task: 8.4 Relative Frequency– A Solidify Understanding Task
Use context to interpret and write conditional statements using relative frequency tables **(S.ID.5)**
Ready, Set, Go Homework: Modeling Data 8.4

Classroom Task: 8.5 Connect the Dots– A Develop Understanding Task
Develop an understanding of the value of the correlation co-efficient **(S.ID.8)**
Ready, Set, Go Homework: Modeling Data 8.5

Classroom Task: 8.6 Making More $ – A Solidify Understanding Task
Estimate correlation and lines of best fit. Compare to the calculated results of linear regressions and correlation the co-efficient **(S.ID.7, S.ID.8)**
Ready, Set, Go Homework: Modeling Data 8.6

Classroom Task: 8.7 Getting Schooled – A Solidify Understanding Task
Use linear models of data and interpret the slope and intercept of regression lines with various units **(S.ID.6, S.ID.7, S.ID.8)**
Ready, Set, Go Homework: Modeling Data 8.7

Classroom Task: 8.8 Rocking the Residuals – A Develop Understanding Task
Use residual plots to analyze the strength of a linear model for data **(S.ID.6)**
Ready, Set, Go Homework: Modeling Data 8.8

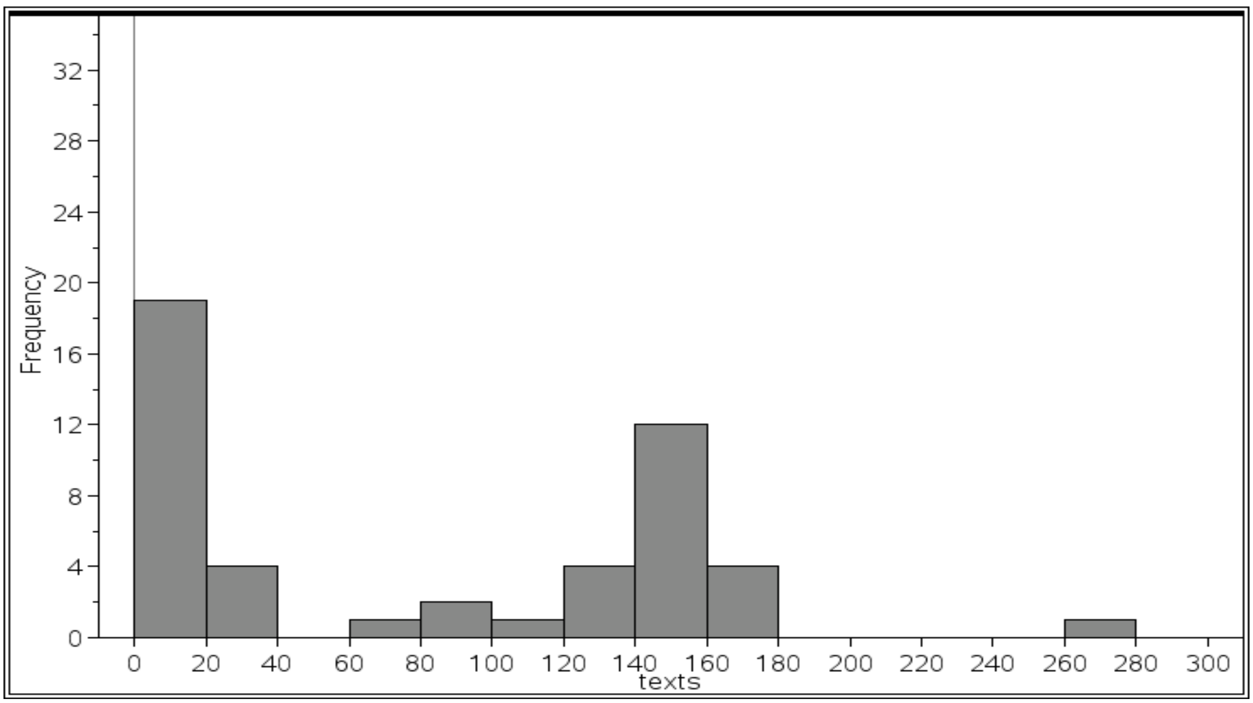
http://www.flickr.com/photos/garryknight/740038165

8.1 Texting by the Numbers

A Solidify Understanding Task

Technology changes quickly and yet has a large impact on our lives. Recently, Rachel was busy chatting with her friends via text message when her mom was trying to also have a conversation with her. Afterward, they had a discussion about what is an appropriate amount of texts to send each day. Since they could not agree, they decided to collect data on the number of texts people send on any given day. They each asked 24 of their friends the following question: "What is the average number of texts you SEND each day?" The data and histogram representing all 48 responses are below:

{150, 5.5, 6, 5, 3, 10, 150, 15, 20, 15, 6, 5, 3, 6, 0, 5, 12, 25, 16, 35, 5, 2, 13, 5, 130, 145, 155, 150, 162, 80, 140, 150, 165, 138, 175, 275, 85, 137, 110, 143, 138, 142, 164, 70, 150, 36, 150, 150}

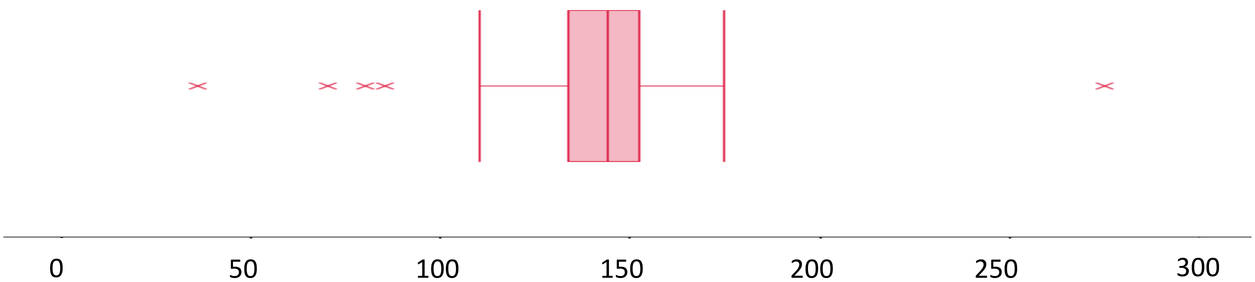


Part I: What information can you conclude based on the histogram above?

Represent the same data by creating a box plot below.

Describe the pros and cons of each representation (histogram and box plot). In other words, what information does each representation highlight? What information does each representation hide or obscure?

Part II: Prior to talking about the data with her mom, Rachel had created a box plot using her own data she collected and it looked quite different than when they combined their data.



Average number of texts sent each day

Describe the data Rachel collected from her friends. What does this information tell you?

What do you think is a reasonable number of texts Rachel can send per day?

Rachel wants to continue sending her normal number of texts (average of 100 per day) and her mom would like her to decrease this by half. Present an argument for each side, using mathematics to justify each person's request.

Name: Modeling Data | 8.1

Ready, Set, Go!

http://www.flickr.com/photos/garryknight/740038

Ready

Topic: Measures of central tendency

Sam's test scores for the term were 60, 89, 83, 99, 95, and 60.

1. Suppose that Sam's teacher decided to base the term grade on the mean.
   a. What grade would Sam receive?

   b. Do you think this is a fair grade? Explain your reasoning.

2. Suppose that Sam's teacher decided to base the term grade on his median score.
   a. What grade would Sam receive?

   b. Do you think this is a fair grade? Explain your reasoning.

3. Suppose that Sam's teacher decided to base the term grade on the mode score.
   a. What grade would Sam receive?

   b. Do you think this is a fair grade? Explain your reasoning.

4. Aiden's test scores for the same term were 30, 70, 90, 90, 91, and 99. Which measure of central tendency would Aiden want his teacher to base his grade on? Justify your thinking.

5. Most teachers base grades on the mean. Do you think this is a fair way to assign grades? Why or why not?

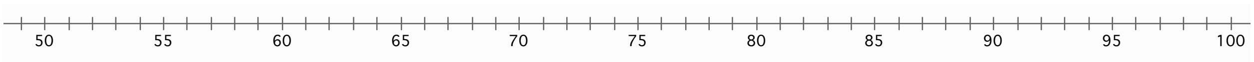
Name: Modeling Data | 8.1

Set

Topic: Examining data distributions in a box-and-whisker plot

60, 64, 68, 68, 72, 76, 76, 80, 80, 80, 84, 84, 84, 84, 88, 88, 88, 92, 92, 96, 96, 96, 96, 96, 96, 96, 100, 100

6. Make a box-and-whisker plot for the following test scores.



7a. How much of the data is represented by the box?
b. How much is represented by each whisker?

8. What does the graph tell you about student success on the test?

Go

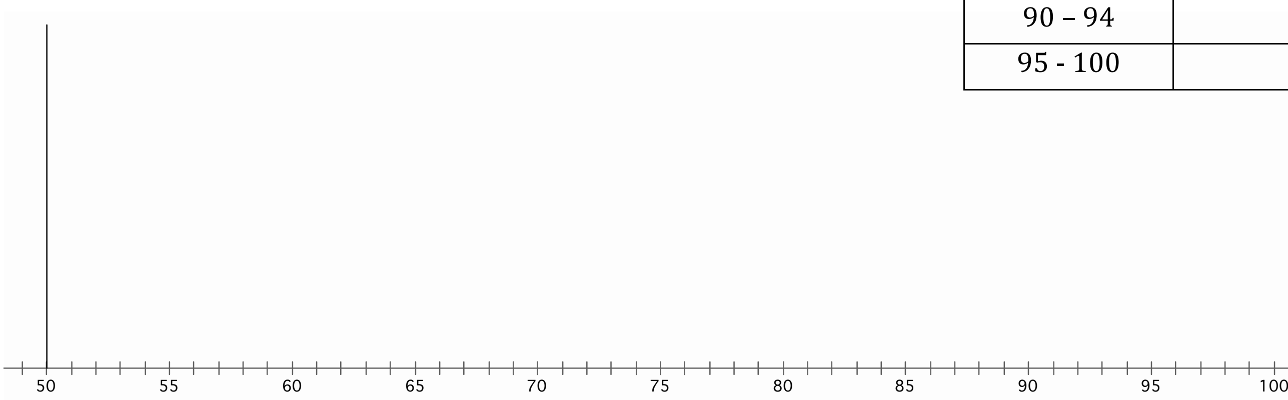
Topic: Drawing histograms.

Use the data from the SET section to answer the following questions

9. Make a frequency table with intervals. Use an interval of 5.

10. Make a histogram of the data using your intervals of 5.

Score	Frequency
60 – 64	
65 – 69	
70 – 74	
75 – 79	
80 – 84	
85 – 89	
90 – 94	
95 - 100	



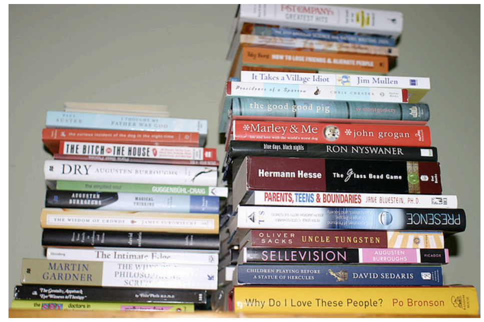
Need Help? Check out these related videos:

http://www.khanacademy.org/math/statistics/e/mean_median_and_mode

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/box-and-whisker-plot

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/histograms

8.2 Data Distribution
A Practice Understanding Task



A lot of information can be obtained from looking at data plots and their distributions. It is important when describing data that we use context to communicate the **shape, center,** and **spread.**

Shape and spread:

- Modes: uniform (evenly spread- no obvious mode), unimodal (one main peak), bimodal (two main peaks), or multimodal (multi locations where the data is relatively higher than others).
- Skewed distribution: when most data is to one side leaving the other with a 'tail'. Data is skewed to side of tail. (if tail is on left side of data, then it is skewed left).
- Outliers: values that stand away from body of distribution.
- Normal distribution: curve is unimodal and symmetric.
- Variability: values that are close together have low variability; values that are spread apart have high variability.

Center:

- Analyze the data and see if one value can be used to describe the data set. Normal distributions make this easy. If not a normal distribution, determine if there is a 'center' value that best describes the data. Bimodal or multimodal data may not have a center that would provide useful data.

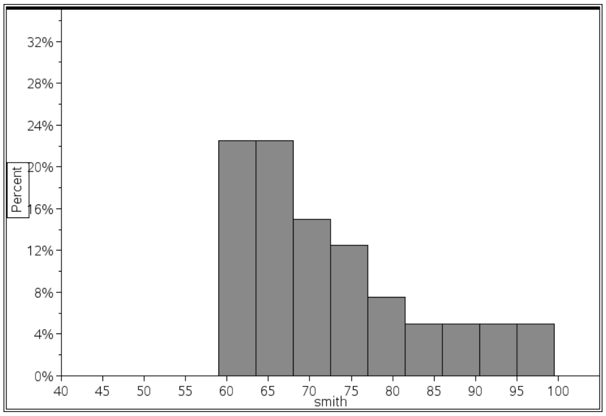
Part I: Use the *Texting By the Numbers* task to describe the shape, center, and spread.

1. Describe the distribution of the histogram that represents the data collected from Rachel and her mom (part I of *Texting by the Numbers Task*).
2. Describe the distribution of the box plot that represents the data collected from Rachel only (part II of *Texting by the Numbers Task*).

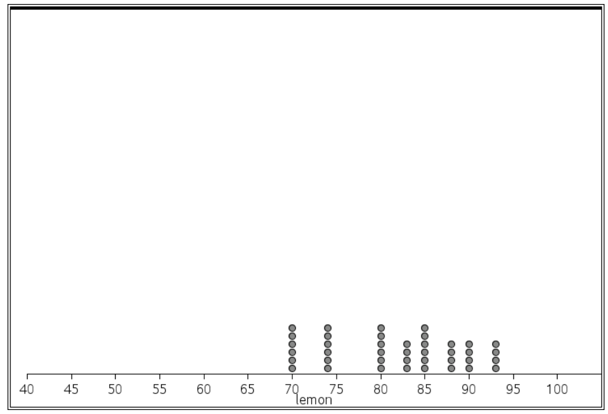
Part II: The following represents test scores from six different classes.

1. Describe the data distribution of each.
2. Compare data distributions between Adams and Smith.
3. Compare data distributions between Smith and Lemon.
4. Compare data distributions between Croft and Hurlea.
5. Compare data distributions between Jones, Adams, and Hurlea.

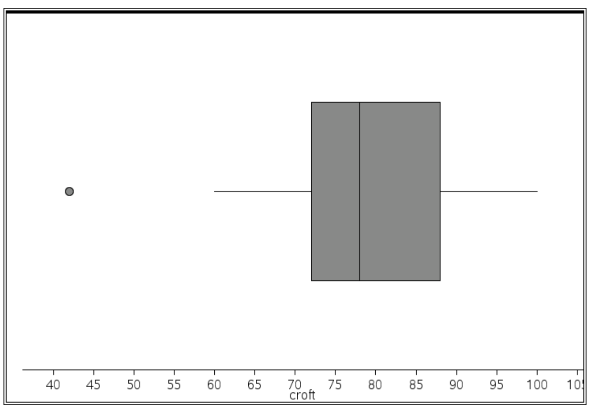
Data set I: Smith's class



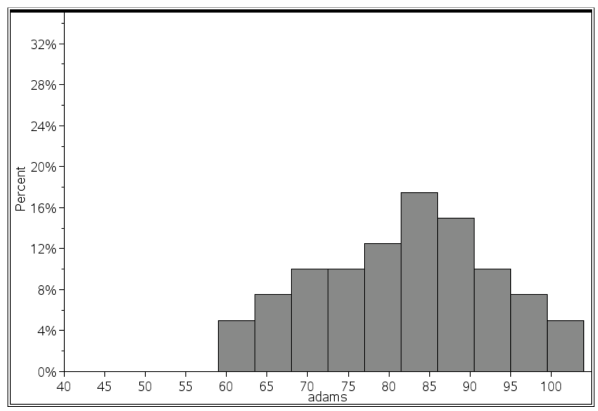
Data set II: Lemon's class



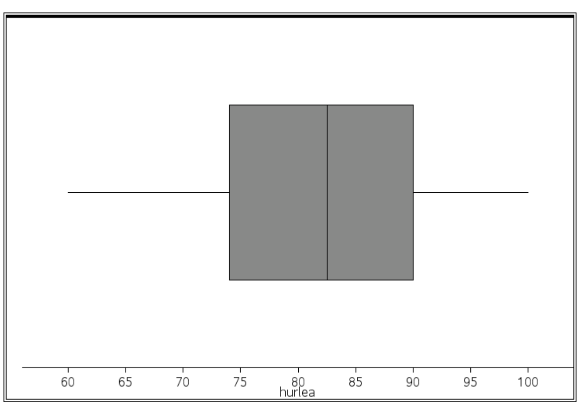
Data set III: Croft's Class



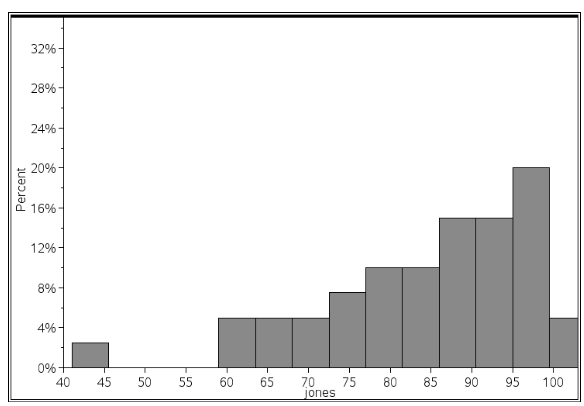
Data set IV: Adam's Class



Data set V: Hurlea's class

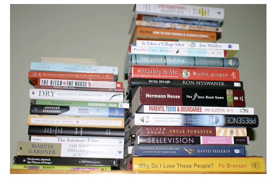


Data set VI: Jones' class



Name: Modeling Data 8.2

Ready, Set, Go!



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Ready

Topic: Sequences in statistics

In problems 1 – 4 you are to select the best answer based on the given data. Below your chosen answer is a confidence scale. Circle the statement that best describes your confidence in the correctness of the answer you chose.

1. Data: 1, 2, 4, 8, 16, 32, _______? The next number in the list will be: _______

a. larger than 32 b. positive c. exactly 64 d. about 63.89

I am certain I am correct. **I am a little unsure.** **I had no idea so I guessed.**

What about the data made you feel the way you did about the answer you marked?

2. Data: 47, -13, -8, 9, -23, 14, ________? The next number in the list will be: _______

a. positive b. negative c. less than 100 d. less than -100

I am certain I am correct. **I am a little unsure.** **I had no idea so I guessed.**

What about the data made you feel the way you did about the answer you marked?

3. Data: -10, ¾, 38, -10, ½, -81, -10, ¼, 93, -10, ______? The next number in the list will be: _____

a. more than 93 b. negative c. a fraction d. a whole number

I am certain I am correct. **I am a little unsure.** **I had no idea so I guessed.**

What about the data made you feel the way you did about the answer you marked?

4. Data: 50, -43, 36, -29, 22, -15, ______? The next number in the list will be: ______

a. odd b. less than 9 c. two-digits d. greater than -15

I am certain I am correct. **I am a little unsure.** **I had no idea so I guessed.**

What about the data made you feel the way you did about the answer you marked?

Name: Modeling Data | 8.2

Set

Topic: Drawing histograms.

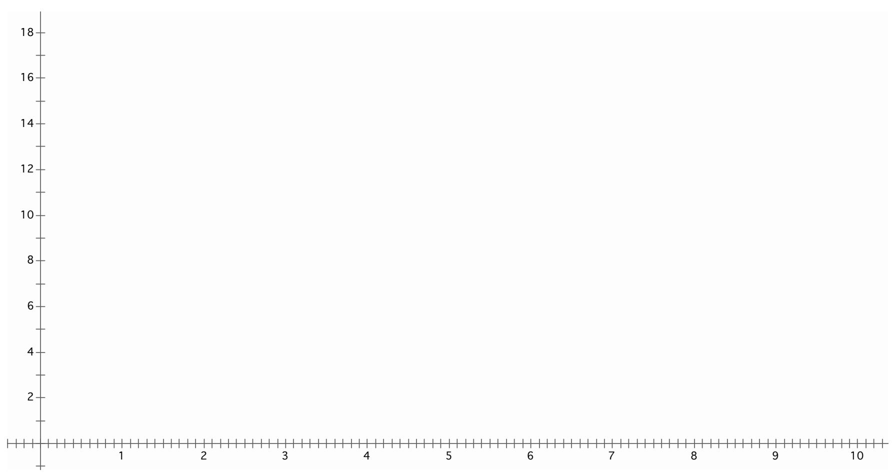
Mr. Austin gave a ten-point quiz to his 9th grade math classes. A total of 50 students took the quiz. Mr. Austin scored the quizzes and listed the scores alphabetically as follows.

1st Period Math	2nd Period Math	3rd Period Math
6, 4, 5, 7, 5, 9, 5, 4, 6, 6, 8, 5, 7, 5, 8, 1, 8, 7, 10, 9	4, 5, 8, 6, 8, 9, 5, 8, 5, 1, 5, 5, 7, 5, 7	9, 8, 10, 5, 9, 7, 8, 9, 8, 5, 8, 10, 8, 8, 5

5. Use the ALL of the quiz data to make a frequency table with intervals. Use an interval of 2.

Score	Frequency
0 - 1	
2 – 3	
4 – 5	
6 – 7	
8 - 10	

6. Use your frequency table to make a histogram for the data



7. Describe the data distribution of the histogram you created. Include words such as: *mode, skewed, outlier, normal, symmetric, center,* and *spread,* if they apply.

Name: Modeling Data 8.2

Go

8. What percent of 97 is 11?

9. What percent of 88 is 132?

10. What percent of 84 is 9?

11. What percent of 88.6 is 70?

12. What is 270% of 60?

13. What is 84% of 25?

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/histograms

http://www.khanacademy.org/math/statistics/v/ck12-org-normal-distribution-problems--qualitative-sense-of-normal-distributions

http://stattrek.com/statistics/two-way-table.aspx

8.3 After School Activity

A Develop Understanding Task

Part I

Rashid is in charge of determining the upcoming after school activity. To determine the type of activity, Rashid asked several students whether they prefer to have a dance or play a game of soccer. As Rashid collected preferences, he organized the data in the following two-way frequency table:

	Girls	Boys	Total
Soccer	14	40	54
Dance	46	6	52
Total	60	46	106

Rashid is feeling unsure of the activity he should choose based on the data he has collected and is asking for help. To better understand how the data is displayed, it is useful to know that the outer numbers, located in the margins of the table, represent the total frequency for each row or column of corresponding values and are called *marginal frequencies* . Values that are part of the 'inner' body of the table are created by the intersection of information from the column and the row and they are called the *joint frequencies.* Using the data in the table, construct a viable argument and explain to Rashid which after school event he should choose.

Part II: Two way frequency tables allow us to organize categorical data in order to draw conclusions. For each set of data below, create a frequency table. When each frequency table is complete, write three sentences about observations of the data, including any trends or associations in the data.

Data set 1: There are 45 total students who like to read books. Of those students, 12 of them like non-fiction and the rest like fiction. Four girls like non-fiction. Twenty boys like fiction.

	Fiction	Nonfiction	Total
Boys			
Girls			
Total			

Observation 1:

Observation 2:

Observation 3:

Data set 2: 35 seventh graders and 41 eighth graders completed a survey about the amount of time they spend on homework each night. 50 students said they spent more than an hour. 12 eighth graders said they spend less than an hour each night.

			Total
More than one hour			
Less than one hour			
Total			

Observation 1:

Observation 2:

Observation 3:

Name: Modeling Data | 8.3

Ready, Set, Go!

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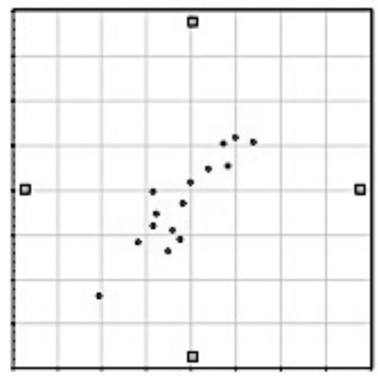
Ready

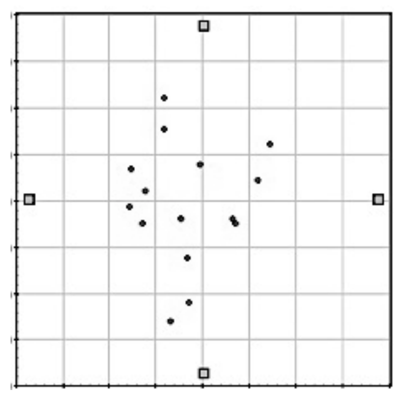
Topic: Interpreting data from a scatter plot

1. The scatter plot compares shoe size and height in adult males. Based on the graph, do you think there is a relationship between a man's shoe size and his height?



Explain your answer.

2. The scatter plot compares left-handedness to birth weight. Based on the graph, do you think being left-handed is related to a person's birth weight?



Explain your answer.

Set

Topic: Two-way frequency tables.

Here is the data from Mr. Austin's ten-point quiz. Students needed to score a 6 or better to pass the quiz.

1st Period Math	2nd Period Math	3rd Period Math
6, 4, 3, 7, 5, 9, 5, 4, 6, 6, 8, 5, 7, 3, 6, 2, 8, 7, 10, 9	3, 3, 8, 6, 6, 9, 5, 8, 5, 3, 5, 5, 7, 5, 7	9, 8, 10, 5, 9, 7, 8, 9, 8, 3, 8, 10, 8, 7, 5

3. Make a two-way frequency table showing how many students passed the quiz and how many failed in each class.

	1st Period	2nd Period	3rd Period	Total
Passed				
Failed				
Total				

4. Use a colored pencil to lightly shade the cells containing the *joint frequency* numbers in the table. The un-shaded numbers are the *marginal frequencies.* (Use these terms to answer the following questions.)

5. If Mr. Austin wanted to see how many students in all 3 classes combined passed the quiz, where would he look?

6. If Mr. Austin wanted to write a ratio of the number of passing students compared to the number of failing students for each class, where would he find the numbers he would need to do this?

7. Make a two-way frequency table that gives the *relative frequencies* of the quiz scores for each class.

	1st Period	2nd Period	3rd Period	Total
Passed				
Failed				
Total				

Go

8. Sophie surveyed all of the 6th grade students at Reagan Elementary School to find out which TV Network was their favorite. She thought that it would be important to know whether the respondent was a boy or a girl so she recorded her information this way.

Animal Planet	Cartoon Network	Disney	Nickelodeon
GGBBBB BGBBBGBBB GGBB BBBBBB	BBBBBBB BBGGBBBG BGBGGGBGG	GGGGGGBBBBBB GBGBGG BBBGGBGG GGGBBBGGGGGB	BBBBGGGGGGGG GGGGGGBB GGGGBGGGGGGGGGBBBB BGGGGGGGG

Sophie planned to use her data to answer the following questions:

I. Are there more girls or boys in the 6th grade?

II. Which network was the boys' favorite?

III. Was there a network that was favored by more than 50% of one gender?

But when she looked at her chart, she realized that the data wasn't telling her what she wanted to know. Her teacher suggested that her data would be easier to analyze if she could organize it into a two-way frequency chart. Help Sophie out by putting the frequencies into the correct cells.

Favorite TV Networks	*Girls*	*Boys*	*Totals*
Animal Planet			
Cartoon Network			
Disney			
Nickelodeon			
Totals			

Now that Sophie has her data organized, use the two-way frequency chart to answer her 3 questions.

a. Are there more girls or boys in the 6th grade?

b. Which network was the boys' favorite?

c. Was there a network that was favored by more than 50% of one gender

Need Help? Check out these related links:

http://stattrek.com/statistics/two-way-table.aspx

8.4 Relative Frequency
A Solidify Understanding Task

Rachel is thinking about the data she and her mom collected for the average number of texts a person sends each day and started thinking that perhaps a two-way table of the data they collected would help convince her mom that she does not send an excessive amount of texts for a teenager. The table separates each data point by age (teenager and adult) and by the average number of texts sent (more than 100 per day or less than 100 per day).

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
Teenager	20	4	24
Adult	2	22	24
Total	22	26	48

Write two observation statements of this two way table.

To further provide evidence, Rachel decided to do some research. She found that only 43% of people with phones send over 100 texts per day. She was disappointed that the data did not support her case and confused because it did not seem to match what she found in her survey. What questions do these statistic raise for you? What data should Rachel look for to support her case?

After looking more closely at the data, Rachel found other percentages within the same data that seemed more accurate with the data she collected from her teenage friends. How might Rachel use the data in the two way table to find percentages that would be useful for her case?

Part II: Once Rachel realized there are a lot of ways to look at a set of data in a two way table, she was self-motivated to learn about *relative frequency tables* and conditional frequencies. When the data is written as a percent, this is called a *relative frequency table*. In this situation, the 'inner' values represent a percent and are called conditional frequencies. The conditional values in a *relative frequency table* can be calculated as percentages of one of the following:

- the whole table (relative frequency of table)
- the rows (relative frequency of rows)
- the columns (relative frequency of column)

Since Rachel wants to emphasize that a person's age makes a difference in the number of texts sent, the first thing she decided to do is focus on the ROW of values so she could write conditional statements about the number of texts a person is likely to send based on their age. This is called a *relative frequency of row* table. Fill in the percentage of teenagers for each of the conditional frequencies in the highlighted row below:

Row

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
Teenager	20	4	24
% of teenagers	___ %	__%	100%
% of Adults	**2** 8%	**22** 92%	**24** 100%
% of People	**22** 46%	**26** 54%	**48** 100%

Since the PERCENTAGES created focus on ROW values, all conditional observations are specific to the information in the row. Complete the following sentence for the *relative frequency of row:*

Of all teenagers in the survey, _______ % average more than 100 texts per day.

Write another statement based on the *relative frequency of row:*

Below is the *relative frequency of column* using the same data. This time, all of the percentages are calculated using the data in the column.

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
Teenagers	**20** 91%	**4** 15%	**24** 50%
Adults	**2** 9%	**22** 85%	**24** 50%
Total	**22** 100%	**26** 100%	**48** 100%

Write two conditional statements using the *relative frequency of column*.

This data represents the *relative frequency of whole table*:

	Average is more than 100 texts sent per day	Average is less than 100 texts sent per day	Total
% of Teenagers	**20** 42%	**4** 8%	**24** 50%
% of Adults	**2** 4%	**22** 46%	**24** 50%
% of Total	**22** 46%	**26** 54%	**48** 100%

Create two conditional distribution statements for the *relative frequency of whole table*

How do *relative frequency tables* impact the way you look at data in two way tables?

Name: Modeling Data | 8.4

Ready, Set, Go!

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Ready

Topic: Linear functions and relationships

Write the explicit linear function for the given information below.

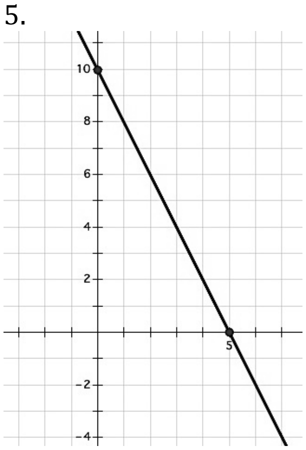
1. (3, 7) (5, 13)

2. Mike earns $11.50 an hour

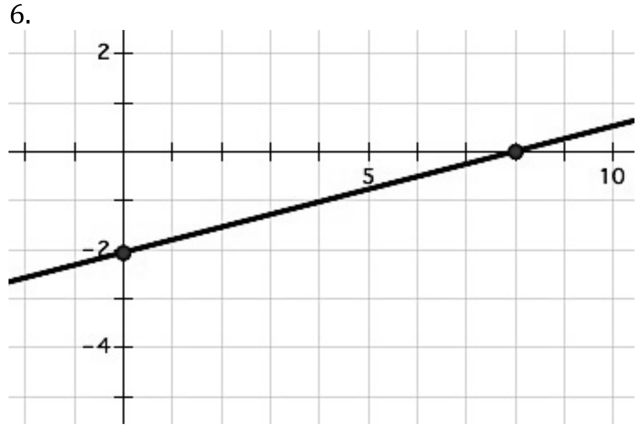
3. (-5, -2) (1, 10)

4. (-2, 12) (6, 8)

5.



6.



Name: Modeling Data | 8.4

Set

Topic: Relative Frequency Tables

For each two-way table below, create the indicated relative frequency table and also provide two observations with regard to the data.

7. This table represents survey results from a sample of students regarding mode of transportation to and from school.

	Walk	Bike	Car Pool	Bus	Total
Boys	37	47	27	122	233
Girls	38	22	53	79	192
Total	75	69	80	201	425

Create the *relative frequency of row table*. Then provide two observation statements.

	Walk	Bike	Car Pool	Bus	Total
Boys					
Girls					
Total	100%	100%	100%	100%	100%

8. The two-way table contains survey data regarding family size and pet ownership.

	No Pets	Own one Pet	More than one pet	Total
Families of 4 or less	35	52	85	172
Families of 5 or more	15	18	10	43
Total	50	70	95	215

Create the *relative frequency of column table*. Then provide two observation statements.

	No Pets	Own one Pet	More than one pet	Total
Families of 4 or less				100%
Families of 5 or more				100%
Total				100%

9. The two-way table below contains survey data about boys and girls shoes.

	Athletic shoes	Boots	Dress Shoe	Total
Girls	21	35	60	116
Boys	50	16	10	76
Total	71	51	70	192

Create the *relative frequency of whole table*. Then provide two observation statements.

	Athletic shoes	Boots	Dress Shoe	Total
Girls				
Boys				
Total				100%

Go

Topic: One variable statistical measures and comparisons

For each set of data determine the mean, median, mode and range. Then create either a box-and-whisker plot or a histogram.

10. 23, 24, 25, 20, 25, 29, 24, 25, 30

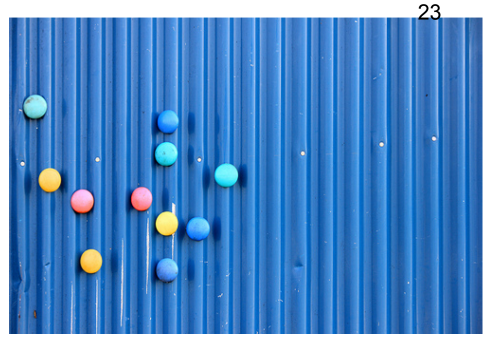
11. 20, 24, 10, 35, 25, 29, 24, 25, 33

12. How do the data sets in problems 10 and 11 compare to one another?

13. 2, 3, 4, 5, 3, 4, 7, 4, 4

14. 1, 1, 3, 5, 5, 10, 5, 1, 14

15. How do the data sets in problems 13 and 14 compare to one another?



8.5 Connect the Dots

A Develop Understanding Task

1. For each set of data:

- Graph on a scatter plot.
- Use technology (graphing calculator or computer) to calculate the correlation coefficient.

Set A

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
1	1.5	2.5	1.9	2.8	3.2	4.5	3.7	1.7	4.8	2.7	2.3

Set B

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
1	1.5	2.5	1.9	2.8	3.2	4.5	3.7	4	4.8	5	4.6

Set C

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
4.7	4.9	4.2	3.9	3.5	3.2	3.1	2.6	3.2	2.1	1.3	0.8

Set D

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
4.7	4.9	3.6	3.9	2.1	4.5	3.1	1.7	3.7	2.1	1.3	1.8

Set E

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4
4.7	4	4.2	3.9	2.8	3.2	4.5	3.7	3.2	4.8	5	4.4

Set F

2	2.3	3.3	3.7	4.2	4.6	4.5	5
1.8	2.22	3.62	4.18	4.88	5.44	5.3	6

Set G

2	2.3	3.3	3.7	4.2	4.6	4.5	5
4.4	4.01	2.71	2.19	1.54	1.02	1.15	0.5

2. Put the scatter plots in order based upon the correlation coefficients.

3. Compare each scatter plot with its correlation coefficient. What patterns do you see?

4. Use the data in Set A as a starting point. Keeping the same *x*-values, modify the y-values to obtain a correlation coefficient as close to 0.75 as you can.

Record your data here.

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4

What did you have to do with the data to get a greater correlation coefficient?

5. This time, again start with the data in Set A. Keep the same x-values, but this time, modify the y values to obtain a correlation coefficient as close to 0.25 as you can.

Record your data here.

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4

What did you have to do with the data to get a correlation coefficient that is closer to 0?

6. One more time: start with the data in Set A. Keep the same x-values, modify the y-values to obtain a correlation coefficient as close to -0.5 as you can.

Record your data here.

2	2.3	3.3	3.7	4.2	4.6	4.5	5	5.5	5.7	6.1	6.4

What did you have to do with the data to get a correlation coefficient that is negative?

7. What aspects of the data does the correlation coefficient appear to describe?

Name: Modeling Data | 8.5

Ready, Set, Go!

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Ready

Topic: Estimating the line of best fit

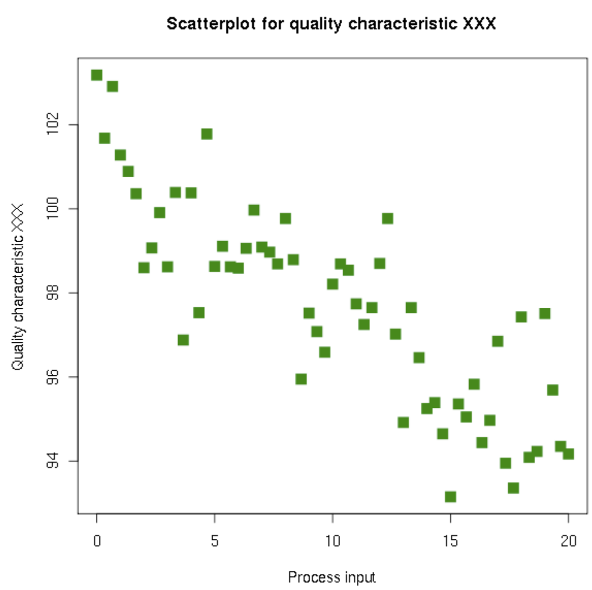
Examine the scatterplot below. Imagine that you drew a straight line through the general pattern of the points, keeping as close as possible to all points with as many points above the line as below.

1. Predict a possible y-intercept and slope for that line.

   a. y-intercept: ________________

   b. slope: ______________________

2. Sketch the line that you imagined for question #1 and write an equation for that line.



© 2012 http://en.wikipedia.org/wiki/File:Scatter_diagram_for_quality_characteristic_XXX.svg

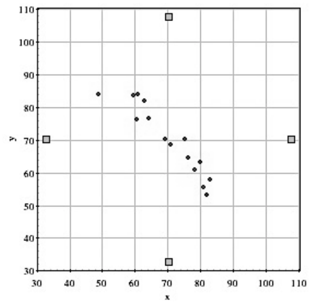
Name: Modeling Data 8.5

Set

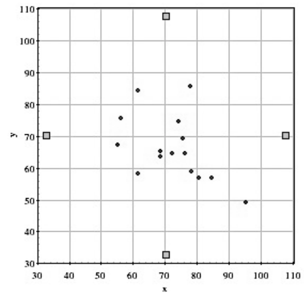
Topic: Estimating the correlation coefficient

Match the scatterplot with its correlation coefficient.

____ 3.



____ 4.



Possible

Correlation Coefficients

a. 0.05

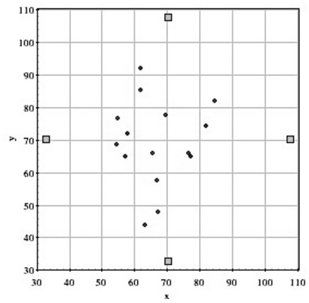
b. 0.97

c. -0.94

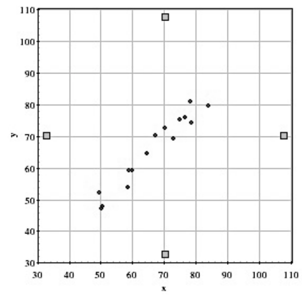
d. -0.49

e. 0.68

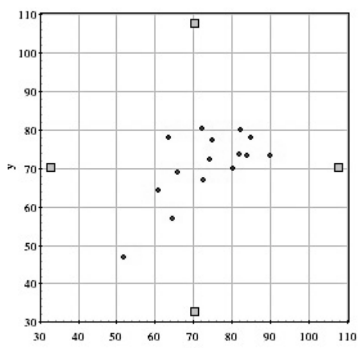
____ 5.



____ 6.



____ 7.



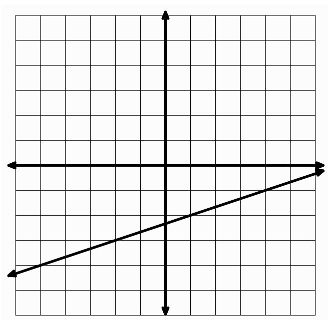
Name: Modeling Data | 8.5

Go

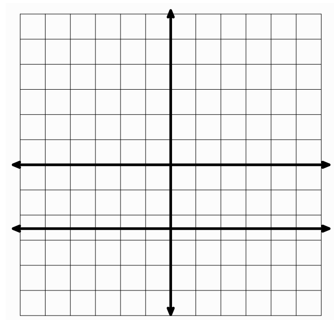
Topic: Visually comparing slopes of lines

Follow the prompt to sketch the graph of a line on the same grid with the given characteristics.

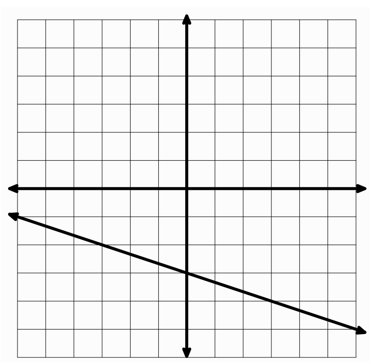
8. A larger slope



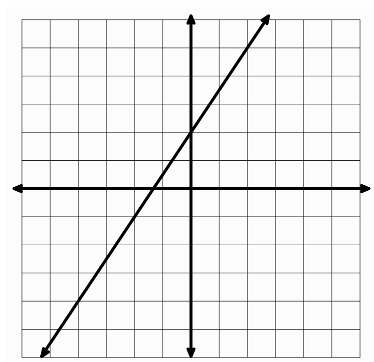
9. A smaller slope



10. A larger y-intercept and a smaller slope

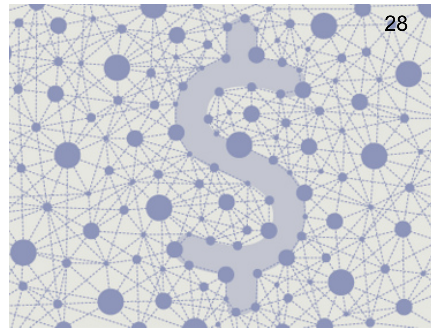


11. Slope is the negative reciprocal



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/fitting-a-line-to-data



8.6 Making More $

A Solidify Understanding Task

Each year the U.S. Census Bureau provides income statistics for the United States. In the years from 1990 to 2005, they provided the data in the tables below. (All dollar amounts have been adjusted for the rate of inflation so that they are comparable from year-to-year.)

Year	Median Income for All Men
2005	41196
2004	41464
2003	40987
2002	40595
2001	41280
2000	41996
1999	42580
1998	42240
1997	40406
1996	38894
1995	38607
1994	38215
1993	37712
1992	37528
1991	38145

Year	Median Income for All Women
2005	23970
2004	23989
2003	24065
2002	23710
2001	23564
2000	23551
1999	22977
1998	22403
1997	21759
1996	20957
1995	20253
1994	19158
1993	18751
1992	18725
1991	18649

1. Create a scatter plot of the data for men. What is your estimate of the correlation coefficient for these data?
   What is the actual correlation coefficient?
   What does it tell you about the relationship between income and years for men?

2. On a separate graph, create a scatter plot of the data for women. What is your estimate of the correlation coefficient for these data?
   What is the actual correlation coefficient?
   What does it tell you about the relationship between income and years for women?

   How does that compare to the data for men?

3. Estimate and draw a line of best fit for each set of data.
   a. Describe how you estimated the line for men. If you chose to run the line directly through any particular points, describe why you selected them.

   b. Describe how you estimated the line for women. If you chose to run the line directly through any particular points, describe why you selected them.

4. Write the equation for each of the two lines in slope intercept form.
   a. Equation for men:

   b. Equation for women:

5. Use technology to calculate a linear regression for each set of data. Add the regression lines to your scatter plots.
   a. Linear regression equation for men:

   b. Linear regression equation for women:

6. Compare your estimated line of best fit to the regression line for men. What does the slope mean in each case? (Include units in your answer.)

7. Compare your estimated line of best fit to the regression line for women. What does the y-intercept mean in each case? (Include units in your answer.)

8. Compare the regression lines for men and women. What do the lines tell us about the income of men vs women in the years from 1991-2005?

9. What do you estimate will be the median income for men and women in 2015?

10. The Census Bureau provided the following statistics for the years from 2006-2011.

Year	Median Income for All Men
2011	37653
2010	38014
2009	38588
2008	39134
2007	41033
2006	41103

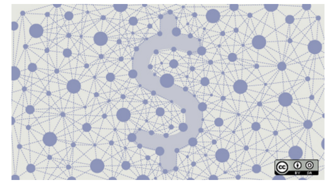
Year	Median Income for All Women
2011	23395
2010	23657
2009	24284
2008	23967
2007	25005
2006	24429

With the addition of these data, what would you now estimate the median income of men in 2015 to be? Why?

11. How appropriate is a linear model for men's and women's income from 1991-2011? Justify your answer.

Name: Modeling Data 8.6

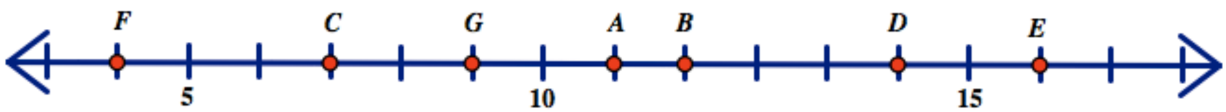
Ready, Set, Go!



Ready

Topic: Finding distance and averages.

Use the number line below to answer the questions.



1. How far away is each of the points on the number line from *point A*?
   (You need to list each point and its distance from *point A*.)

2. What is the total of all the distances from *point A* that you found in exercise number one?

3. What is the average distance that any of the given *points B through G* are from *point A*?

4. Which point on the number line is located the average distance away from *point A*?

5. Label another location on the number line that is the average distance away from *point A.*
   (Call it *point X*)

6. How far away is each of the points on the number line from *point D*?
   (You need to list each point and its distance from *point D*.)

7. What is the total of all the distances from *point D* that you found in exercise number six?

8. What is the average distance that any of the six other points are from *point D*?

9. Is there a point on the number line located the average distance away from *point D*?

10. Label another location on the number line that is the average distance away from *point D*?
    (Call it *point Y*)

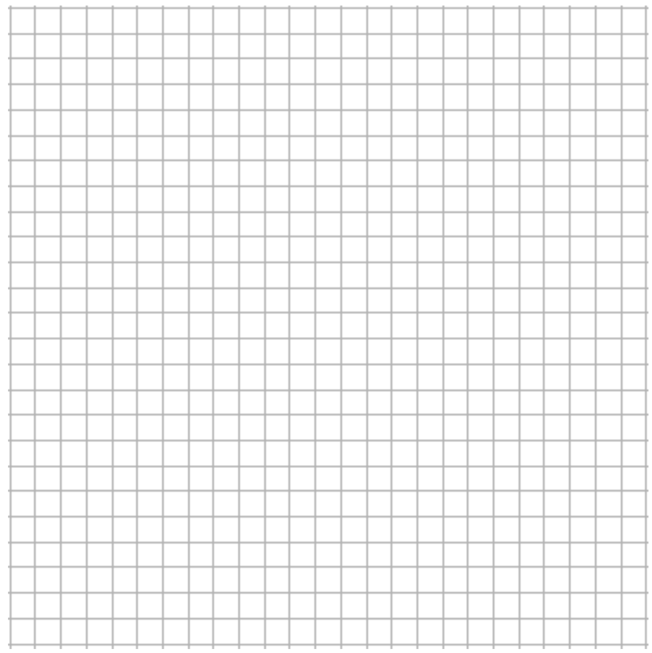
Name: Modeling Data 8.6

Set

Topic: Scatter Plots and line of best fit or trend lines.

11. Create a scatter plot for the data in the table.

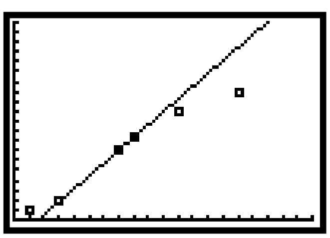
English Score	History Score
60	65
53	59
44	57
61	61
70	67

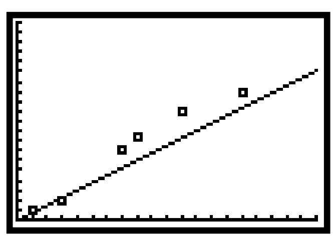


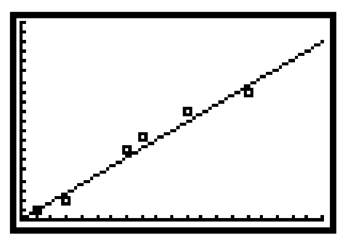
12. Do the English and History scores have a positive or negative correlation?

13. Do English and History scores have a strong or weak correlation?

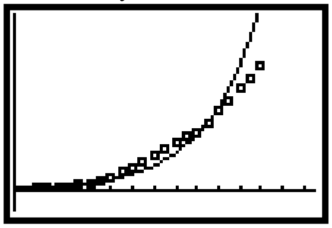
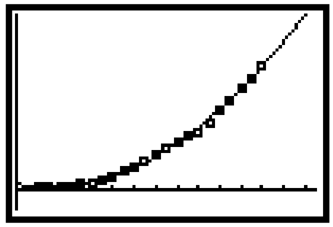
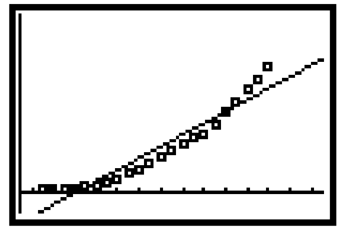
14. Which of the graphs below shows the best model for the data and will create the best predictions? Circle your choice and say why it is the best model for the data.



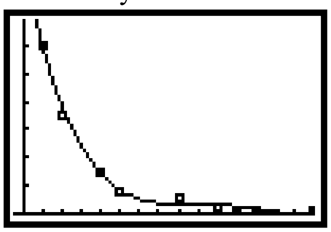
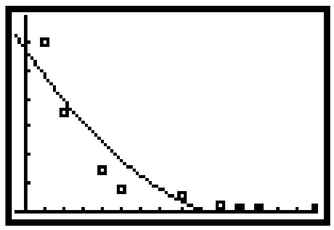
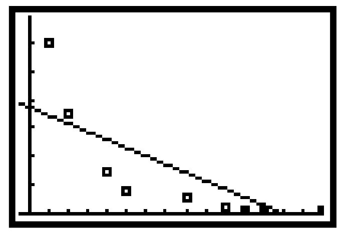




15. Which of the graphs below shows the best model for the data and will create the best predictions? Circle your choice and say why it is the best model for the data.

16. Which of the graphs below shows the best model for the data and will create the best predictions? Circle your choice and say why it is the best model for the data.

Go

Topic: Creating explicit functions for arithmetic and geometric sequences.

In each problem below an input connected output are given along with either the common difference or the common ratio. Use this information to create an explicit function for the sequence.

17. $f(2) = 7$, common difference = 3

18. $g(1) = 8$, common ratio = 2

19. $h(6) = 3$, common ratio = -3

20. $r(5) = -3$, common difference = 7

21. $g(7) = 1$, common difference = -9

22. $g(1) = 5$, common ratio = $\frac{1}{2}$

8.7 Getting Schooled
A Solidify Understanding Task

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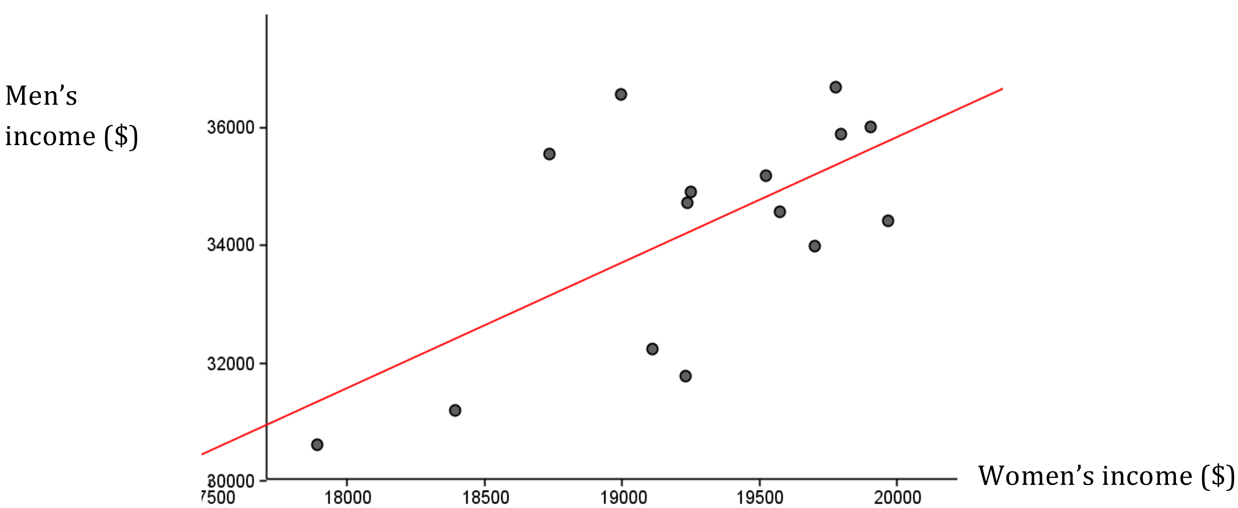
In *Getting More $,* Leo and Araceli noticed a difference in men's and women's salaries. Araceli thought that it was unfair that women were paid less than men. Leo thought that there must be some good reason for the discrepancy, so they decided to dig deeper into the Census Bureau's income data to see if they could understand more about these differences.

First, they decided to compare the income of men and women that graduated from high school (or equivalent), but did not pursue further schooling. They created the scatter plot below, with the *x* value of a point representing the average woman's salary for some year and the *y* value representing the average man's salary for the same year. For instance, the year 2011 is represented on the graph by the point (17887, 30616). You can find this point on the graph in the bottom left corner.



1. Based upon the graph, estimate the correlation coefficient.

2. Estimate the average income for men in this time period. Describe how you used the graph to find it.

3. What is the average income for women in this time period? Describe how you used the graph to find it.

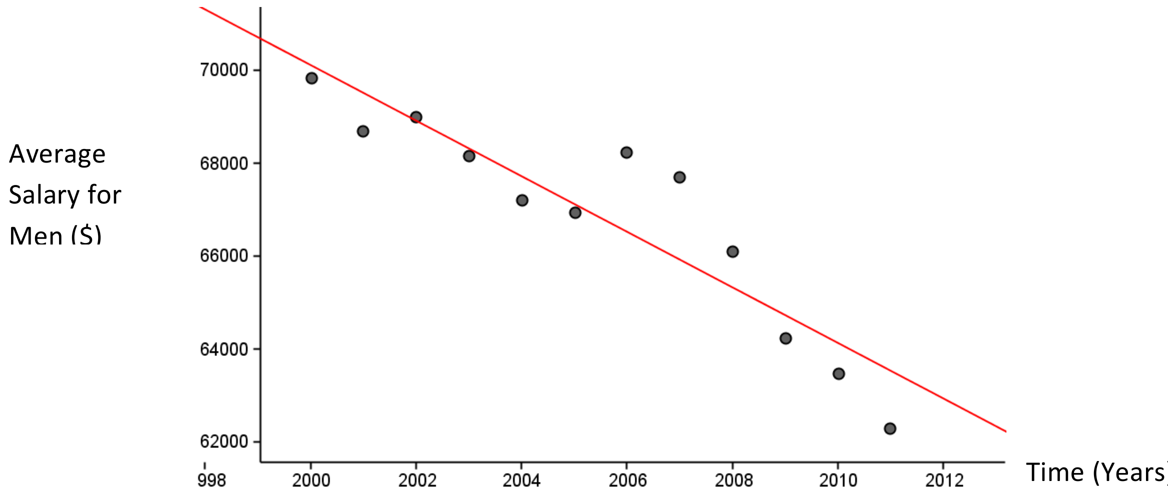
4. Leo and Araceli calculated the linear regression for these data to be $y = 2.189x - 6731.8$. What does the slope of this regression line mean about the income of men compared to women? Use precise units and language.

"Hmmmm," said Araceli, "It's just as I suspected. The whole system is unfair to women." "No, wait," said Leo, "Let's look at incomes for men and women with bachelor's degrees or more. Maybe it has something to do with levels of education."

5. Leo and Araceli started with the data for men with bachelor's degrees or more. They found the correlation coefficient for the average salary vs year from 2000-2011 was r = -.9145.

   Predict what the graph might look like and draw it here. Be sure to scale and label the axes and put 12 points on your graph.

The actual scatter plot for salaries for men with bachelor's degrees from 2000-2011 is below. How did you do?



6. Both Leo and Araceli were surprised at this graph. They calculated the regression line and got $y = -598.25x + 1266626.34$. What does this equation say about the income of men with bachelor's degrees from 2000-2011?

7. Leo wondered why the y-intercept in the equation was $1,266,626.34 and yet the graph seems to cross the y axis around $72,000. What would you tell Leo to resolve his concern?

Next, they turned their attention to the data for women with bachelor's degrees or more from 2000-2011. Here's the data:

Year	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001	2000
Income for Women ($)	41338	42409	42746	42620	44161	44007	42690	42539	42954	42871	42992	43293

Analyze these data by creating a scatter plot, interpreting the correlation coefficient and the regression line. Draw the graph and report the results of your analysis below:

Now that you have analyzed the results for women, compare the results for men and women with bachelor's degrees and more over the period from 2000-2011.

Leo believes that the difference in income between men and women may be explained by differences in education, but Araceli believes there must be other factors such as discrimination. Based on the data in this task and *Getting More $,* make a convincing case to support either Leo or Araceli.

What other data that would be useful in making your case? Explain what you would look for and why.

Ready, Set, Go!

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Ready

Topic: Finding distances and averages.

The graph below has several points and shows the line ***y=x*** use this graph to answer each question.

1. The vertical distance between *point N* and the line ***y=x*** is labeled on the graph. Find all of the vertical distances between the points and the line $y = x$.

    B:

    D:

    E:

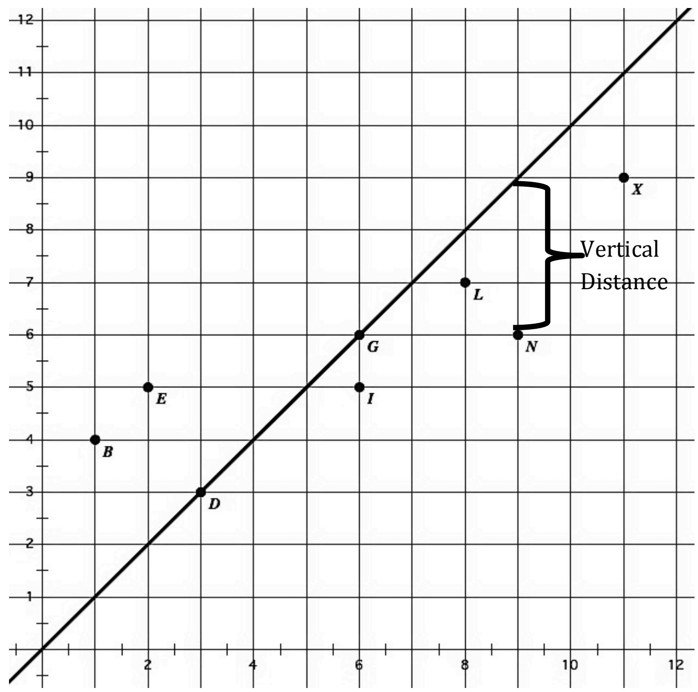
    G:

    I:

    L:

    N:

    X:



2. What is the *sum of all the distances* that the points are away from the line ***y=x***?

3. What is the *average vertical distance* that any of the points are away form the line **y=x**?

4. Is the line on the graph the line of best fit? Explain why or why not. If it is not the best then draw a line that is better fit to the data.

5. Estimate the correlation coefficient for this set of data points.
If you have a way to calculate it exactly then do so. (Using a graphing calculator or data software.)

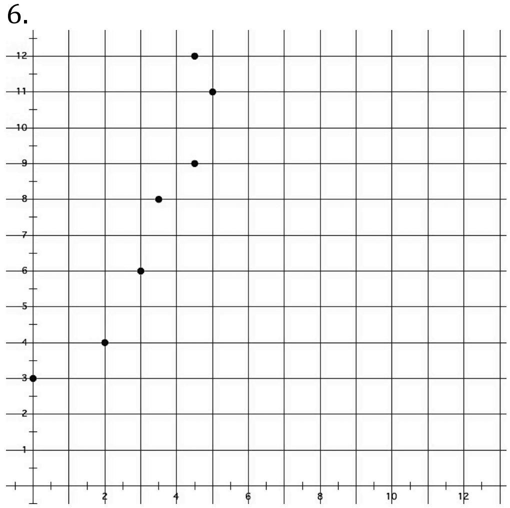
Name: Modeling Data | 8.7

Set

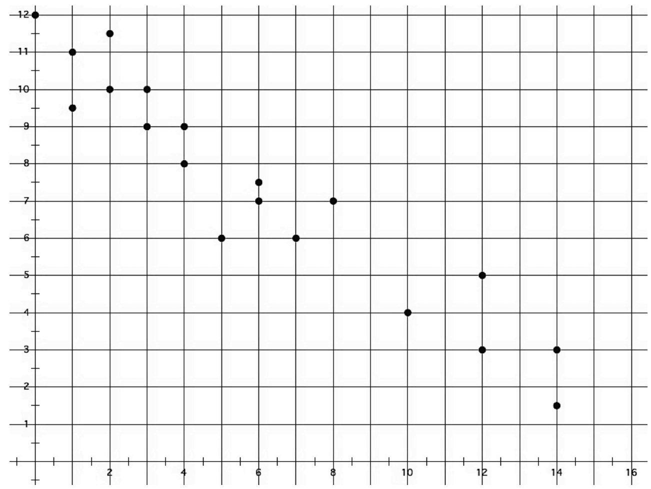
Topic: Creating and analyzing scatter plots.

Determine whether a linear or an exponential model would be best for the given scatter plots. Then sketch a model on the graph that could be used to make predictions.

6.

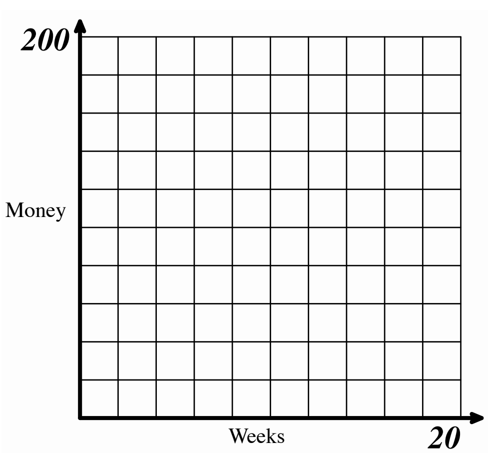


7.



8a. Use the data to make a scatter plot.

Weeks since school started	Money in savings
1	200
3	175
4	162
7	120
10	87
13	57
20	5



b. Is the correlation of the graph positive or negative? Why?

c. What would you estimate the correlation coefficient to be? Why? (If you have a calculator or software that can calculate it precisely then do so.)

d. Create a regression line and find the regression equation. What is the regression equation?

e. What does the slope of the regression equation mean in terms of the variables?

f. Most school years are 36 weeks. If the rate of spending is kept the same how much more money needs to be saved during the summer in order for there to be money to last all 36 weeks.

Name: Modeling Data | 8.7

Go

Topic: Data and statistics, when to use two way tables when to use scatter plots.

9. In what situations does it make the most sense to use a two-way table and look at residual frequencies to make decisions or conclusions?

10. In what situations does it make the most sense to use a scatter plot and a linear or exponential model to analyze and make decisions or draw conclusions?

For each of the representations below label as a *function*, *not a function.* If, not a function say why. If it is a function then label as *linear, exponential or neither*.

11.

x	f(x)
0	5
1	169
2	333
3	497

12.

X	Y
1	15
2	25
3	15
2	30

13.

x	h(x)
2	5
3	10
4	20
5	40

14. $g(x) = 4 - 12x$

15. $s(t) = 3 \cdot 4^{t-1}$

16. The amount of medicine in the blood stream of a cat as time passes. The initial dose of medicine is 80mm and the medicine brakes down at 35% each hour.

17.

Time	0	1	2	3	4
Money in Bank	250	337.50	455.63	615.09	830.38

8.8 Rockin' the Residuals

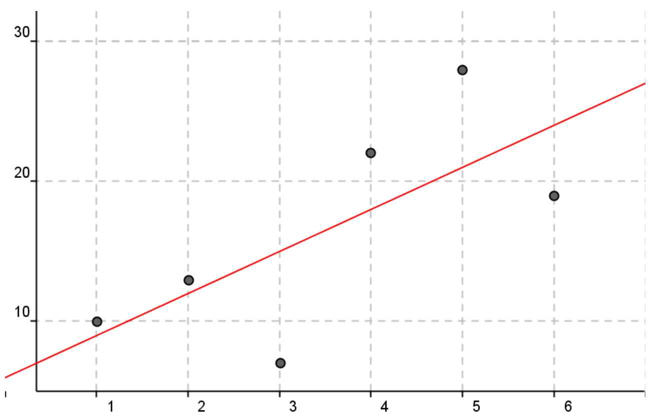
A Solidify Understanding Task

The correlation coefficient is not the only tool that statisticians use to analyze whether or not a line is a good model for the data. They also consider the residuals, which is to look at the difference between the observed value (the data) and the predicted value (the y-value on the regression line). This sounds a little complicated, but it's not really. The residuals are just a way of thinking about how far away the actual data is from the regression line.

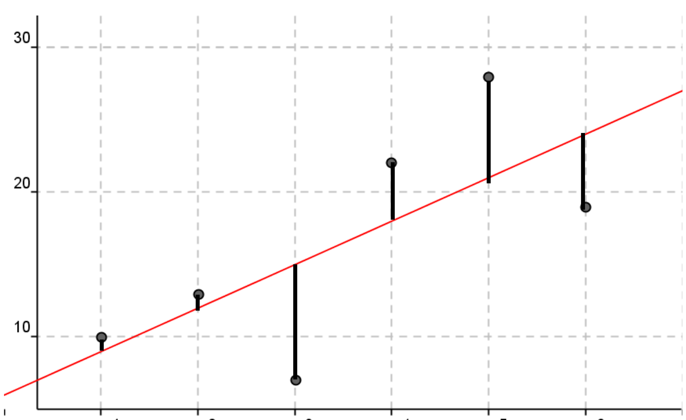
Start with some data:

x	1	2	3	4	5	6
y	10	13	7	22	28	19

Create a scatter plot and graph the regression line. In, this case the line is $y = 3x + 6$.



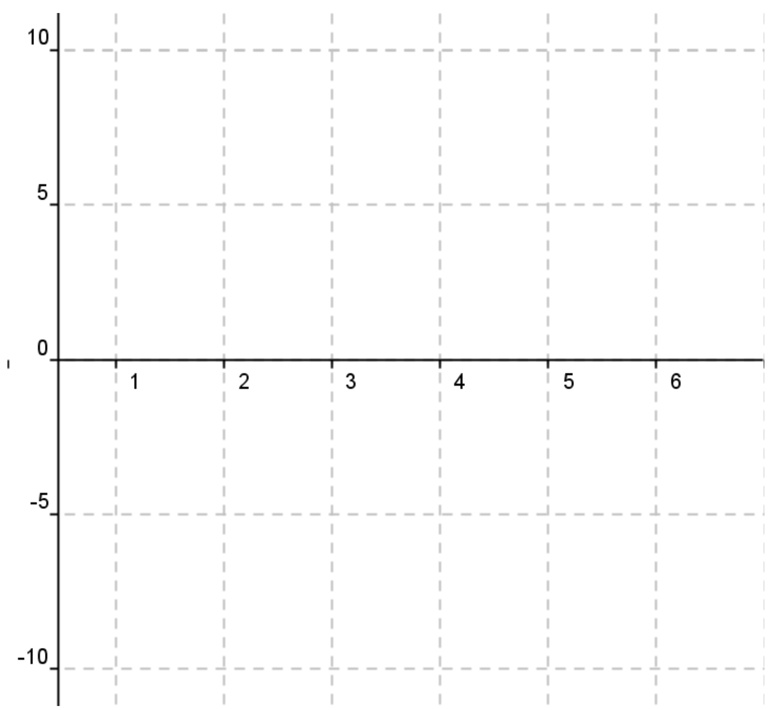
Draw a line from each data point to the regression line, like the segments drawn from each point below.



1. The residuals are the lengths of the segments. How can you calculate the length of each segment to get the residuals?

2. Generally, if the data point is above the regression line the residual is positive, if the data point is below the line, the residual is negative. Knowing this, use your plan from #1 to create a table of residual values using each data point.

3. Statisticians like to look at graphs of the residuals to judge their regression lines. So, you get your chance to do it. Graph the residuals here.



Now, that you have constructed a residual plot, think about what the residuals mean and answer the following questions.

4. If a residual is large and negative, what does it mean?

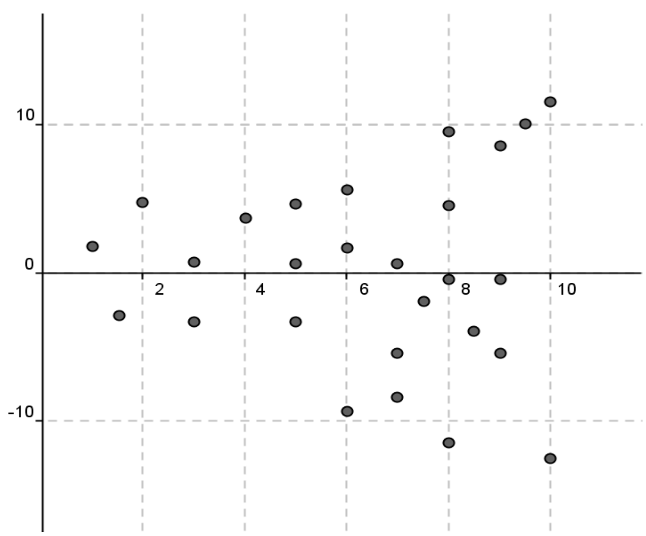
5. What does it mean if a residual is equal to 0?

6. If someone told you that they estimated a line of best fit for a set of data points and all of the residuals were positive, what would you say?

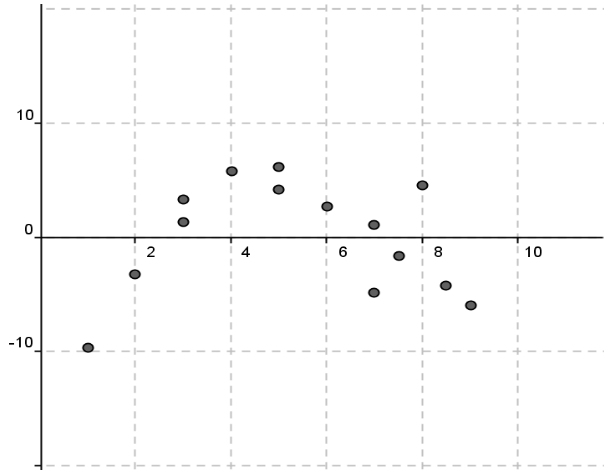
7. If the correlation coefficient for a data set is equal to 1, what will the residual plot look like?

Statisticians use residual plots to see if there are patterns in the data that are not predicted by their model. What patterns can you identify in the following residual plots that might indicate that the regression line is not a good model for the data? Based on the residual plot are there any points that may be considered outliers?

8.



9.



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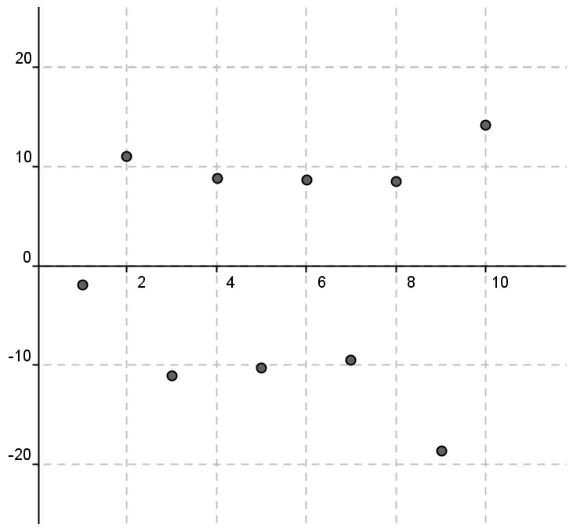
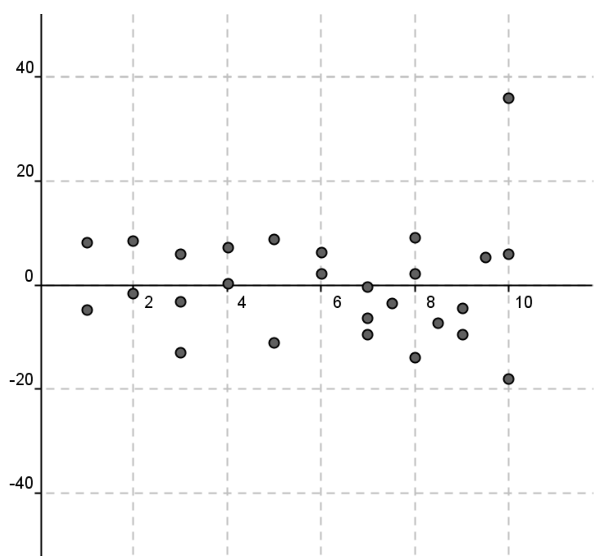


11.



Name: Modeling Data | 8.8

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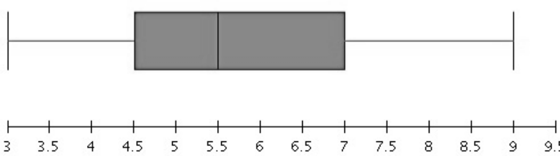
Ready, Set, Go!

Ready

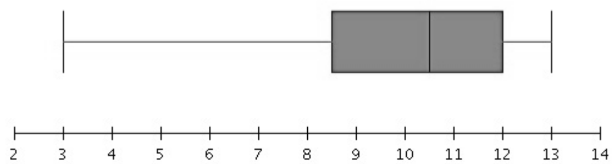
Topic: Describe the spread of the data.

Given the box-and-whisker plots describe the spread of the data set. Provide specifics about the median, range, interquartile range and so forth.

1.

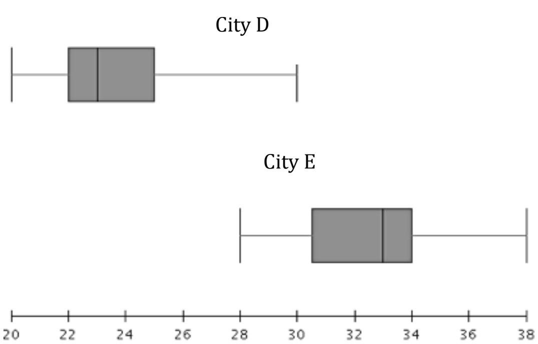


2.



3. If the box-and-whisker plots above represent the results of two different classes on the same assessment, which class did better? Why?

4. The two box-and-whisker plots below show the low temperatures for two cities in the United States.



a. Which city would be considered the coldest City D or City E? Why?

b. Do these cities ever experience the same temperature? How do you know?

c. Is there any way to know the exact temperature for any given day from the box and whisker plots?

d. What advantage if any could a scatter plot of temperature data have over a box and whisker plot?

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Set

Topic: Residuals, residual plots and correlation coefficients.

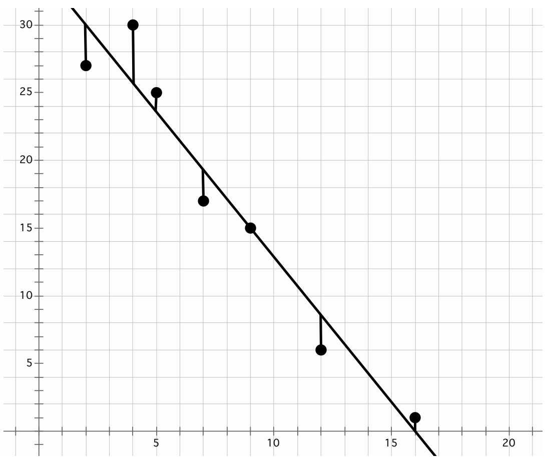
The Data Sheets below are scatter plots that have the regression line and the residuals indicated.

5a. Mark on the graph where $(\overline{x}, \overline{y})$ would be located.

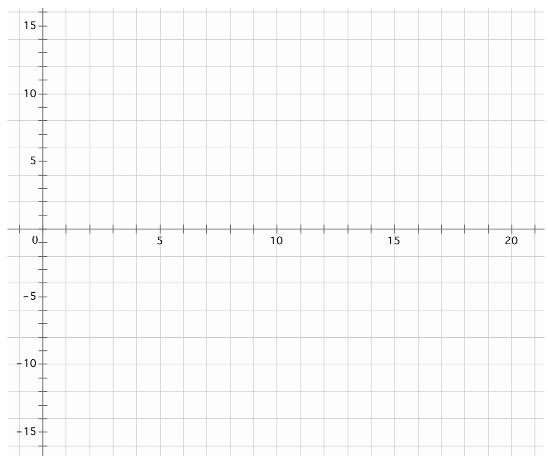
b. Use this given plot to create a residual plot.

c. What would you predict the correlation coefficient to be?

Data Sheet 1



Residual Plot 1

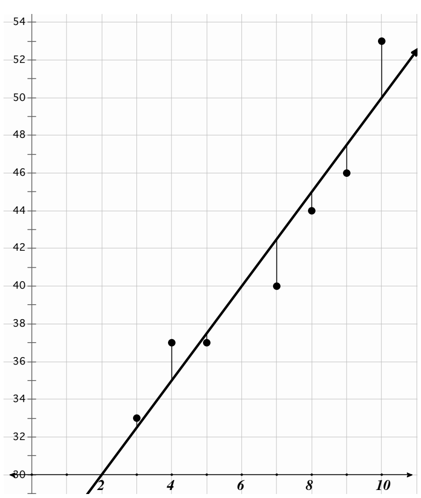


6a. Mark on the graph where $(\overline{x}, \overline{y})$ would be located.

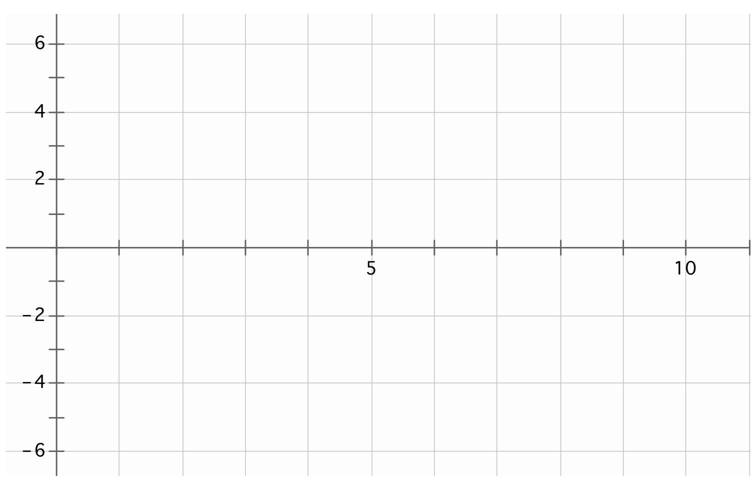
b. Use this given plot to create a residual plot.

c. What would you predict the correlation coefficient to be?

Data Sheet 2



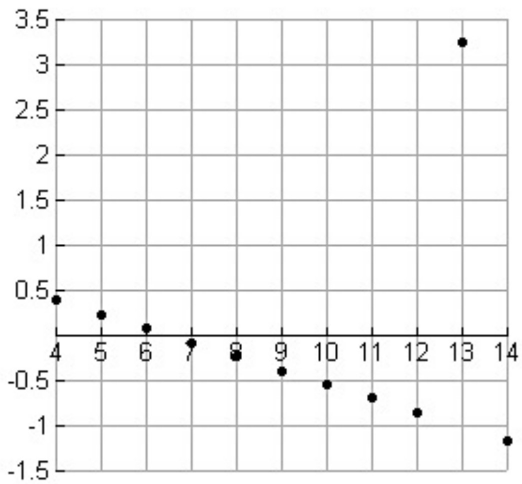
Residual Plot 2



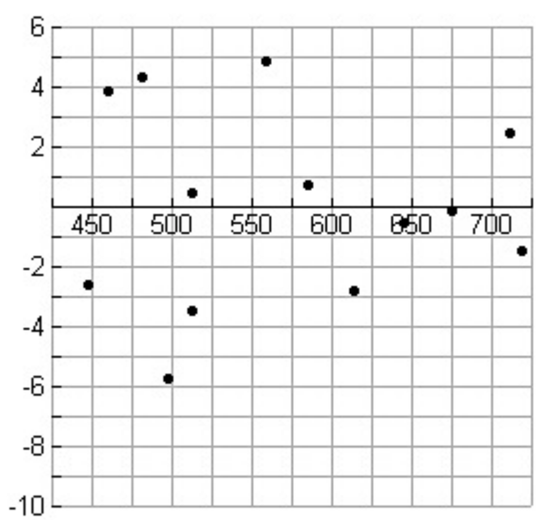
Name: Modeling Data 8.8

The following graphs are residual plots. Analyze the residual plots to determin how well the prediction line (line of best fit) describes the data.

7. Plot 1 Analysis



8. Plot 2 Analysis



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Go

Topic: Geometric constructions.

9. Construct an isosceles triangle with a compass and straight edge.

10. Construct a square using compass and straight edge..

11. Use a compass and straight edge to construct a hexagon inscribed in a circle.